Name _____ Class _____ Date _____

Practice 11-2

Arithmetic Sequences

Find the 43rd term of each sequence.

1. 12, 14, 16, 18, . . .
2. 13.1, 3.1, −6.9, −16.9, . . .
3. 19.5, 19.9, 20.3, 20.7, . . .
4. 27, 24, 21, 18, . . .
5. 2, 13, 24, 35, . . .
6. 21, 15, 9, 3, . . .
7. 1.3, 1.4, 1.5, 1.6, . . .
8. −2.1, −2.3, −2.5, −2.7, . . .
9. 45, 48, 51, 54, . . .

Is the given sequence arithmetic? If so, identify the common difference.

10. 2, 3, 5, 8, . . .
11. 0, −3, −6, −9, . . .
12. 0.9, 0.5, 0.1, −0.3, . . .
13. 3, 8, 13, 18, . . .
14. 14, −15, −44, −73, . . .
15. 3.2, 3.5, 3.8, 4.1, . . .
16. −34, −28, −22, −16, . . .
17. 2.3, 2.5, 2.7, 2.9, . . .
18. 127, 140, 153, 166, . . .

Find the missing term of each arithmetic sequence.

19. . . . 23, ▭, 49, . . .
20. 14, ▭, 28, . . .
21. . . . 29, ▭, 33, . . .
22. . . . 14, ▭, 15, . . .
23. . . . −45, ▭, −39, . . .
24. . . . −5, ▭, −2, . . .
25. −2, ▭, 2, . . .
26. . . . −6, ▭, 2, . . .
27. −34, ▭, 77, . . .
28. . . . −45, ▭, −12, . . .
29. −2, ▭, 456, . . .
30. . . . 34, ▭, 345, . . .

Find the arithmetic mean a_n of the given terms.

31. $a_{n-1} = 2, a_{n+1} = 7$
32. $a_{n-1} = 13.2, a_{n+1} = 15.8$
33. $a_{n-1} = 29, a_{n+1} = -11$
34. $a_{n-1} = \frac{2}{5}, a_{n+1} = \frac{4}{5}$
35. $a_{n-1} = 15, a_{n+1} = -17$
36. $a_{n-1} = -6, a_{n+1} = -7$

37. Each year, a volunteer organization expects to add 5 more people to the number of shut-ins for whom the group provides home maintenance services. This year, the organization provides the service for 32 people.
    a. Write a recursive formula for the number of people the organization expects to serve each year.
    b. Write the first five terms of the sequence.
    c. Write an explicit formula for the number of people the organization expects to serve each year.
    d. How many people would the organization expect to serve in the 20th year?

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Practice 11-3

Geometric Sequences

Find the missing term of each geometric sequence.

1. 4, ▭, 16, . . .
2. 9, ▭, 16, . . .
3. 2, ▭, 8, . . .
4. 3, ▭, 12, . . .
5. 2, ▭, 50, . . .
6. 4, ▭, 5.76, . . .

Is the given sequence geometric? If so, identify the common ratio and find the next two terms.

7. 3, 9, 27, 81, . . .
8. 4, 8, 16, 32, . . .
9. 4, 8, 12, 16, . . .
10. 4, −8, 16, −32, . . .
11. 1, 0.5, 0.25, 0.125, . . .
12. 100, 30, 9, 2.7, . . .
13. −5, 0, 5, 10, . . .
14. 64, −32, 16, −8, . . .
15. 1, 4, 9, 16, . . .

Identify each sequence as *arithmetic*, *geometric*, or *neither*. Then find the next two terms.

16. $9, 3, 1, \frac{1}{3}, \ldots$
17. 1, 0, −2, −5, . . .
18. 2, −2, 2, −2, . . .
19. −3, 2, 7, 12, . . .
20. 1, −2, −5, −8, . . .
21. 1, −2, 3, −4, . . .

Write the explicit formula for each sequence. Then generate the first five terms.

22. $a_1 = 3, r = -2$
23. $a_1 = 5, r = 3$
24. $a_1 = -1, r = 4$
25. $a_1 = -2, r = -3$
26. $a_1 = 32, r = -0.5$
27. $a_1 = 2187, r = \frac{1}{3}$
28. $a_1 = 9, r = 2$
29. $a_1 = -4, r = 4$
30. $a_1 = 0.1, r = -2$

31. When a pendulum swings freely, the length of its arc decreases geometrically. Find each missing arc length.
    a. 20th arc is 20 in.; 22nd arc is 18.5 in.
    b. 8th arc is 27 mm; 10th arc is 3 mm

32. The deer population in an area is increasing. This year, the population was 1.025 times last year's population of 2537.
    a. Assuming that the population increases at the same rate for the next few years, write an explicit formula for the sequence.
    b. Find the expected deer population for the fourth year of the sequence.

33. You enlarge a picture to 150% of its size several times. After the first increase, the picture is 1 in. wide.
    a. Write an explicit formula to model the size after each increase.
    b. How wide is the photo after the 2nd increase?
    c. How wide is the photo after the 3rd increase?
    d. How wide is the photo after the 12th increase?

Practice 11-4

Arithmetic Series

For each sum, find the number of terms, the first term, and the last term. Then evaluate the series.

1. $\sum_{n=1}^{4}(n-1)$
2. $\sum_{n=2}^{6}(2n-1)$
3. $\sum_{n=3}^{8}(n+25)$
4. $\sum_{n=2}^{5}(5n+3)$
5. $\sum_{n=1}^{4}(2n+0.5)$
6. $\sum_{n=1}^{6}(3-n)$
7. $\sum_{n=5}^{10}n$
8. $\sum_{n=1}^{4}(-n-3)$
9. $\sum_{n=3}^{6}(3n+2)$

Write the related series for each finite sequence. Then evaluate each series.

10. $1, 3, 5, \ldots, 15$
11. $5, 8, 11, \ldots, 26$
12. $4, 9, 14, 19, \ldots, 44$
13. $10, 25, 40, 55, 70, 85$
14. $17, 25, 33, 41, 49, 57, 65$
15. $125, 126, 127, \ldots, 131$

Use summation notation to write each arithmetic series for the specified number of terms.

16. $1 + 3 + 5 + \ldots; n = 7$
17. $2.3 + 2.6 + 2.9 + \ldots; n = 5$
18. $4 + 8 + 12 + \ldots; n = 4$
19. $10 + 7 + 4 + \ldots; n = 6$
20. $3 + 7 + 11 + \ldots; n = 8$
21. $15 + 25 + 35 + \ldots; n = 7$

Tell whether each list is a *series* or a *sequence*. Then tell whether it is *finite* or *infinite*.

22. $7, 12, 17, 22, 27$
23. $3 + 5 + 7 + 9 + \ldots$
24. $8, 8.2, 8.4, 8.6, 8.8, 9.0, \ldots$
25. $1 + 5 + 9 + 13 + 17$
26. $40, 20, 10, 5, 2.5, 1.25, \ldots$
27. $10 + 20 + 30 + 40 + 50$

Each sequence has six terms. Evaluate each related series.

28. $1, 0, -1, \ldots, -4$
29. $4, 5, 6, \ldots, 9$
30. $-7, -9, -11, \ldots, -17$
31. $-6, -7, -8, \ldots, -11$
32. $0, 0.3, 0.6, \ldots, 1.5$
33. $5, 7, 9, \ldots, 15$

34. An embroidery pattern calls for 5 stitches in the first row and for three more stitches in each successive row. The 25th row, which is the last row, has 77 stitches. Find the total number of stitches in the pattern.

35. A marching band formation consists of 6 rows. The first row has 9 musicians, the second has 11, the third has 13 and so on. How many musicians are in the last row and how many musicians are there in all?

Practice 11-5

Geometric Series

Decide whether each infinite geometric series *diverges* or *converges*. State whether each series has a sum.

1. $3 + \frac{3}{2} + \frac{3}{4} + \ldots$
2. $4 + 2 + 1 + \ldots$
3. $17 + 15.3 + 13.77 + \ldots$
4. $6 + 11.4 + 21.66 + \ldots$
5. $-20 - 8 - 3.2 - \ldots$
6. $50 + 70 + 98 + \ldots$

Evaluate each infinite series that has a sum.

7. $\sum_{n=1}^{\infty}5\left(\frac{2}{3}\right)^{n-1}$
8. $\sum_{n=1}^{\infty}(-2.1)^{n-1}$
9. $\sum_{n=1}^{\infty}\left(-\frac{1}{2}\right)^{n-1}$
10. $\sum_{n=1}^{\infty}2\left(\frac{5}{3}\right)^{n-1}$

Evaluate each infinite geometric series.

11. $8 + 4 + 2 + 1 + \ldots$
12. $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots$
13. $120 + 96 + 76.8 + 61.44 + \ldots$
14. $1000 + 750 + 562.5 + 421.875 + \ldots$

Determine whether each series is *arithmetic* or *geometric*. Then evaluate the series to the given term.

15. $2 + 5 + 8 + 11 + \ldots; S_9$
16. $\frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \ldots; S_8$
17. $-3 + 6 - 12 + 24 - \ldots; S_{10}$
18. $-2 + 2 + 6 + 10 + \ldots; S_{12}$

Evaluate the finite series for the specified number of terms.

19. $40 + 20 + 10 + \ldots; n = 10$
20. $4 + 12 + 36 + \ldots; n = 15$
21. $15 + 12 + 9.6 + \ldots; n = 40$
22. $27 + 9 + 3 + \ldots; n = 100$
23. $0.2 + 0.02 + 0.002 + \ldots; n = 8$
24. $100 + 200 + 400 + \ldots; n = 6$

25. This month, Julia deposits \$400 to save for a vacation. She plans to deposit 10% more each successive month for the next 11 months. How much will she have saved after the 12 deposits?

26. Suppose your business made a profit of \$5500 the first year. If the profit increases 20% per year, find the total profit over the first 5 yr.

27. The end of a pendulum travels 50 cm on its first swing. Each swing after the first travels 99% as far as the preceding one. How far will the pendulum travel before it stops?

28. A seashell has chambers that are each 0.82 times the length of the next chamber. The outer chamber is 32 mm around. Find the total length of the shell's spiraled chambers.

29. The first year a toy manufacturer introduces a new toy, its sales total \$495,000. The company expects its sales to drop 10% each succeeding year. Find the total expected sales in the first 6 yr. Find the total expected sales if the company offers the toy for sale for as long as anyone buys it.

Selection Set

Practice 11.2

4. 27, 24, 21, 18...

$d = -3$

$a_{43} = 27 + (43-1)(-3)$

$= 27 + 42(-3)$

$= 27 - 126$

$a_{43} = -99$

8. $-2.1, -2.3, -2.5, -2.7 \ldots$

$d = -0.2$

$a_{43} = -2.1 + (43-1)(-0.2)$

$= -2.1 + 42(-0.2)$

$= -2.1 - 8.4$

$a_{43} = -10.5$

12. Yes arithmetic

Common difference -0.4

16. $-34, -28, -22, -16 \ldots$

Yes arithmetic

Common difference 6

20. $14, \_\_\_, 28$

$\frac{14+28}{2} = \frac{42}{2} = 21$

24. $\ldots, -5, \_\_\_, -2 \ldots$

$\frac{-5+(-2)}{2} = \frac{-7}{2} = -3.5$

28. $\ldots, -45, \_\_\_, -12 \ldots$

$\frac{-45+(-12)}{2} = \frac{-57}{2} = -28.5$

32. $a_{n-1} = 13.2 \quad a_{n+1} = 15.8$

$a = \frac{13.2+15.8}{2}$

$= \frac{29.0}{2} = 14.5$

36. $a_{n-1} = -6 \quad a_{n+1} = -7$

$a_n = \frac{-6+-7}{2} = \frac{-13}{2} = -6.5$

3a. $P = 2537(1.025)^{2-1}$

b. $P = 2537(1.025)^{4-1}$

$= 2537(1.025)^3 \approx 2732$

3 1a. $\sqrt{20 \text{ in} \cdot 18 \text{ in}}$

$= \sqrt{360 \text{ in}^2}$

$= \sqrt{370} \approx 19.2 \text{ in}$

b. $\sqrt{27 \text{ mm} \cdot 3 \text{ mm}}$

$= \sqrt{81} = 9 \text{ mm}$

Practice 11.3

3. $2, \_\_\_, 8$

geometric mean $= \sqrt{2 \cdot 8}$

$= \sqrt{16}$

geom. mean $= 4$

6. $4, \_\_\_, 5.76$

$= \sqrt{4 \cdot 5.76}$

$= \sqrt{23.04}$

$= 4.8$

9. 4, 8, 12, 16

not geometric

12. 100, 30, 9, 2.7

geometric $r = 0.3$

100, 30, 9, 2.7, 0.81, 0.243

15. 1, 4, 9, 16

not geometric

18. 2, -2, 2, -2...

geometric $r = -1$

2, -2, 2, -2, 2, -2...

21. 1, -2, 3, -4...

Neither

1, -2, 3, -4, 5, -6

24. $a_1 = -1 \quad r = 4$

$a_n = -1(4)^{n-1}$

$a_1 = -1$
$a_2 = -4$
$a_3 = -16$
$a_4 = -64$
$a_5 = -256$

work

$a_2 = -1(4)^{2-1}$

$= -1(4)$

$= -4$

$a_3 = -1(4)^{3-1}$

$= -1(4)^2$

$a_3 = -16$

$a_4 = -1(4)^{4-1}$

$= -1(4)^3$

$a_4 = -64$

$a_5 = -1(4)^{5-1}$

$= -1(4)^4$

$a_5 = -256$

28. $a_1 = 9, \quad r = 2$

$a_n = 9(2)^{n-1}$

$a_1 = 9 \quad a_2 = 9 \cdot 2^1 = 18$

$a_3 = 9(2)^2 = 9(4) = 36$

$a^4 = 9(2)^3 = 9 \cdot 8 = 72$

$2^5 = 9(2)^4 = 9 \cdot 16 = 144$

9, 18, 36, 72, 144

Practice 11.4

3. $\sum_{n=3}^{8}(n+25)$

terms $= 8-3+1 = 6$

$a_1 = (3+25) = 28$

$a_6 = (8+25) = 33$

$S_6 = \frac{6}{2}(28+33)$

$= 3(61)$

$S_6 = 183$ (Sum of first 6 terms)

6. $\sum_{n=1}^{6}(3-n)$

terms $= 6$

$a_1 = 3-1 = 2$

$a_6 = 3-6 = -3$

$S_6 = \frac{6}{2}(2+(-3))$

$= 3(-1)$

$S_6 = -3$

21. $15+25+35\ldots$

$d = 10$

$a_n = 15+(n-1)10$

$= 15+10n-10$

$a_n = 10n+5$

$\sum_{n=1}^{7}(10n+5)$

9. $\sum_{n=3}^{6}(3n+2)$

terms $= 6-3+1 = 4$

$a_1 = 3\cdot 3+2 = 11$

$a_4 = 3\cdot 6+2 = 20$

$S_4 = \frac{4}{2}(11+20)$

$= 2(31)$

$S_4 = 62$

12. $4+9+14+19+24+29+34+39+44$

$S_9 = \frac{9}{2}(4+44)$

$= \frac{9}{2}(48)$

$= 9(24)$

$S_9 = 216$

15. $125+126+127+128+129+130+131$

$S_7 = \frac{7}{2}(125+131)$

$= \frac{7}{2}(256)$

$S_7 = 896$

18. common difference $= 4$

$a_n = 4+(n-1)4$

$= 4+4n-4$

$a_n = 4n$

$\sum_{n=1}^{4}4n$

11.4 (Cont.)

24. sequence, infinite

27. series, finite

30. $S_6 = \frac{6}{2}(-7+(-17))$

$= 3(-24)$

$S_6 = -72$

33. $S_6 = \frac{6}{2}(5+15)$

$= 3(20)$

$S_6 = 60$

Practice 11.5

3. $\frac{153}{17} = 0.9$

$0.9 < 1$, therefore converges, has a sum.

6. $\frac{70}{50} = 1.4$

$1.4 > 1$ therefore diverges, no sum

11.5 (cont.)

7. $\sum_{n=1}^{\infty}5\left(\frac{2}{3}\right)^{n-1}$

$S = \frac{a_1}{1-r} = \frac{5}{1-\frac{2}{3}} = \frac{5}{\frac{1}{3}} = 15$

$S = 15$

9. $\sum_{n=1}^{\infty}\left(-\frac{1}{2}\right)^{n-1}$

$r = -\frac{1}{2}$, $a_1 = 1$

$S = \frac{1}{1-(-\frac{1}{2})} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$

$S = \frac{2}{3}$

12. $r = \frac{1}{3}$ $a_1 = 1$

$S = \sum_{n=1}^{\infty}1\left(\frac{1}{3}\right)^{n-1}$

$S = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$

$S = 1.5$

15. arithmetic, $d = 3$

$S_9 = \frac{9}{2}(2+\_\_\_)$

$a_9 = 2+(9-1)3$

$a_9 = 2+24 = 26$

$S_9 = \frac{9}{2}(2+26)$

$= \frac{9}{2}(28)$

$S_9 = 126$

11.5 (cont)

18. arithmetic $d=4$

$a_1 = -2$

$a_{12} = -2 + (12-1)(4)$

$= -2 + 11(4)$

$a_{12} = 42$

$S_{12} = \frac{12}{2}(-2+42)$

$= 6(40)$

$S_{12} = 240$

21. $r = 0.8$, $a_1 = 15$

$S_{40} = \frac{15(1-0.8^{40})}{1-0.8}$

$= \frac{15(1-0.8^{40})}{.2}$

$S_{40} \approx 74.99$

24. $r = 2$

$a_1 = 100$

$S_6 = \frac{100(1-2^6)}{1-2}$

$= \frac{100(-63)}{-1}$

$S_6 = 6300$

27. $S = \frac{50}{1-0.99}$

$= \frac{50}{0.01}$

$S = 5000$ cm