PRE-CALCULUS CHAPTER 5 Review

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DATE: ______________

1. Solve: $\cos 3x + \cos x = 0$ on $[0, 2\pi)$

Use the Sum to Product formula

$\cos 3x + \cos x = 0$

$2\cos\left[\frac{3x+x}{2}\right]\cos\left[\frac{3x-x}{2}\right] = 0$

$2\cos 2x \cos x = 0$

$\cos 2x = 0$

$2x = \frac{\pi}{2}$ $\quad 2x = \frac{3\pi}{2}$

$x = \frac{\pi}{4} + \pi n$ $\quad x = \frac{3\pi}{4} + \pi n$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

2. Verify the identities. You may only work one side.

$$\frac{\sin 7x - \sin 5x}{\cos 7x - \cos 5x} = -\cot 6x$$

$$\frac{2\cos\left[\frac{7x+5x}{2}\right]\sin\left[\frac{7x-5x}{2}\right]}{-2\sin\left[\frac{7x+5x}{2}\right]\sin\left[\frac{7x-5x}{2}\right]}$$

$$\frac{\cos 6x}{-\sin 6x}$$

$-\cot 6x$

3. $\dfrac{\cos^3 u+\sin^3 u}{\cos u+\sin u}=1-\sin u\cos u$

$\cancel{(\cos u+\sin u)(\cos^2 u \ldots)}$

$$\frac{(\cos u+\sin u)(\cos^2 u-\cos u\sin u+\sin^2 u)}{\cos u+\sin u}$$

$\cos^2 u+\sin^2 u-\sin u\cos u$

$1-\sin u\cos u$

4. $\sin\theta+\cos\theta=\dfrac{\tan\theta+1}{\sec\theta}$

$$\frac{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$$

$\sin\theta+\cos\theta$

5. $\dfrac{\sec^2 x}{\cot x}-\tan^3 x=\tan x$

$$\frac{1+\tan^2 x}{\cot x}-\tan^3 x$$

$$\frac{1}{\cot x}+\frac{\tan^2 x}{\cot x}-\tan^3 x$$

$\tan x+\tan^3 x-\tan^3 x$

$\tan x$

6. $\dfrac{\sec x-\cos x}{\tan x}=\sin x$

$$\frac{\sec x}{\tan x}-\frac{\cos x}{\tan x}$$

$$\frac{\frac{1}{\cos x}}{\frac{\sin x}{\cos x}}-\frac{\frac{\cos x}{1}}{\frac{\sin x}{\cos x}}$$

$$\frac{1}{\sin x}-\frac{\cos^2 x}{\sin x}$$

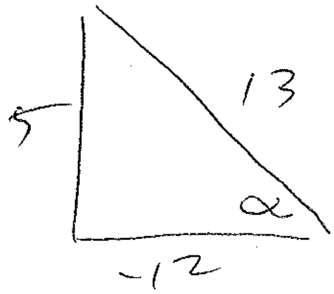
$$\frac{1-\cos^2 x}{\sin x}$$

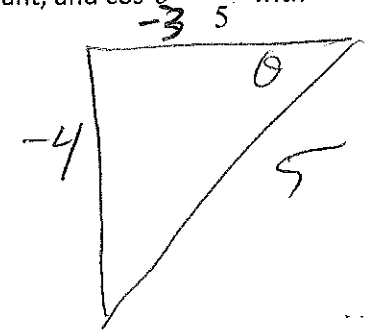
$$\frac{\sin^2 x}{\sin x}$$

$\sin x$

7. Suppose θ lies in the third quadrant, α lies in the second quadrant, and $\cos\theta = \frac{-3}{5}$ with

$\tan\alpha = \frac{-5}{12}$





Find the following:

a. $\sin(\alpha+\theta)$ b) $\cos\frac{\alpha}{2}$ c) $\tan 2\theta$

(a) $\sin(\alpha+\theta) = \sin\alpha\cos\theta + \cos\alpha\sin\theta$

$$= \left(\frac{5}{13}\right)\left(\frac{-3}{5}\right) + \left(\frac{-12}{13}\right)\left(\frac{-4}{5}\right)$$

$$= \frac{-15}{65} + \frac{48}{65} = \frac{33}{65}$$

(b) $\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}} = \sqrt{\frac{1+\left(\frac{-12}{13}\right)}{2}} = \sqrt{\frac{\frac{13}{13}-\frac{12}{13}}{2}} = \sqrt{\frac{\frac{1}{13}}{2}}$

$$= \sqrt{\frac{1}{26}} = \frac{1}{\sqrt{26}} = \frac{\sqrt{26}}{26}$$

(c) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{4}{3}\right)}{1-\left(\frac{4}{3}\right)^2} = \frac{\frac{8}{3}}{1-\frac{16}{9}} = \frac{\frac{8}{3}}{-\frac{7}{9}} = \boxed{-\frac{24}{7}}$

8. Write: $\cos\left(\phi+\frac{\pi}{2}\right)-\cos\left(\phi-\frac{\pi}{2}\right)$ as a product.

$u=\phi+\frac{\pi}{2}$

$v=\phi-\frac{\pi}{2}$

$\cos u - \cos v$

$-2\sin\left[\frac{u+v}{2}\right]\sin\left[\frac{u-v}{2}\right]$

$-2\sin\left[\frac{\phi+\frac{\pi}{2}+\phi-\frac{\pi}{2}}{2}\right]\sin\left[\frac{\phi+\frac{\pi}{2}-\phi+\frac{\pi}{2}}{2}\right]$

$-2\sin\left[\frac{2\phi}{2}\right]\sin\left[\frac{\pi}{2}\right]$

$-2\sin\phi$

9. Write: $\cos\frac{3\pi}{4}-\cos\frac{\pi}{4}$ as a product and then solve

$\cos\frac{3\pi}{4}-\cos\frac{\pi}{4}=-2\sin\left[\frac{\frac{3\pi}{4}+\frac{\pi}{4}}{2}\right]\sin\left[\frac{\frac{3\pi}{4}-\frac{\pi}{4}}{2}\right]$

$=-2\sin\left[\frac{\pi}{2}\right]\sin\left[\frac{\pi}{4}\right]$

$=-2(1)\frac{\sqrt{2}}{2}$

$=-\sqrt{2}$

10. Write $\cos(18°)\cos(64°)$ as a sum.

$\cos 18^\circ\cos(64^\circ)=\frac{1}{2}\left[\cos(18-64)+\cos(18+64)\right]$

$\frac{1}{2}\left[\cos(-46^\circ)+\cos(82^\circ)\right]$

OR

$\frac{1}{2}\left[\cos(46^\circ)+\cos(82^\circ)\right]$