Hour: _____

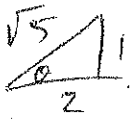
Honors Algebra 2B Final Exam Review Part 2

Name: KEY

Date: _____

1. Find the exact value of $\tan\left(\frac{-7\pi}{6}\right)$. $-\frac{\sqrt{3}}{3}$

2. Find the exact value of $\sin\left(\arctan\frac{1}{2}\right)$. $\frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$



3. Find all solutions to $2\sin^2 x - 3\sin x = -1$ on the interval $[0, 2\pi)$.

$(2\sin x - 1)(\sin x - 1) = 0$

$2\sin x = 1$ $\quad \sin x = 1$

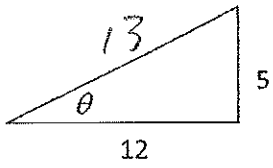
$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

4. Given $\cos\theta = \frac{\sqrt{7}}{4}$, where $0 < \theta < \pi$, determine the exact value for $\cos 2\theta$.

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{\sqrt{7}}{4}\right)^2 - \left(\frac{3}{4}\right)^2 = \frac{7}{16} - \frac{9}{16} = \frac{-2}{16} = \frac{-1}{8}$

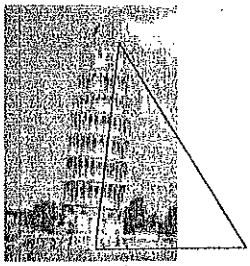
5. Use the figure to find the exact value of $\sin\frac{\theta}{2}$.

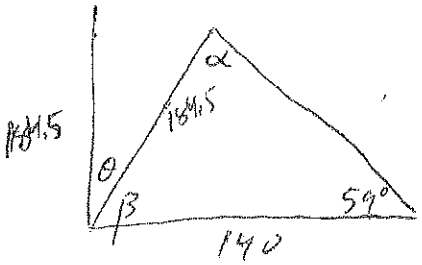


$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-\frac{12}{13}}{2}} = \sqrt{\frac{1}{26}} = \frac{\sqrt{26}}{26}$

6. What is the domain of the function $f(x) = \sin\frac{1}{x}$? $(-\infty, 0) \cup (0, \infty)$

7.





The original height of the Leaning Tower of Pisa was 184.5 feet. At a distance of 140 feet from the base of the tower, the angle of elevation is 59°. How far is the tower leaning from the original vertical position (The arc distance)?

$\frac{\sin 59^\circ}{184.5} = \frac{\sin\alpha}{140}$

$\sin\alpha = 0.65043$

$\alpha = 40.574^\circ$

$\beta = 180 - 40.574 - 59$

$\beta = 80.426^\circ$

$\theta = 90 - 80.426 = 9.5737^\circ$

$\theta = 9.5737^\circ \times \frac{\pi}{180^\circ} = 0.1671$

$s = r\theta$

$= 184.5(0.1671)$

$= 30.83$ FEET

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8. Use the half angle identity to find the exact value of :

a. $\sin\left(\frac{3\pi}{8}\right)$

$\sin\frac{\frac{3\pi}{4}}{2} = \sqrt{\frac{1-\cos\frac{3\pi}{4}}{2}}$

$= \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}$

b) $\cos\left(\frac{11\pi}{12}\right)$

$\cos\frac{\frac{11\pi}{6}}{2} = \sqrt{\frac{1+\cos\frac{11\pi}{6}}{2}}$

$= \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$

9. Use the Binomial Theorem to expand and simplify $(x^2 - 3i)^4$.

1 4 6 4 1

$(x^2)^4 - 4(x^2)^3(3i) + 6(x^2)^2(3i)^2 - 4(x^2)(3i)^3 + (3i)^4$

$x^8 - 12x^6i - 54x^4 + 108x^2i + 81$

10. Evaluate the sum

$\sum_{k=1}^{9} \log\left(\frac{k+1}{k}\right)$ ... $\log\left(\frac{2}{1}\right) + \log\left(\frac{3}{2}\right) + \log\left(\frac{4}{3}\right) + \log\left(\frac{5}{4}\right) + \log\left(\frac{6}{5}\right) + \log\left(\frac{7}{6}\right) + \log\left(\frac{8}{7}\right) + \log\left(\frac{9}{8}\right) + \log\left(\frac{10}{9}\right)$

$\log\left(\frac{2}{1}\cdot\frac{3}{2}\cdot\frac{4}{3}\cdot\frac{5}{4}\cdot\frac{6}{5}\cdot\frac{7}{6}\cdot\frac{8}{7}\cdot\frac{9}{8}\cdot\frac{10}{9}\right) = \log 10 = 1$

11. Find the sum of the first 30 positive multiples of 4

$\sum_{n=1}^{30} 4n$

$a_1 = 4$

$a_{30} = 120$

$n = 30$

$S_{30} = \frac{30}{2}(4+120)$

1860

12. Evaluate ${}_nC_r$

a) ${}_{22}C_5$

$\frac{22!}{(22-5)!\,5!} = \frac{22!}{17!\,5!}$

26334

b) ${}_{30}C_{18}$

$\frac{30!}{(30-18)!\,18!} = \frac{30!}{12!\,18!}$

86493225

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13. Given: $x^5 - 32 = 0$, solve for the five roots

$\theta = 0$

$r = 32$

$\sqrt[5]{32}\left(\cos\frac{0+2\pi k}{5} + i\sin\frac{0+2\pi k}{5}\right)$

$k = 0, 1, 2, 3, 4$

2

$0.618 + 1.902i$ $\quad 0.618 - 1.902i$

$-1.618 + 1.176i$ $\quad -1.618 - 1.176i$

14. Verify the identity

(a) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\cos(2\theta+\theta)$

$\cos 2\theta\cos\theta - \sin 2\theta\sin\theta$

$(2\cos^2\theta - 1)\cos\theta - 2\sin\theta\cos\theta\sin\theta$

$2\cos^3\theta - \cos\theta - 2\sin^2\theta\cos\theta$

$\cos^3\theta - \cos\theta - 2(1-\cos^2\theta)\cos\theta$

$2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$

$4\cos^3\theta - 3\cos\theta$

(b) $\dfrac{\sin 7x - \sin 5x}{\cos 7x - \cos 5x} = -\cot 6x$

$\dfrac{2\cos\left(\frac{7x+5x}{2}\right)\sin\left(\frac{7x-5x}{2}\right)}{-2\sin\left(\frac{7x+5x}{2}\right)\sin\left(\frac{7x-5x}{2}\right)}$

$\dfrac{-\cos 6x}{\sin 6x}$

$-\cot 6x$

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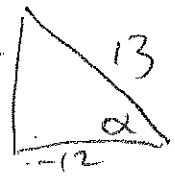
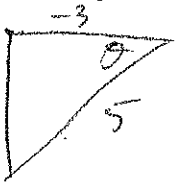
15. Suppose θ lies in the third quadrant, α lies in the second quadrant, and $\cos\theta = \frac{-3}{5}$ with $\tan\alpha = \frac{-5}{12}$





Find the following:

b. $\sin(\alpha+\theta)$

$\sin\alpha\cos\theta + \cos\alpha\sin\theta$

$\frac{5}{13}\left(\frac{-3}{5}\right) + \left(\frac{-12}{13}\right)\left(\frac{-4}{5}\right)$

$\frac{-15}{65} + \frac{48}{65} = \frac{33}{65}$

b) $\cos\frac{\alpha}{2}$

$\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}}$

$= \sqrt{\frac{1-\frac{12}{13}}{2}}$

$= \sqrt{\frac{1}{26}}$

$= \frac{\sqrt{26}}{26}$

c) $\tan 2\theta$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\left(\frac{4}{3}\right)}{1-\left(\frac{4}{3}\right)^2} = \frac{\frac{8}{3}}{1-\frac{16}{9}} = \frac{-24}{7}$

18. Use sigma notation to write the sum: $\frac{1}{2}+\frac{2}{4}+\frac{6}{8}+\frac{24}{16}+\frac{120}{32}+\frac{720}{64}$.

$\sum_{n=1}^{6} \frac{n!}{2^n}$

19. If possible, find the sum of the infinite series $\sum_{n=0}^{\infty} 3\left(\frac{1}{5}\right)^n$.

$S = \frac{3}{1-\frac{1}{5}} = \frac{15}{4}$

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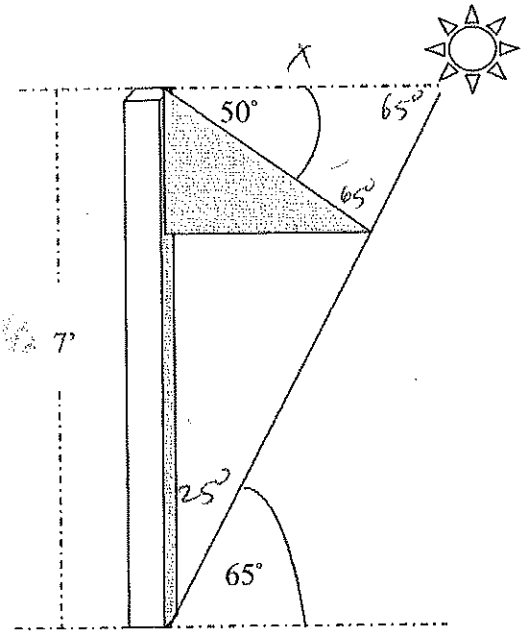
20. Mr. C has a sliding glass door leading to his backyard. Mr. C's wife asked him to construct an awning above the door to prevent direct sunlight from coming in the sliding glass door and waking their napping children. The door is 7' tall, the angle of elevation to the sun when the sunlight could potentially enter the house is 65°, the angle of depression for the proposed awning is 50°. How long must the awning be to prevent the sunlight from entering the door (The hypotenuse of the right triangle awning)?



$$\frac{7}{\sin 65^\circ} = \frac{x}{\sin 25^\circ}$$

$$3.26' = x$$

3.26'

21. Given: $y = 4\sin\left(\frac{x}{2} + \pi\right) + 3$

List **ALL** of the pertinent information for the graph, but DO NOT GRAPH IT.

AMPLITUDE: 4

PERIOD: 4π

LEFT: 2π

UP: 3

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22. Telephone numbers in the United States have ten digits—the area code followed by the local telephone number. How many numbers are possible within each area code? Note: a local telephone number cannot begin with a 0 or 1.

$8 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

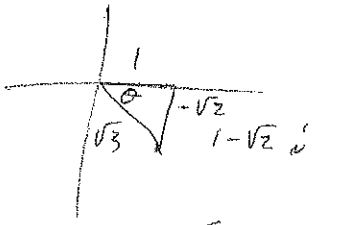
8×10^6

23. Find: $\left(1-\sqrt{2}i\right)^{22}$

$(\sqrt{3})^{22}\left(\cos(22(-0.955)) + i\sin(22(-0.955))\right)$

$(\sqrt{3})^{22}(-0.562 - 0.831i)$

$= 99556.6 - 147209.2i$



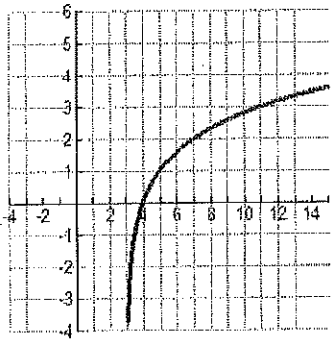
$r = \sqrt{3}$

$\tan\theta = -\sqrt{2}$

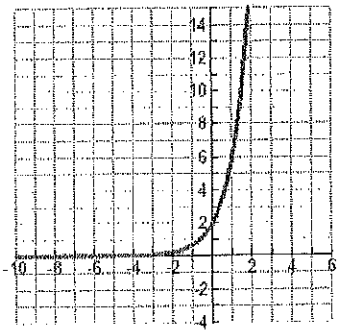
$\theta = -0.955$

24. Graph the following and label all of the important information:

a. $y = \log_2(x-3)$



b. $y = 2 \cdot 3^x$



c. $y = 4^{-x}$