Honors Algebra 2B Welcome Back! Name: Key

Hour: ___________

1. Solve.

a) $\frac{x}{x-2}+\frac{3}{x-5}=-1$

$\frac{x(x-5)}{(x-2)(x-5)}+\frac{3(x-2)}{(x-2)(x-5)}$

$\frac{x^2-5x+3x-6}{x^2-7x+10}=-1$

$x^2-2x-6=-x^2+7x-10$

$2x^2-9x+4=0$

$(2x-1)(x-4)=0$

$x=4,\frac{1}{2}$

b) $\sqrt{2x+3}+\sqrt{x-2}=2$

$(\sqrt{2x+3})^2=(2-\sqrt{x-2})^2$

$2x+3 = 4-4\sqrt{x-2}+x-2$

$2x+3=2+x-4\sqrt{x-2}$

$(x+1)^2=(-4\sqrt{x-2})^2$

$x^2+2x+1=16(x-2)$

$x^2+2x+1=16x-32$

$(x^2-14x+33)=0 \quad (x-11)(x-3)=0$

No solution

c) $4|3-2x|\le 16$

$3-2x\le 4$

$-2x\le 1$

$x\ge -\frac{1}{2}$

$3-2x\ge -4$

$-2x\ge -7$

$x\le \frac{7}{2}$

$[-\frac{1}{2},\frac{7}{2}]$

2. Suppose you work in a lab. You need a 15% hydrochloric acid (HCl) solution for a certain test, but your supplier only stocks solutions of concentration 10% and 30% HCl. Rather than pay the hefty surcharge to have the supplier make a custom 15% HCl solution, you decide to mix the stock solutions to make your own 15% HCl solution. You need 10 liters of the 15% HCl solution. How much of each stock solution should you use?

$x+y=10$

$.1x+.3y=.15(10)$

x = 10% solution

y = 30% solution

~~x = 12.5 L~~

$y=2.5$ L

$x=7.5$ L

True or False:

3. FALSE $x < a \to \sqrt{x} < \sqrt{a}$

4. FALSE $(1+i)^2 = 0$

5. FALSE $(a+b)^n = a^n + b^n$

6. FALSE $x < a \to -x < -a$

7. Given $f(x) = x - 2$ and $g(x) = 6 - 2x$, find $(f+g)(-2)$

$(f+g)(x) = -x + 4$

$-(-2) + 4 = 6$

8. Given $f(x) = 4 - 2x^2$ and $g(x) = 2 - x$. Find $(f \circ g)(x)$

$4 - 2(2-x)^2$

$4 - 2(4 - 4x + x^2)$

$4 - 8 + 8x - 2x^2$

$-2x^2 + 8x - 4$

9. Given $f(x) = 3x^3 - 1$, find $f^{-1}(x)$.

$x = 3y^3 - 1$

$\cancel{x}x + 1 = 3y^3$

$\frac{x+1}{3} = y^3$

$\sqrt[3]{\frac{x+1}{3}} = f(x)^{-1}$ OR

$f^{-1}(x) = \frac{\sqrt[3]{9x+9}}{3}$

10. The height (in feet) y of a ball thrown by a "wild and crazy" Honors Algebra 2 student is given by $y = -2x^2 + 16x + 90$, where x is the time in seconds.

$x = \frac{-16}{2(-4)} = 4$ $\quad$ $y = -2(4)^2 + 16(4) + 90$ $= 122$ FT

a) What is the maximum height of the ball?

b) How tall is the building? 90 FT

c) What is the domain of this function?

$0 = -2x^2 + 16x + 90$

$0 = x^2 - 8x - 45$

$x = \frac{8 \pm \sqrt{64 + 180}}{2}$

$= \frac{8 \pm \sqrt{244}}{2}$

$= \frac{8 \pm 2\sqrt{61}}{2} = 4 \pm \sqrt{61}$

D: $[0, 4 + \sqrt{61}]$

11. Let $g(x) = x^2 - 2x - 1$.

a) Find the domain and range of the function. D: $x = \mathbb{R}$ $\quad$ R: $y \geq -2$

$\frac{2}{2(1)} = 1$

$y = (1)^2 - 2(1) - 1$

$y = -2$

b) Find $g\left(-\frac{1}{2}\right)$. $\frac{1}{4}$

c) When does $g(x) = 16$. $16 = x^2 - 2x - 1$ $\quad$ $0 = x^2 - 2x - 17$ $\quad$ $x = 1 \pm 3\sqrt{2}$

d) Find $\frac{g(x+h) - g(x)}{h}$, $h \neq 0$. $\frac{(x+h)^2 - 2(x+h) - 1 - (x^2 - 2x - 1)}{h}$ $= 2x + h - 2$

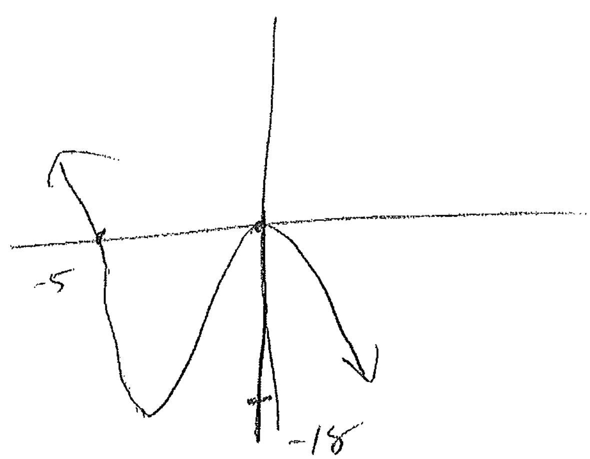
e) Algebraically find the interval on which $g(x) > 0$.

$0 < x^2 - 2x - 1$

$(-\infty, 1 - \sqrt{2}) \cup (1 + \sqrt{2}, \infty)$

12. Sketch the graph of the function $f(x) = -x^3 - 5x^2$ without using your calculator.

$-x^2(x + 5)$



13. Consider a line containing the points $A(1,2)$ and B, where B is a point on the curve $h(x)=4x^3-9x^2+12x-5$. Express the slope of $\overline{AB}$ as a function of x.

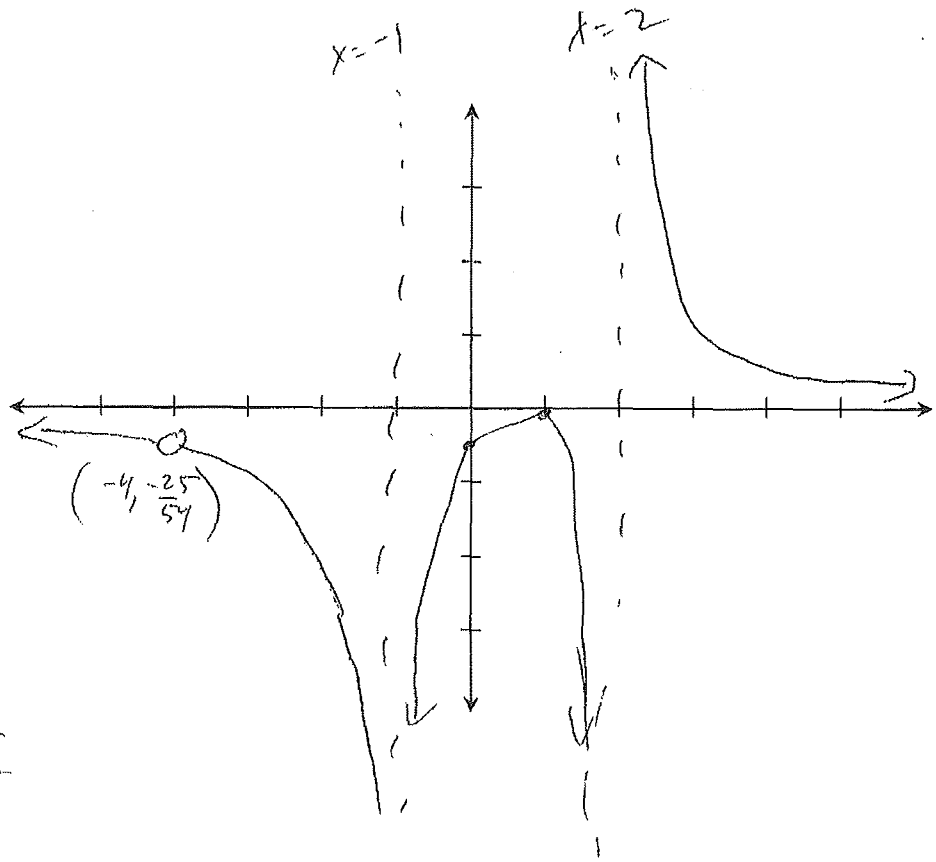
$$m = \frac{4x^3-9x^2+12x-5 \quad -2}{x-\text{[scribbled]}} = 4x^2-5x+7$$

14. Graph the function $h(x)=\dfrac{x^3+2x^2-7x+4}{x^4+4x^3-3x^2-14x-8}$ by hand. Label all the important features of the graph.

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -7 & 4 \\ & & 1 & 3 & -4 \\ \hline & 1 & 3 & -4 & \underline{|0} \end{array}$$

$$\frac{(x-1)(x-1)(x+4)}{(x+1)(x+1)(x+4)(x-2)}$$

$$\begin{array}{r|rrrrr} -4 & 1 & 4 & -3 & -14 & -8 \\ & & -4 & 0 & 12 & 8 \\ \hline -1 & 1 & 0 & -3 & -2 & \underline{|0} \\ & & -1 & 1 & 2 & \\ \hline & 1 & -1 & -2 & \underline{|0} & \end{array}$$



$$\frac{(x-1)(x-1)}{(x+1)(x+1)(x-2)}$$

$$\frac{(-4-1)(-4-1)}{(-4+1)(-4+1)(-4-2)} = \frac{25}{-54}$$

15. Let $f(x) = -2x^9 + 6x^8 - 6x^7 + 2x^6 + 50x^5 - 150x^4 + 150x^3 - 50x^2$.

a. List the possible rational zeros of f. $\dfrac{\pm 1, \pm 2, \pm 5, \pm 10, \pm 25, \pm 50}{\pm 1, \pm 2}$

b. Find all rational zeros of f.

$-2x^2(x^7 - 3x^6 + 3x^5 - x^4 - 25x^3 + 75x^2 - 75x + 25)$

$x = 0, 0$

$= 1, 1, 1$

1	1	-3	3	-1	-25	75	-75	25
		1	-2	1	0	-25	50	-25
1	1	-2	1	0	-25	50	-25	0
		1	-1	0	0	-25	25	
1	1	-1	0	0	-25	25	0	
		1	0	0	0	-25		
	1	0	0	0	-25	0		

c. Find all real zeros of f.

$x^4 - 25 = 0$

$(x^2 - 5)(x^2 + 5) = 0$

$x^2 = 5 \quad x = \pm\sqrt{5}$

d. Find all zeros of f.

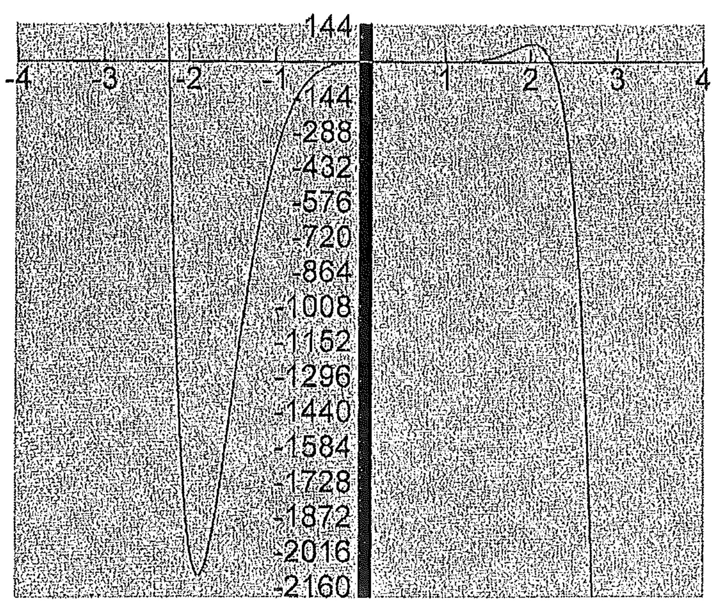
$x^2 + 5 = 0 \quad x = \pm i\sqrt{5}$

$x^2 = -5$

e. Write f as the product of linear factors.

$-2x^2(x-1)(x-1)(x-1)(x-\sqrt{5})(x+\sqrt{5})(x-i\sqrt{5})(x+i\sqrt{5})$

f. Sketch the graph of f.



16. Expand.

a) $\ln\left(\frac{x^2-1}{x^3}\right)$

$\ln\left(\frac{(x-1)(x+1)}{x^3}\right)$

$\ln(x-1)+\ln(x+1)-3\ln x$

b) $\log_7 \sqrt{x^2\left(x^2-2x-15\right)}$

$\log_7 x\sqrt{(x-5)(x+3)}$

$\log_7 x+\frac{1}{2}\log_7(x-5)+\frac{1}{2}\log_7(x+3)$

17. Condense.

a) $\frac{2}{3}\log a-3\log b-4\log c$

$\log\left(\frac{\sqrt[3]{a^2}}{b^3c^4}\right)$

b) $\frac{1}{2}\left[\ln(x+1)+2\ln(x-1)\right]+3\ln x$

$\frac{1}{2}\ln(x+1)(x-1)^2+\ln x^3$

$\ln x^3(x-1)\sqrt{x+1}$

$\ln\left((x^4-x^3)\sqrt{x+1}\right)$

18. Solve.

a) $\log x=1-\log(x-3)$

$\log x+\log(x-3)=1$

$\log x(x-3)=1$

$x^2-3x=10$

$x^2-3x-10=0$

$(x-5)(x+2)$

$x=5, \cancel{-2}$

b) $\log\left(x^2\right)=\left(\log x\right)^2$

$2\log x=(\log x)^2$

$0=(\log x)^2-2\log x$

$0=\log x(\log x-2)$

$x=1,\ 100$

c) $e^{2x}+2e^x-15=0$

$(e^x+5)(e^x-3)=0$

$\cancel{e^x+5=0}$ $\quad e^x-3=0$

$\cancel{e^x=-5}$ $\quad e^x=+3$

$x=1.099$

d) $5^{2x^2+3x}=25^{6-x}$

$5^{2x^2+3x}=\left(5^2\right)^{6-x}$

$5^{2x^2+3x}=5^{12-2x}$

$2x^2+3x=12-2x$

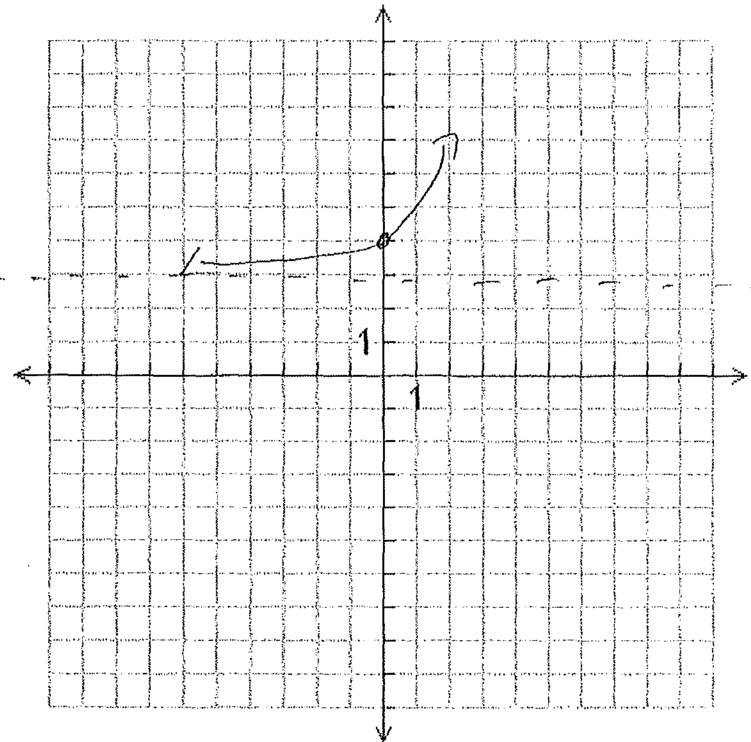
$2x^2+5x-12=0$

$(2x-3)(x+4)=0$

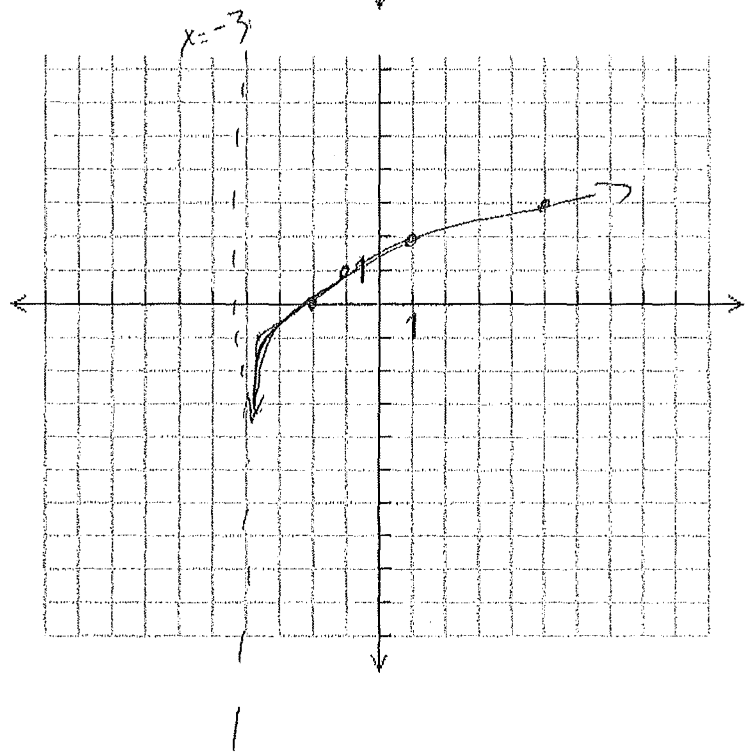
$x=\frac{3}{2},-4$

19. Sketch the graph of the function. Identify any zeros or asymptotes.

a. $f(x) = 2^x + 3$



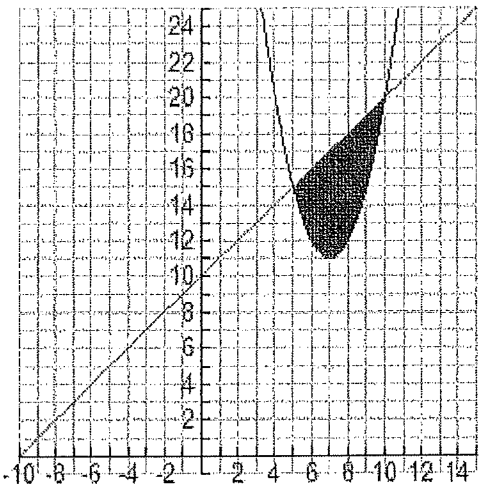
b. $h(x) = \log_2(x+3)$



$2^y = x+3$

x	y
-2	0
-1	1
1	2
5	3
$-2\frac{1}{2}$	-1

20. Find the maximum height between the given parabola and line between the interval where they intersect.



(5,15) (10, 20) VERTEX (7,11)

$y = a(x-7)^2 + 11$

$15 = a(5-7)^2 + 11$

$4 = 4a$

$1 = a$

$y = (x-7)^2 + 11$

$m = \frac{20-15}{10-5} = \frac{5}{5} = 1$

$y = x^2 - 14x + 49 + 11$

$y = x + 10$

$y = x^2 - 14x + 60$

$(x+10) - (x^2 - 14x + 60)$

$-x^2 + 15x - 50$

$x = \frac{-15}{2(-1)} = \frac{15}{2}$

$-\left(\frac{15}{2}\right)^2 + 15\left(\frac{15}{2}\right) - 50 = \frac{25}{4}$