LESSON 1

1. Prepare Spinner 1 and Spinner 2 for the chance experiment described on this second scenario card. (Recall that Spinner 2 has six equal sectors.)

   Prepare Spinner 2 as described. You can use the same Spinner 1 used for Scenario Card 1.

2. What is the sample space for the chance experiment described on this scenario card?

   There are 18 outcomes on this scenario card. Students can list all of the outcomes or describe them. Here is the list (the first number is the outcome from Spinner 1, and the second number is the outcome from Spinner 2);

   \[(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)\]

3. Based on the sample space, determine the outcomes and the probabilities for each of the events on this scenario card. Complete the table below.

<table>
<thead>
<tr>
<th>Event</th>
<th>Outcomes</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outcome is an odd number on Spinner 2.</td>
<td>((1, 1), (1, 3), (1, 5), (2, 1), (2, 3), (2, 5), (3, 1), (3, 3), (3, 5))</td>
<td>(\text{The probability is } \frac{9}{18} \text{ which is } 0.5).</td>
</tr>
<tr>
<td>Outcome is an odd number on Spinner 1 and an even number on Spinner 2.</td>
<td>((1, 2), (1, 4), (1, 6), (3, 2), (3, 4), (3, 6))</td>
<td>(\text{The probability is } \frac{6}{18} \text{ which is approximately } 0.333).</td>
</tr>
<tr>
<td>Outcome is the sum of 7 from the numbers received from Spinner 1 and Spinner 2.</td>
<td>((1, 6), (2, 5), (3, 4))</td>
<td>(\text{The probability is } \frac{3}{18} \text{ which is approximately } 0.167).</td>
</tr>
<tr>
<td>Outcome is an even number on Spinner 2.</td>
<td>((1, 2), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 2), (3, 4), (3, 6))</td>
<td>(\text{The probability is } \frac{9}{18} \text{ which is } 0.5).</td>
</tr>
<tr>
<td>Outcome is the sum of 2 from the numbers received from Spinner 1 and Spinner 2.</td>
<td>((1, 1))</td>
<td>(\text{The probability is } \frac{1}{18} \text{ which is approximately } 0.056).</td>
</tr>
</tbody>
</table>

*Note that although each event is different, some events are subsets of another event. As a result, students want to assign a larger number to the event with more outcomes. Expect that students obtain scores of 4 or 5 for each turn. As an extension, students may be asked to revise the descriptions of the events on the strategy cards in order to make the game more challenging.*
4. Assign the numbers 1-5 to the events described on the scenario card.

The following assignments would be based on the 5 assigned to the event with the greatest probability (the most likely outcome), 4 to the event with the next largest, etc.:

<table>
<thead>
<tr>
<th>Five Events of Interest: Scenario 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outcome is an odd number on Spinner 2.</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

5. Determine at least three final scores based on the numbers you assigned to the events.

Responses will vary. Provided are three final scores based on outcomes from carrying out the game.

Player: Scott

<table>
<thead>
<tr>
<th>Trial</th>
<th>Outcome from Spinner 1</th>
<th>Outcome from Spinner 2</th>
<th>Points (see Problem 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

Final Score: 23 points

Player: Scott

<table>
<thead>
<tr>
<th>Trial</th>
<th>Outcome from Spinner 1</th>
<th>Outcome from Spinner 2</th>
<th>Points (see Problem 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Final Score: 22 points

Player: Scott

<table>
<thead>
<tr>
<th>Trial</th>
<th>Outcome from Spinner 1</th>
<th>Outcome from Spinner 2</th>
<th>Points (see Problem 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

Final Score: 23 points
6. Alan also included a fair coin as one of the scenario tools. Develop a scenario card (Scenario Card 3) that uses the coin and one of the spinners. Include a description of the chance experiment and descriptions of five events relevant to the chance experiment.

*Answers will vary. Encourage students to be creative with this part of their assignment. Anticipate language similar to that used in the examples. A sample response card is included.*

*The following is an example of a completed Scenario Card 3:*

<table>
<thead>
<tr>
<th><strong>Scenario Card 3</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tools:</strong></td>
</tr>
<tr>
<td>Fair coin (head or tail)</td>
</tr>
<tr>
<td>Spinner 1 <em>(three equal sectors with the number 1 in one sector, the number 2 in the second sector, and the number 3 in the third sector)</em></td>
</tr>
<tr>
<td><strong>Directions (chance experiment):</strong></td>
</tr>
<tr>
<td><em>Toss fair coin, and spin Spinner 1. Record the head or tail from your toss and the number from your spin.</em></td>
</tr>
<tr>
<td><strong>Five Events of Interest:</strong></td>
</tr>
<tr>
<td><strong>Outcome is an odd number on Spinner 1.</strong></td>
</tr>
<tr>
<td>5</td>
</tr>
</tbody>
</table>
7. Determine the sample space for your chance experiment. Then, complete the table below for the five events on your scenario card. Assign the numbers 1–5 to the descriptions you created.

Evaluate this chart based on the sample space and the descriptions developed by students. To evaluate, encourage several students to explain their game scenario cards to other students, or provide a sample of the scenario cards developed for students to try. The sample response is based on the scenario card presented in Problem 6.

<table>
<thead>
<tr>
<th>Event</th>
<th>Outcomes</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outcome is an odd number on Spinner 1.</td>
<td>$(1, H), (1, T), (3, H), (3, T)$</td>
<td>The probability is $\frac{4}{6}$, which is approximately 0.667.</td>
</tr>
<tr>
<td>Outcome is a prime number on Spinner 1.</td>
<td>$(2, H), (2, T), (3, H), (3, T)$</td>
<td>The probability is $\frac{4}{6}$, which is approximately 0.667.</td>
</tr>
<tr>
<td>Outcome is a tail.</td>
<td>$(1, T), (2, T), (3, T)$</td>
<td>The probability is $\frac{3}{6}$, which is 0.5.</td>
</tr>
<tr>
<td>Outcome is a head and is not an even number on Spinner 1.</td>
<td>$(1, H), (3, H)$</td>
<td>The probability is $\frac{2}{6}$, which is approximately 0.333.</td>
</tr>
<tr>
<td>Outcome is a tail and a 1 on Spinner 1.</td>
<td>$(1, T)$</td>
<td>The probability is $\frac{1}{6}$, which is approximately 0.167.</td>
</tr>
</tbody>
</table>

8. Determine a final score for your game based on five turns.

<table>
<thead>
<tr>
<th>Turn</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

Answers vary based on the descriptions developed by students. Note: If time permits, encourage selected students to explain their games to other members of the class.
LESSON 2

1. The Waldo School Board asked eligible voters to evaluate the town’s library service. Data are summarized in the following table:

<table>
<thead>
<tr>
<th>Age (in years)</th>
<th>Male</th>
<th>Female</th>
<th>Male</th>
<th>Female</th>
<th>Male</th>
<th>Female</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>18–25</td>
<td>10</td>
<td>8</td>
<td>5</td>
<td>7</td>
<td>5</td>
<td>5</td>
<td>17</td>
<td>18</td>
</tr>
<tr>
<td>26–40</td>
<td>30</td>
<td>28</td>
<td>25</td>
<td>30</td>
<td>20</td>
<td>30</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>41–65</td>
<td>30</td>
<td>32</td>
<td>26</td>
<td>21</td>
<td>15</td>
<td>10</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>66 and Older</td>
<td>21</td>
<td>25</td>
<td>8</td>
<td>15</td>
<td>2</td>
<td>10</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

a. What is the probability that a randomly selected person who completed the survey rated the library as good?

\[
\frac{104}{515} \approx 0.357
\]

b. Imagine talking to a randomly selected male voter who had completed the survey. How do you think this person rated the library services? Explain your answer.

*Answers will vary. A general look at the table indicates that most male voters rated the library as good. As a result, I would predict that this person would rate the library as good.*

c. Use the given data to construct a two-way table that summarizes the responses on gender and rating of the library services. Use the following template as your guide:

<table>
<thead>
<tr>
<th></th>
<th>Good</th>
<th>Average</th>
<th>Poor</th>
<th>Do Not Use</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>91</td>
<td>64</td>
<td>42</td>
<td>44</td>
<td>241</td>
</tr>
<tr>
<td>Female</td>
<td>93</td>
<td>73</td>
<td>55</td>
<td>53</td>
<td>274</td>
</tr>
<tr>
<td>Total</td>
<td>184</td>
<td>137</td>
<td>97</td>
<td>97</td>
<td>515</td>
</tr>
</tbody>
</table>
d. Based on your table, answer the following:
   
i. A randomly selected person who completed the survey is male. What is the probability he rates the library services as good?
   \[
   \frac{91}{241} \approx 0.378
   \]

   ii. A randomly selected person who completed the survey is female. What is the probability she rates the library services as good?
   \[
   \frac{93}{274} \approx 0.339
   \]

e. Based on your table, answer the following:
   
i. A randomly selected person who completed the survey rated the library services as good. What is the probability this person is male?
   \[
   \frac{91}{184} \approx 0.495
   \]

   ii. A randomly selected person who completed the survey rated the library services as good. What is the probability this person is female?
   \[
   \frac{93}{274} \approx 0.339
   \]

f. Do you think there is a difference in how male and female voters rated library services? Explain your answer.

   *Answers will vary.* Yes, I think there is a difference. For example, 37.9% of the male voters rated the library services as good, but only 33.9% of the female voters rated the services this way. There are also differences between the ratings from male and female voters for the other categories.
2. **Obedience School for Dogs** is a small franchise that offers obedience classes for dogs. Some people think that larger dogs are easier to train and, therefore, should not be charged as much for the classes. To investigate this claim, dogs enrolled in the classes were classified as large (30 pounds or more) or small (under 30 pounds). The dogs were also classified by whether or not they passed the obedience class offered by the franchise. 45% of the dogs involved in the classes were large. 60% of the dogs passed the class. Records indicate that 40% of the dogs in the classes were small and passed the course.

a. Complete the following hypothetical 1000 two-way table:

<table>
<thead>
<tr>
<th></th>
<th>Passed the Course</th>
<th>Did Not Pass the Course</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large Dogs</td>
<td>200</td>
<td>250</td>
<td>450</td>
</tr>
<tr>
<td>Small Dogs</td>
<td>400</td>
<td>150</td>
<td>550</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>600</strong></td>
<td><strong>400</strong></td>
<td><strong>1,000</strong></td>
</tr>
</tbody>
</table>

b. Estimate the probability that a dog selected at random from those enrolled in the classes passed the course.

\[
\frac{600}{1000} = 0.600, \text{ meaning 60\% passed the course.}
\]

c. A dog was randomly selected from the dogs that completed the class. If the selected dog was a large dog, what is the probability this dog passed the course?

\[
\frac{200}{450} \approx 0.444, \text{ meaning that approximately 44.4\% of large dogs passed the course.}
\]

d. A dog was randomly selected from the dogs that completed the class. If the selected dog is a small dog, what is the probability this dog passed the course?

\[
\frac{400}{550} \approx 0.727, \text{ meaning that approximately 72.7\% of small dogs passed the course.}
\]

e. Do you think dog size and whether or not a dog passes the course are related?

*Answers will vary. Yes, there is a noticeably greater probability that a dog passed the obedience class if a dog is small than if the dog is large.*

f. Do you think large dogs should get a discount? Explain your answer.

*Answers will vary. No, large dogs should not get a discount. Large dogs are not as likely to have passed the obedience class as small dogs.*
LESSON 3

Oostburg College has a rather large marching band. Engineering majors were heard bragging that students majoring in engineering are more likely to be involved in the marching band than students from other majors.

1. If the above claim is accurate, does that mean that most of the band is engineering students? Explain your answer.

   No. It means that if a randomly selected student is an engineering major, the probability this person is in the marching band is greater than if this person was not an engineering major.

2. The following graph was prepared to investigate the above claim:

   ![Graph showing frequency of students in the marching band based on major.]

   Based on the graph, complete the following two-way frequency table:

<table>
<thead>
<tr>
<th></th>
<th>In the Marching Band</th>
<th>Not in the Marching Band</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engineering Major</td>
<td>40</td>
<td>135</td>
<td>175</td>
</tr>
<tr>
<td>Not Engineering Major</td>
<td>120</td>
<td>510</td>
<td>630</td>
</tr>
<tr>
<td>Total</td>
<td>160</td>
<td>645</td>
<td>805</td>
</tr>
</tbody>
</table>
3. Let $M$ represent the event that a randomly selected student is in the marching band. Let $E$ represent the event that a randomly selected student is an engineering major.
   a. Describe the event represented by the complement of $M$.
      
      A randomly selected student is not in the marching band.
   
   b. Describe the event represented by the complement of $E$.
      
      A randomly selected student is not majoring in engineering.
   
   c. Describe the event $M$ and $E$ ($M$ intersect $E$).
      
      A randomly selected student is majoring in engineering and is in the marching band.
   
   d. Describe the event $M$ or $E$ ($M$ union $E$).
      
      A randomly selected student is majoring in engineering or is in the marching band.
   
4. Based on the completed two-way frequency table, determine the following, and explain how you got your answer:
   a. The probability that a randomly selected student is in the marching band
      
      \[
      \frac{160}{805} \approx 0.199
      \]
      
      I compared the number of students in the marching band to the total number of students.
   
   b. The probability that a randomly selected student is an engineering major
      
      \[
      \frac{175}{805} \approx 0.217
      \]
      
      I compared the number of engineering majors to the total number of students.
   
   c. The probability that a randomly selected student is in the marching band and an engineering major
      
      \[
      \frac{40}{805} \approx 0.05
      \]
      
      I found the number of students who are in the band and are engineering majors and compared it to the total number of students.
   
   d. The probability that a randomly selected student is in the marching band and not an engineering major
      
      \[
      \frac{120}{805} \approx 0.149
      \]
      
      I found the number of students who are in the band and are NOT engineering majors and compared it to the total number of students.
5. Indicate if the following conditional probabilities would be calculated using the rows or the columns of the two-way frequency table:

   a. A randomly selected student is majoring in engineering. What is the probability this student is in the marching band?

      This probability is based on the row Engineering Major.

   b. A randomly selected student is not in the marching band. What is the probability that this student is majoring in engineering?

      This probability is based on the column Not in the Marching Band.

6. Based on the two-way frequency table, determine the following conditional probabilities:

   a. A randomly selected student is majoring in engineering. What is the probability that this student is in the marching band?

      \[
      \frac{40}{175} \approx 0.229
      \]

   b. A randomly selected student is not majoring in engineering. What is the probability that this student is in the marching band?

      \[
      \frac{120}{630} \approx 0.190
      \]
7. The claim that started this investigation was that students majoring in engineering are more likely to be in the marching band than students from other majors. Describe the conditional probabilities that would be used to determine if this claim is accurate.

Given a randomly selected student is an engineering major, what is the probability the student is in the marching band. Also, given a randomly selected student is not an engineering major, what is the probability the student is in the marching band.

8. Based on the two-way frequency table, calculate the conditional probabilities identified in Problem 7.

The probabilities were calculated in Problem 6. Approximately 0.229 (or 22.9%) of the engineering students are in the marching band. Approximately 0.190 (or 19.0%) of the students not majoring in engineering are in the marching band.

9. Do you think the claim that students majoring in engineering are more likely to be in the marching band than students for other majors is accurate? Explain your answer.

The claim is accurate based on the conditional probabilities.

10. There are 40 students at Oostburg College majoring in computer science. Computer science is not considered an engineering major. Calculate an estimate of the number of computer science majors you think are in the marching band. Explain how you calculated your estimate.

The probability that a randomly selected student who is not majoring in engineering is in the marching band is 0.190. As a result, you would estimate that 19% of the 40 computer science majors are in the marching band. Since 40(0.190) = 7.6, I would expect that 7 or 8 computer science majors are in the marching band.
LESSON 4

1. Consider the following questions:
   a. A survey of the students at a Midwest high school asked the following questions:
      "Do you use a computer at least 3 times a week to complete your schoolwork?"
      "Are you taking a mathematics class?"
      Do you think the events “a randomly selected student uses a computer at least 3 times a week” and “a randomly selected student is taking a mathematics class” are independent or not independent? Explain your reasoning.
      Anticipate students indicate that using a computer at least 3 times per week and taking a mathematics class are not independent. However, it is also acceptable for students to make a case for independence. Examine the explanations students provide to see if they understand the meaning of independence.
   b. The same survey also asked students the following:
      "Do you participate in any extracurricular activities at your school?"
      "Do you know what you want to do after high school?"
      Do you think the events “a randomly selected student participates in extracurricular activities” and “a randomly selected student knows what she wants to do after completing high school” are independent or not independent? Explain your reasoning.
      Answers will vary. Anticipate students indicate that students involved in extracurricular activities are often students who want to attend college. It is likely the events are not independent.
   c. People attending a professional football game in 2013 completed a survey that included the following questions:
      "Is this the first time you have attended a professional football game?"
      "Do you think football is too violent?"
      Do you think the events “a randomly selected person who completed the survey is attending a professional football game for the first time” and “a randomly selected person who completed the survey thinks football is too violent” are independent or not independent? Explain your reasoning.
      Answers will vary. Anticipate that students indicate that people who attend football more often are more likely to not think the game is too violent. It is likely the events are not independent. Again, examine the explanations students provide if they indicate the events are not independent.
2. Complete the table below in a way that would indicate the two events “uses a computer” and “is taking a mathematics class” are independent.

<table>
<thead>
<tr>
<th></th>
<th>Uses a Computer at Least 3 Times a Week for Schoolwork</th>
<th>Does Not Use a Computer at Least 3 Times a Week for Schoolwork</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>In a Mathematics Class</td>
<td>420</td>
<td>280</td>
<td>700</td>
</tr>
<tr>
<td>Not in a Mathematics Class</td>
<td>180</td>
<td>120</td>
<td>300</td>
</tr>
<tr>
<td>Total</td>
<td>600</td>
<td>400</td>
<td>1,000</td>
</tr>
</tbody>
</table>

The values in the table were based on 0.60 of the students not in mathematics use a computer at least 3 times a week for school (0.60 \times 300). Also, 0.60 of the students in mathematics use a computer at least 3 times a week for school (0.60 \times 700).

3. Complete the following hypothetical 1000 table. Are the events “participates in extracurricular activities” and “know what I want to do after high school” independent or not independent? Justify your answer.

<table>
<thead>
<tr>
<th></th>
<th>Participates in Extracurricular Activities</th>
<th>Does Not Participate in Extracurricular Activities</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Know What I Want to Do</td>
<td>550</td>
<td>250</td>
<td>800</td>
</tr>
<tr>
<td>After High School</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Do Not Know What I Want</td>
<td>50</td>
<td>150</td>
<td>200</td>
</tr>
<tr>
<td>To Do After High School</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>600</td>
<td>400</td>
<td>1,000</td>
</tr>
</tbody>
</table>

The events “student participates in extracurricular activities” and “student knows what I want to do after high school” are not independent. Students could indicate that the events are not independent in several ways. For example, \( \frac{50}{200} \) (the probability that a randomly selected student who does not know what he wants to do after high school participates in extracurricular activities) does not equal \( \frac{550}{800} \) (the probability that a randomly selected student who does know what he wants to do after high school participates in extracurricular activities).
4. The following hypothetical 1000 table is from Lesson 2:

<table>
<thead>
<tr>
<th></th>
<th>No Household Member Smokes</th>
<th>At Least One Household Member Smokes</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student Has Asthma</td>
<td>73</td>
<td>120</td>
<td>193</td>
</tr>
<tr>
<td>Student Does Not Have Asthma</td>
<td>506</td>
<td>301</td>
<td>807</td>
</tr>
<tr>
<td>Total</td>
<td>579</td>
<td>421</td>
<td>1,000</td>
</tr>
</tbody>
</table>

The actual data from the entire population are given in the table below.

<table>
<thead>
<tr>
<th></th>
<th>No Household Member Smokes</th>
<th>At Least One Household Member Smokes</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student Has Asthma</td>
<td>69</td>
<td>113</td>
<td>182</td>
</tr>
<tr>
<td>Student Does Not Have Asthma</td>
<td>473</td>
<td>282</td>
<td>755</td>
</tr>
<tr>
<td>Total</td>
<td>542</td>
<td>395</td>
<td>937</td>
</tr>
</tbody>
</table>

a. Based on the hypothetical 1000 table, what is the probability that a randomly selected student who has asthma has at least one household member who smokes?

\[ \frac{120}{193} \approx 0.622 \]

b. Based on the actual data, what is the probability that a randomly selected student who has asthma has at least one household member who smokes (round your answer to 3 decimal places)?

\[ \frac{113}{182} \approx 0.621 \]

c. Based on the hypothetical 1000 table, what is the probability that a randomly selected student who has no household member who smokes has asthma?

\[ \frac{73}{579} \approx 0.126 \]
d. Based on the actual data, what is the probability that a randomly selected student who has no household member who smokes has asthma?

\[
\frac{69}{542} \approx 0.127
\]

e. What do you notice about the probabilities calculated from the actual data and the probabilities calculated from the hypothetical 1000 table?

The conditional probabilities differ only due to rounding in constructing the hypothetical 1000 table from probability information based on the actual data. When an actual data table is available, it can be used to calculate probabilities. When only probability information is available, constructing a hypothetical 1000 table from that information and using it to compute other probabilities will give the same answers as if the actual data were available.

5. As part of the asthma research, the investigators wondered if students who have asthma are less likely to have a pet at home than students who do not have asthma. They asked the following two questions:

“Do you have asthma?”
“Do you have a pet at home?”

Based on the responses to these questions, you would like to set up a two-way table that you could use to determine if the following two events are independent or not independent:

Event 1: A randomly selected student has asthma.
Event 2: A randomly selected student has a pet at home.

a. How would you label the rows of the two-way table?

Anticipate students indicate for the rows “Has Asthma” and “Does Not Have Asthma.” Students might use these labels for the columns rather than the rows, which is also acceptable.

b. How would you label the columns of the two-way table?

Anticipate students indicate for the columns “Has a Pet” and “Does Not Have a Pet.”

c. What probabilities would you calculate to determine if Event 1 and Event 2 are independent?

Answers may vary. Row conditional probabilities or column conditional probabilities would have to be equal if the events are independent. For column conditional probabilities (based on the definition of rows and columns above), this would mean that the probability that a randomly selected student who has a pet has asthma is equal to the probability that a randomly selected student who does not have a pet has asthma.
LESSON 5

1. On a flight, some of the passengers have frequent-flier status, and some do not. Also, some of the passengers have checked baggage, and some do not. Let the set of passengers who have frequent-flier status be $F$ and the set of passengers who have checked baggage be $C$. On the Venn diagrams provided, shade the regions representing the following instances:

a. Passengers who have frequent-flier status and have checked baggage

b. Passengers who have frequent-flier status or have checked baggage

c. Passengers who do not have both frequent-flier status and checked baggage

d. Passengers who have frequent-flier status or do not have checked baggage
2. For the scenario introduced in Problem 1, suppose that, of the 400 people on the flight, 368 have checked baggage, 228 have checked baggage but do not have frequent-flier status, and 8 have neither frequent-flier status nor checked baggage.

   a. Using a Venn diagram, calculate the following:
      
      i. The number of people on the flight who have frequent-flier status and have checked baggage
         
         *Number of passengers with frequent-flier status and checked baggage:
         
         \[ 368 - 228 = 140 \]

      ii. The number of people on the flight who have frequent-flier status
         
         *Number of passengers with frequent-flier status:
         
         \[ 400 - 8 - 228 = 164 \]

   b. In the Venn diagram provided below, write the probabilities of the events associated with the regions marked with a star (*).

   \[ \text{Answer:} \]

   _Diagram showing Venn diagram with probabilities marked._
3. When an animal is selected at random from those at a zoo, the probability that it is North American (meaning that its natural habitat is in the North American continent) is 0.65, the probability that it is both North American and a carnivore is 0.16, and the probability that it is neither American nor a carnivore is 0.17.

   a. Using a Venn diagram, calculate the probability that a randomly selected animal is a carnivore.

   
   
   
   
   
   \[
P(C) = 0.16 + 0.18 = 0.34
   \]

   

   
   
   
   

   b. Complete the table below showing the probabilities of the events corresponding to the cells of the table.

<table>
<thead>
<tr>
<th></th>
<th>North American</th>
<th>Not North American</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carnivore</td>
<td>0.16</td>
<td>0.18</td>
<td>0.34</td>
</tr>
<tr>
<td>Not Carnivore</td>
<td>0.49</td>
<td>0.17</td>
<td>0.66</td>
</tr>
<tr>
<td>Total</td>
<td>0.65</td>
<td>0.35</td>
<td>1.00</td>
</tr>
</tbody>
</table>
4. This question introduces the mathematical symbols for *and*, *or*, and *not*.

Considering all the people in the world, let $A$ be the set of Americans (citizens of the United States), and let $B$ be the set of people who have brothers.

- The set of people who are Americans and have brothers is represented by the shaded region in the Venn diagram below.

This set is written $A \cap B$ (read $A$ intersect $B$), and the probability that a randomly selected person is American and has a brother is written $P(A \cap B)$.

- The set of people who are Americans or have brothers is represented by the shaded region in the Venn diagram below.

This set is written $A \cup B$ (read $A$ union $B$), and the probability that a randomly selected person is American or has a brother is written $P(A \cup B)$.

- The set of people who are not Americans is represented by the shaded region in the Venn diagram below.

This set is written $A^c$ (read $A$ complement), and the probability that a randomly selected person is not American is written $P(A^c)$. 
Now, think about the cars available at a dealership. Suppose a car is selected at random from the cars at this dealership. Let the event that the car has manual transmission be denoted by $M$, and let the event that the car is a sedan be denoted by $S$. The Venn diagram below shows the probabilities associated with four of the regions of the diagram.

![Venn Diagram](image)

a. What is the value of $P(M \cap S)$?
   
   0.12

b. Complete this sentence using and or or:
   
   $P(M \cap S)$ is the probability that a randomly selected car has a manual transmission \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ and is a sedan.

c. What is the value of $P(M \cup S)$?
   
   0.09 + 0.12 + 0.60 = 0.81

d. Complete this sentence using and or or:
   
   $P(M \cup S)$ is the probability that a randomly selected car has a manual transmission \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ or is a sedan.

e. What is the value of $P(S^c)$?
   
   1 - (0.6 + 0.12) = 0.28

f. Explain the meaning of $P(S^c)$.

   $P(S^c)$ is the probability that a randomly selected car is not a sedan.
1. When an avocado is selected at random from those delivered to a food store, the probability that it is ripe is 0.12, the probability that it is bruised is 0.054, and the probability that it is ripe and bruised is 0.019.
   a. Rounding your answers to the nearest thousandth where necessary, find the probability that an avocado randomly selected from those delivered to the store is
      i. Not bruised.
         \[ 1 - 0.054 = 0.946 \]
      ii. Ripe given that it is bruised.
         \[ P(\text{ripe given bruised}) = \frac{P(\text{ripe and bruised})}{P(\text{bruised})} = \frac{0.019}{0.054} \approx 0.352 \]
      iii. Bruised given that it is ripe.
         \[ P(\text{bruised given ripe}) = \frac{P(\text{bruised and ripe})}{P(\text{ripe})} = \frac{0.019}{0.12} \approx 0.158 \]
   b. Which is larger, the probability that a randomly selected avocado is bruised given that it is ripe or the probability that a randomly selected avocado is bruised? Explain in words what this tells you.
      \[ P(\text{bruised given ripe}) = 0.158 \text{ and } P(\text{bruised}) = 0.054. \text{ Therefore } P(\text{bruised given ripe}) \text{ is greater than } P(\text{bruised}), \text{ which tells you that ripe avocados are more likely to be bruised than avocados in general.} \]
   c. Are the events "ripe" and "bruised" independent? Explain.
      \[ \text{No, because } P(\text{bruised given ripe}) \text{ is different from } P(\text{bruised}). \]

2. Return to the probability information given in Problem 1. Complete the hypothetical 1000 table given below, and use it to find the probability that a randomly selected avocado is bruised given that it is not ripe. (Round your answer to the nearest thousandth.)

<table>
<thead>
<tr>
<th></th>
<th>Ripe</th>
<th>Not Ripe</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bruised</td>
<td>19</td>
<td>35</td>
<td>54</td>
</tr>
<tr>
<td>Not Bruised</td>
<td>101</td>
<td>845</td>
<td>946</td>
</tr>
<tr>
<td>Total</td>
<td>120</td>
<td>880</td>
<td>1,000</td>
</tr>
</tbody>
</table>

\[ P(\text{bruised given not ripe}) = \frac{35}{880} \approx 0.040 \]
3. According to the U.S. census website (www.census.gov), based on the U.S. population in 2010, the probability that a randomly selected man is 65 or older is 0.114, and the probability that a randomly selected woman is 65 or older is 0.146. In the questions that follow, round your answers to the nearest thousandth:
   a. If a man is selected at random and a woman is selected at random, what is the probability that both people selected are 65 or older? (Hint: Use the multiplication rule for independent events.)
      \[(0.114)(0.146) \approx 0.017\]
   b. If two men are selected at random, what is the probability that both of them are 65 or older?
      \[(0.114)(0.114) \approx 0.013\]
   c. If two women are selected at random, what is the probability that neither of them is 65 or older?
      \[\text{If one woman is selected at random, the probability that she is not 65 or older is } 1 - 0.146 = 0.854.\]
      \[\text{So, if two women are selected at random, the probability that neither of them is 65 or older is} \]
      \[(0.854)(0.854) \approx 0.729.\]
4. In a large community, 72% of the people are adults, 78% of the people have traveled outside the state, and 11% are adults who have not traveled outside the state.
   a. Using a Venn diagram or a hypothetical 1000 table, calculate the probability that a randomly selected person from the community is an adult and has traveled outside the state.

\[
\begin{array}{c}
A \\
0.11 \\
0.72 \\
T \\

0.78 \\
\end{array}
\]

\[
P(\text{adult and traveled out of state}) = 0.72 - 0.11 = 0.61
\]
b. Use the multiplication rule for independent events to decide whether the events “is an adult” and “has traveled outside the state” are independent.

\[ P(\text{adult and traveled out of state}) = 0.61 \]
\[ P(\text{adult})P(\text{traveled out of state}) = (0.72)(0.78) = 0.5616 \]

*Since these two quantities are not equal, the two events are not independent.*

5. In a particular calendar year, 10% of the registered voters in a small city are called for jury duty. In this city, people are selected for jury duty at random from all registered voters in the city, and the same individual cannot be called more than once during the calendar year.

a. What is the probability that a registered voter is not called for jury duty during a particular year?

0.90

b. What is the probability that a registered voter is called for jury duty two years in a row?

\[(0.10)(0.10) = 0.01\]

6. A survey of registered voters in a city in New York was carried out to assess support for a new school tax. 51% of the respondents supported the school tax. Of those with school-age children, 56% supported the school tax, while only 45% of those who did not have school-age children supported the school tax.

a. If a person who responded to this survey is selected at random, what is the probability that

i. The person selected supports the school tax?

0.51

ii. The person supports the school tax given that she does not have school-age children?

0.45

b. Are the two events “has school-age children” and “supports the school tax” independent? Explain how you know this.

*These two events are not independent because the probability of support given no school-age children is not the same as the probability of support.*

c. Suppose that 35% of those responding to the survey were over the age of 65 and that 10% of those responding to the survey were both over age 65 and supported the school tax. What is the probability that a randomly selected person who responded to this survey supported the school tax given that she was over age 65?

\[ P(\text{support given over age 65}) = \frac{P(\text{support and over age 65})}{P(\text{over age 65})} \]

\[ = \frac{0.10}{0.35} \]

\[ \approx 0.286 \]
1. Of the works of art at a large gallery, 59% are paintings, and 83% are for sale. When a work of art is selected at random, let the event that it is a painting be $A$ and the event that it is for sale be $B$.

   a. What are the values of $P(A)$ and $P(B)$?

   \[
P(A) = 0.59 \quad P(B) = 0.83
   \]

   b. Suppose you are told that $P(A$ and $B) = 0.51$. Find $P(A$ or $B)$.

   \[
P(A$ or $B) = P(A) + P(B) - P(A$ and $B) = 0.59 + 0.83 - 0.51 = 0.91
   \]

   c. Suppose now that you are not given the information in part (b), but you are told that the events $A$ and $B$ are independent. Find $P(A$ or $B)$.

   \[
P(A$ and $B) = P(A)P(B) = (0.59)(0.83) = 0.4897
   \]

   So, $P(A$ or $B) = P(A) + P(B) - P(A$ and $B) = 0.59 + 0.83 - 0.4897 = 0.9303$. 
2. A traveler estimates that, for an upcoming trip, the probability of catching malaria is 0.18, the probability of catching typhoid is 0.13, and the probability of catching neither of the two diseases is 0.75.

   a. Draw a Venn diagram to represent this information.

   ![Venn Diagram]

   b. Calculate the probability of catching both of the diseases.

   \[ P(M \text{ or } T) = 1 - 0.75 = 0.25 \]

   By the addition rule:

   \[ P(M \text{ or } T) = P(M) + P(T) - P(M \text{ and } T) \]

   \[ 0.25 = 0.18 + 0.13 - P(M \text{ and } T) \]

   \[ 0.25 = 0.31 - P(M \text{ and } T) \]

   \[ P(M \text{ and } T) = 0.06 \]

   c. Are the events “catches malaria” and “catches typhoid” independent? Explain your answer.

   \[ P(M \text{ and } T) = 0.06 \]

   \[ P(M)P(T) = (0.18)(0.13) = 0.0234 \]

   Since these quantities are different, the two events are not independent.
3. A deck of 40 cards consists of the following:

- 10 black cards showing squares, numbered 1–10
- 10 black cards showing circles, numbered 1–10
- 10 red cards showing X’s, numbered 1–10
- 10 red cards showing diamonds, numbered 1–10

A card will be selected at random from the deck.

a. i. Are the events “the card shows a square” and “the card is red” disjoint? Explain.

   Yes. There is no red card that shows a square.

ii. Calculate the probability that the card will show a square or will be red.

\[
P(\text{square or red}) = P(\text{square}) + P(\text{red})
\]

\[
= \frac{10}{40} + \frac{20}{40} = \frac{30}{40} = \frac{3}{4}
\]

b. i. Are the events “the card shows a 5” and “the card is red” disjoint? Explain.

   No. There are red fives in the deck.

ii. Calculate the probability that the card will show a 5 or will be red.

\[
P(\text{5 or red}) = P(\text{5}) + P(\text{red}) - P(\text{5 and red})
\]

\[
= \frac{4}{40} + \frac{20}{40} - \frac{2}{40} = \frac{22}{40} = \frac{11}{20}
\]
4. The diagram below shows a spinner. When the pointer is spun, it is equally likely to stop on 1, 2, or 3. The pointer will be spun three times. Expressing your answers as fractions in lowest terms, find the probability, and explain how the answer was determined that the total of the values from all three spins is

a. 9.

The only way to get a total of 9 is to spin a 3, 3 times. Since the probability of spinning a 3 is \( \frac{1}{3} \),

\[
P(\text{total is 9}) = P(3, 3, 3) = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{27}.
\]

b. 8.

There are 3 ways to get a total of 8. Since the probability of spinning a 1, 2, and 3 are all equally likely \( \frac{1}{3} \):

\[
P(\text{total is 8}) = P(3, 3, 2) + P(3, 2, 3) + P(2, 3, 3)
\]

\[
= \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}
\]

\[
= \frac{1}{27} + \frac{1}{27} + \frac{1}{27}
\]

\[
= \frac{3}{27}
\]

\[
= \frac{1}{9}
\]
c. 7.

There are 6 ways to get a total of 7. Since the probability of spinning a 1, 2, and 3 are all equally likely \(\frac{1}{3}\):

\[
P(\text{total is 7}) = P(3, 3, 1) + P(3, 1, 3) + P(1, 3, 3) + P(3, 2, 2) + P(2, 3, 2) + P(2, 2, 3)
\]
\[
= 6 \left( \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \right)
\]
\[
= \frac{6}{27}
\]
\[
= \frac{2}{9}
\]

5. A number cube has faces numbered 1 through 6, and a coin has two sides—heads and tails. The number cube will be rolled once, and the coin will be flipped once. Find the probabilities of the following events. (Express your answers as fractions in lowest terms.)

a. The number cube shows a 6.

\[
\frac{1}{6}
\]

b. The coin shows heads.

\[
\frac{1}{2}
\]

c. The number cube shows a 6, and the coin shows heads.

\[
\frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}
\]

d. The number cube shows a 6, or the coin shows heads.

\[
P(6 \text{ or heads}) = P(6) + P(\text{heads}) - P(6 \text{ and heads})
\]
\[
= \frac{1}{6} + \frac{1}{2} - \frac{1}{12} = \frac{2}{12} + \frac{6}{12} - \frac{1}{12} = \frac{7}{12}
\]
6. Kevin will soon be taking exams in math, physics, and French. He estimates the probabilities of his passing these exams to be as follows:

- Math: 0.9
- Physics: 0.8
- French: 0.7

Kevin is willing to assume that the results of the three exams are independent of each other. Find the probability of each event.

a. Kevin will pass all three exams.

\[(0.9)(0.8)(0.7) = 0.504\]

b. Kevin will pass math but fail the other two exams.

\[(0.9)(0.2)(0.3) = 0.054\]

c. Kevin will pass exactly one of the three exams.

\[
P(\text{passes exactly one}) = P(\text{passes math, fails physics, fails French}) \\
+ P(\text{fails math, passes physics, fails French}) \\
+ P(\text{fails math, fails physics, passes French}) \\
= (0.9)(0.2)(0.3) + (0.1)(0.8)(0.3) + (0.1)(0.2)(0.7) \\
= 0.092\]