Chapter 1 Answers

Practice 1-1
1. 47, 53  2. 1.00001, 1.00001  3. 42, 54  4. –64, 128  5. 22, 29  6. 63.5, 63.75  7. Sample: 2 or 3  8. 51 or 49  9. 6 or 8  10. A or AA  11. D or G  12. Y or A  13. any hexagon  14. circle with 8 equally spaced diameters  15. a 168.75° angle  16. 21 handshakes  17. h = \frac{n(a - 1)}{2}  18. 34  19. Sample: The farther out you go, the closer the ratio gets to a number that is approximately 0.618.

Practice 1-2
1. 2.
2. 3.
3. 4.
4. 5. \[ \text{A, C, D} \]  6. \[ \text{C} \]  7. \[ \text{D} \]  8. \[ \text{B} \]  9. \[ \text{A} \]  10. \[ \text{plane ABC} \]

Practice 1-3
1. 2. any two of the following: \[ \text{ABD, DBC, CBE, ABE, ECD, ADE, ACE, ACD} \]  3. Points E, B, and D are collinear.  4. yes  5. yes  6. no  7. no  8. yes  9. no  10. yes  11. yes  12. yes  13. no  14. yes  15. yes  16. \[ \text{G, LM} \]  17. \[ \text{the empty set} \]  18. \[ \text{KP} \]  19. \[ \text{M} \]  20. \[ \text{plane ABD} \]  21. Sample: plane \[ \text{ABC} \]

Practice 1-4
1. true  2. false  3. true  4. false  5. false  6. false  7. \[ \text{JK, HG} \]  8. \[ \text{EH} \]  9. any three of the following pairs: \[ \text{EF and FH, EH and GJ, HG and JE, HG and FK, JK and EH, JK and FG, JE and FG, EH and FK, JE and KG, EH and KG, JH and KE, JH and HG, JE and GE, JK and KE, KG and KE, KG and KB, KB and KE, KB and JH, KB and JM, KB and JG, KE and GH, KE and JK, KG and KH, KG and KE, JH and KE, JH and JG, JG and KE, JG and JH, JG and KG, JG and KG, KG and KE, KG and KB, KB and KE, KB and JH, KB and JM, KB and JG, KE and GH, KE and JK, KG and KH, KG and KE}; \[ \text{planes A and B} \]  11. planes A and C; planes B and C  12. planes A and C  13. planes B and C  14. Sample: \[ \text{ED} \]  15. 6  16. \[ \text{EF and ED or EG and ED} \]  17. \[ \text{FE, FD} \]  18. \[ \text{GF, GD} \]  19. yes

20. Sample:

21. Sample:
Chapter 1 Answers (continued)

Practice 1-5
1. 4  
2. 12  
3. 20  
4. 6  
5. 22  
6. –10 or 6  
7. –1 or 1  
8. 3; 4; no  
9. 6; 6; yes  
10. C or –2  
11. 15  
12. 31  
13. 14  
14. \( x = 11 \frac{3}{2}; AB = 31; \\
BC = 31 \)  
15. \( x = 35 \frac{3}{2}; AB = 103; BC = 103 \)

Practice 1-6
1. any three of the following: \( \angle O, \angle MOP, \angle POM, \angle 1 \)  
2. \( \angle AOB \)  
3. \( \angle EOC \)  
4. \( \angle DOC \)  
5. 51  
6. 90  
7. 17  
8. 107  
9. 141  
10. 68  
11. \( \angle ABD, \angle DBE, \angle EBF, \angle DBF, \angle FBC \)  
12. \( \angle ABE, \angle DBC \)  
13. \( \angle ABE, \angle EBC \)  
14. \( x = 5; 8; 21; 13 \)  
15. \( x = 9; 85; 35; 120 \)

Practice 1-7
1. 

2. 

3. 

4. 

5. 

6. 

7. 

8. 

9. 

10. 

11. 

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Chapter 1 Answers (continued)

12.

13.

14. true 15. false 16. false 17. true 18. true

Practice 1-8

1.–5.

6. \(5\sqrt{2} = 7.1\)  7. \(2\sqrt{17} = 8.2\)  8. 12  9. 8
10. 12  11. \(\sqrt{26} = 5.1\)  12. (5, 5)  13. \((-1, 1)\)
14. \((10, -5)\)  15. \((-2, 6)\)  16. \((-0.3, 3.4)\)
17. \((2\frac{7}{8}, -3\frac{3}{4})\)  18. (5, -2)  19. (4, 10)
20. \((-3, 4)\)  21. yes; \(AB = BC = CD = DA = 6\)
22. \(\sqrt{401} = 20.025\)
23.

24. 24.7  25. (3.5, 3)

Practice 1-9

1. 792 in.\(^2\)  2. 3240 in.\(^2\)  3. 2.4 m\(^2\)  4. \(32\pi\)
5. 16\(\pi\)  6. 7.8\(\pi\)  7. 26 cm; 42 cm\(^2\)  8. 46 cm; 42 cm\(^2\)
9. 29 in.; 42 in.\(^2\)  10. 40 ft; 51 ft\(^2\)  11. 40 m; 99 m\(^2\)
12. 40 m; 91 m\(^2\)  13. 68; 285  14. 26; 22  15. 30; 44
16. 156.25\(\pi\)  17. 10,000\(\pi\)  18. \(\frac{25}{16}\)  19. 48; 28
20. 36  21. 26; 13

Reteaching 1-1

1.

2.

3.

4.

5.

6.

Reteaching 1-2

1a.
Chapter 1 Answers (continued)

1b.

Front Top Right

2a.

Front Top Right

2b.

Front Top Right

3a.

Front Top Right

3b.

Front Top Right

4a.

Front Top Right

4b.

Front Top Right

5.

Front Top Right

6.

Front Right

Reteaching 1-3
1.–6. Check students’ work.

7.

Reteaching 1-4
Samples:
1. $\overline{AB}$ and $\overline{EF}$  
2. $\overline{AB}$ and $\overline{DH}$  
3. $\overline{AC}$ and $\overline{AE}$  
4. plane $ABD$ and plane $EFH$  
5. plane $CDH$ and plane $BFD$  
6. plane $CDH$, plane $ACG$, and plane $ABD$  
7. $\overline{EG}$ and $\overline{BF}$

8.

9.

10.
Chapter 1 Answers (continued)

11.

12.

13.

Reteaching 1-5
1. 40 km  2. 25 km  3. 30 km  4. 30 km  5. 60 km  6. Ryan  7. 100 km; 25 + 15 + 30 + 30 = 100

Reteaching 1-6
1a. Each angle is 108°. 1b. obtuse  2a. Each angle is 72°. 2b. acute  3a. Each angle is 108°. 3b. obtuse  4. 90°; right  5. 55°; acute  6. 110°; obtuse  7. 50°; acute  8. 90°; right  9. 100°; obtuse

Reteaching 1-7
Samples:
1.

2.

3.

Reteaching 1-8
1.

2. $XY = 10.6$, $YZ = 4.1$, $XZ = 7.2$  
3. $4.1 + 7.2 = 11.3$, $11.3 > 10.6$  
4. 12.2  
5. 8.1  6. 9.9  7. 11.4  8. 9.2  9. 8.5  10. 18.0  11. 8.5  12. 17

Reteaching 1-9
1a. 1 by 60, 2 by 30, 3 by 20, 4 by 15, 5 by 12, and 6 by 10; Check students’ drawings.  
1b. 122, 64, 46, 38, 34, and 32  
2a. 1 by 36, 2 by 18, 3 by 12, 4 by 9 and 6 by 6; Check students’ drawings.  
2b. 1 by 36  3. 3 cm by 6 cm  
4. Sample:  diameter = 8 units;  
        radius = 4 units
Chapter 1 Answers (continued)

5. Sample: 50 units$^2$  
6. Sample: 50.3 units$^2$  
7. They are close.

Enrichment 1-1
1. 32; 32; 32  
2. 16  
3. 8

4.

<table>
<thead>
<tr>
<th>Round</th>
<th>Round of</th>
<th>Games in Round</th>
<th>Total Games Played</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>32</td>
<td>32</td>
</tr>
<tr>
<td>2</td>
<td>32</td>
<td>16</td>
<td>48</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
<td>8</td>
<td>56</td>
</tr>
<tr>
<td>4</td>
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<td>4</td>
<td>60</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>2</td>
<td>62</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>1</td>
<td>63</td>
</tr>
</tbody>
</table>

5. The number in the fourth column is equal to the sum of the numbers in the third column up to the given row.
6. 16  
7. $1 + 2 + 2^2 + \ldots + 2^n = 2^{n+1} - 1$
8. 127  
9. 3

Enrichment 1-2
1. 4  
2. 7

3. 

4.

5. 

6. 

7.

8. 

9. 

10. $V = 9$ units$^3$; S.A. = 32 units$^2$  
11. $V = 9$ units$^3$; S.A. = 34 units$^2$  
12. $V = 9$ units$^3$; S.A. = 34 units$^2$  
13. $V = 7$ units$^3$; S.A. = 28 units$^2$  
14. $V = 11$ units$^3$; S.A. = 34 units$^2$  
15. $V = 14$ units$^3$; S.A. = 44 units$^2$  
16. Sample: The foundation drawings; you can find the sum of the numbers on each square.
17. Sample: When there are no valleys in the cube arrangement, you can count the total number of squares used in all six views of an orthographic projection to find the surface area.
Chapter 1 Answers (continued)

**Enrichment 1-3**

1. No; they will be parallel. 2. No; they will be skew.
3. $S_1$ and $S_2$; $T$ and $B$; $S_3$ and $S_4$ 4. $S_3$ 5. $S_4$
6. line $l$ or $L_3$ 7. line $b$ 8. line $n$ or $L_2$
9. line $a$ 10. $S_2$ 11. 0 12. 0 13. $S_3$
18. Check students’ work. 19. Check students’ work.

**Enrichment 1-4**

1. north–south 2. east–west 3. east–west
4. north–south 5. always 6. never 7. never
8. always 9. never 10. 1000 ft 11. Sample: If one airplane is flying east–west and one is flying north–south, then one was out of its proper altitude range. Sample: If both are flying east–west at the same cruising altitude, then the air traffic controller instructed one of the airplanes incorrectly.
12. First Avenue, Second Avenue, or Third Avenue
13. Park Street, First Avenue, Second Avenue, or Grove Street 14. Second Avenue or First Avenue 15. No; the streets are parallel. 16. Carter Street, Oak Street, Hill Street, Pine Street, and Crosstown Road

**Enrichment 1-5**

1. 1 mi 2. 9 mi 3. 5 mi 4. ABDE, 12 mi; ABCFE, 14 mi 5. F; more shorter connections 6. I; only one long connection 7. link E with I; it is short and would increase the number of connections to I.

**Enrichment 1-6**

1. $-\frac{4}{2}$ 2. $\frac{1}{2}$ 3. $\frac{1}{2}$ 4. III 5. II 6. I
7. $4\frac{1}{2}$ units 8. $5\frac{1}{2}$ units 9. 1 unit

**Enrichment 1-7**

1.–6. 9. hexagon $AB'B'F'$ and quadrilateral $IC'D'E'$
10. hexagon $CDEE'D'C'$ and quadrilateral $A'F'IB'$

**Chapter Project**

**Activity 1: Paper Folding**
- Table: 4; 8; 16
- 32; 64; multiply by 2

**Activity 2: Creating**
Check students’ work.

**Activity 3: Writing**
Check students’ work.

**Activity 4: Researching**
Check students’ work.
**Checkpoint Quiz 1**

1. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$
2. 49, 56
3. G
4. Any three points are coplanar.
5. intersecting
6. skew
7. parallel

**Checkpoint Quiz 2**

1. 25
2. (3, 0)
3. B
4. 16 ft
5. $64\pi$ ft$^2$

**Chapter Test, Form A**

1. Add 5; 32, 37.
2. Multiply by $-\frac{1}{2}$, $-\frac{1}{16}$, $\frac{1}{32}$.
3. Add 4, then 5, then 6, and so on; 29, 37.
4. Add another side of a hexagon; check students’ work.
5. The relationship $(x - 1)(x + 1) = x^2 - 1$ is true for all whole numbers $x$. No counterexample exists.
6. Sample:

   ![Hexagon Diagram]

7. Sample:

   ![Diagram]

8. Possible answers: A, O, and B; E, O, and F; C, O, and D

9. E, G, O, and F
10. Sample: A, D, F, and B
11a. O
11b. $EF$
11c. O
11d. G
12. Yes; they are vertical angles.
13. No; there are no markings.
14. Yes; they are adjacent and $m\angle AOB + m\angle DOC = 180$.
15. No; you cannot conclude this from the diagram.
16a. 11
16b. 22
16c. 40
16d. 56
16e. 56
17. 34
18. 90
19. 146
20. 21
21. 112
22. 63°; acute
23. 56
24. 56
25. 34
26. 90°; right
27. 56

**Chapter Test, Form B**

1. Multiply by 3; 81, 243.
2. Add 11, add 9, add 11, add 9, and so on; 44, 55.
3. Insert one more dot; a circle with three dots, a circle with four dots.
4. The relationship $\frac{(x - 1) + (x + 1)}{2} = x$ is true for all whole numbers $x$. No counterexample exists.
5. Sample:

6. Sample:

   ![Diagram]

7. M, E, B, and D
8. C, M, and O or M, B, and E
9. Sample: A, F, O, and D
10a. M
10b. M
10c. F
10d. O
11. $\angle AOB$
12. $\angle DOE$
13. $\angle COD$
14. $\angle AOE$ and $\angle BOC$ or $\angle AOB$ and $\angle COE$
15a. 5
15b. 9
15c. 48
16. 25
17. 25
Chapter 1 Answers (continued)

18. 130
19. 180
20. 140°; obtuse
21. 65°; acute
22.
23.
24.
25.
26. 5
27. 17.3
28. (9, 18)
29. 58 in.; 168 in.²
30. 38 cm; 78 cm²
31. 8π in.; 16π in.²
32. 30π cm; 225π cm²

Alternative Assessment, Form C

TASK 1: Scoring Guide
Samples:
a. Exterior rays form a right angle for each figure. The total number of rays doubles with each successive figure. Two parallel segments connect the innermost rays.
b. Other answers are possible. For example, instead of considering the total number of rays (2, 4, 8, 16), consider the rays in the interior of the right angle: 0, 2, 6, . . . In that case, you can argue that 12 comes next, giving a total of 14 rays, not 16, in figure 4.
c. same as in part b, except with 32 rays (If considering the rays in the interior of the right angle, Figure 5 will have 20 rays.)

3 Student gives a correct explanation, clear descriptions, and an accurate drawing.

2 Student gives explanations, descriptions, and a drawing that are generally clear but may contain a few errors.
1 Student makes significant errors in explanations and descriptions.
0 Student makes little or no attempt.

TASK 2: Scoring Guide
a. Sample:

b. No; any three points are coplanar.
c. Sample: A, B, and C

3 Student gives an accurate diagram, correct answers, and a valid explanation.
2 Student draws a diagram that contains some errors but gives answers to parts b and c that are correct.
1 Student draws a diagram that contains significant errors and gives answers to parts b and c that are incorrect.
0 Student makes little or no attempt.

TASK 3: Scoring Guide
Sample:

3 Student constructs the figures correctly.
2 Student constructs the figures, but the constructions may contain some minor errors.
1 Student misses some key ideas, resulting in significant errors in the constructions.
0 Student makes little or no attempt.
Chapter 1 Answers (continued)

TASK 4: Scoring Guide

Foundation drawing:

Orthographic drawing:

<table>
<thead>
<tr>
<th>Front</th>
<th>Top</th>
<th>Right</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3  Student draws completely accurate figures.
2  Student draws generally accurate figures, but these may contain a minor error.
1  Student draws figures containing significant errors.
0  Student makes little or no attempt.

Cumulative Review

14. Sample: 1, 2, 3, 4, 5, 6, ... and 1, 2, 3, 1, 2, 3, 1, ... The first is the natural numbers. In the second, the numbers 1, 2, and 3 repeat.
15. Yes; the markings show they are congruent.
16. No; there are no markings.
17. Yes; you can conclude the angles are supplementary from the diagram.
18. sometimes
19. never
20. Not acceptable; too general; Sample: a ruler is a tool used for measuring lengths and is marked showing uniform units.

21. Sample:

22. Sample:

23. \((-4, -2)\)  24. \((0, -3)\)  25. \(\sqrt{29}\)
Chapter 2 Answers

Practice 2-1
1. Sample: It is 12:00 noon on a rainy day. 2. Sample: The car will not start because of a dead battery. 3. Sample: If you are strong, then you drink milk. 4. If a rectangle is a square, then it has four sides the same length. 5. If you are tired, then you did not sleep. 6. If you are tired, then you did not sleep. 7. If a number is an integer, then it is a whole number; true. 8. Drinking Sustain makes you train harder and run faster. 9. If you are strong, then you drink milk. 10. It is not freezing outside. 11. Shannon lives in the smallest state in the United States. 12. On Thursday, the track team warms up by jogging 2 miles.

Practice 2-2
1. Two angles have the same measure if and only if they are congruent. 2. \(2x - 5 = 11\) if and only if \(x = 8\). 3. The converse, “If \(|n| = 17\), then \(n = 17\)” is not true. 4. A figure has eight sides if and only if it is an octagon. 5. If a whole number is a multiple of 5, then its last digit is either 0 or 5. If a whole number has a last digit of 0 or 5, then it is a multiple of 5. 6. If two lines are perpendicular, then the lines form four right angles. If two lines form four right angles, then the lines are perpendicular. 7. If you live in Texas, then you live in the largest state in the contiguous United States. If you live in the largest state in the contiguous United States, then you live in Texas. 8. Sample: Other objects, such as spheres, fit this description. 9. Sample: Other objects, such as spheres, are round. 10. Sample: Baseball also fits this definition. 11. Sample: Pleasing, smooth, and rigid all are too vague.

Practice 2-3
1. \(\angle A\) and \(\angle B\) are complementary. 2. Football practice is canceled for Monday. 3. \(\triangle DEF\) is a right triangle. 4. If you liked the movie, then you enjoyed yourself. 5. If two lines are not parallel, then they intersect at a point. 6. If you vacation at the beach, then you like Florida. 7. not possible 8. Tamika lives in Nebraska.

Practice 2-4

Practice 2-5
1. 30 2. 15 3. 20 4. 6 5. 16 6. 9 7. \(m \angle A = 135\), \(m \angle B = 45\) 8. \(m \angle A = 36\), \(m \angle B = 144\) 9. \(m \angle A = 75\), \(m \angle B = 15\) 10. \(m \angle A = 10\), \(m \angle B = 80\) 11. \(m \angle PMO = 55\), \(m \angle PMQ = 125\), \(m \angle QMN = 55\) 12. \(m \angle BOD = m \angle COE = 90\); \(m \angle BOC = m \angle COD = 45\); \(m \angle AOB = m \angle DOE = 45\) 13. \(m \angle BWC = m \angle CWD\), \(m \angle AWB + m \angle BWC = 180\); \(m \angle CWD + m \angle DWA = 180\); \(m \angle AWB = m \angle AWD\)

Reteaching 2-1
1.–3. Check students’ work. 4. If you hear thunder, then you see lightning; statement: true; converse: false. 5. If you pants are jeans, then they are blue; statement: false; converse: true. 6. If you are eating a tangerine, then you are eating an orange fruit; statement: true; converse: false. 7. If a number is an integer, then it is a whole number; statement: true; converse: false. 8. If a triangle has one angle greater than 90°, then it is an obtuse triangle; statement: true; converse: true. 9. If \(n^2 = 64\), then \(n = 8\); statement: true; converse: false. 10. If you got an A for the quarter, then you got an A on the first test; statement: false; converse: false. 11. If a figure has four sides, then it is a square; statement: true; converse: false. 12. If \(x = 144\), then \(\sqrt{x} = 12\); statement: true; converse: true.

Reteaching 2-2
1. If \(n = 15\) or \(n = -15\), then \(|n| = 15\). If \(|n| = 15\), then \(n = 15\) or \(n = -15\). 2. If two segments are congruent, then they have the same measure. If two segments have the same measure, then they are congruent. 3. If you live in California, then you live in the most populated state in the United States. If you live in the most populated state in the United States, then you live in California. 4. If an integer is a multiple of 10, then its last digit is 0. If an integer’s last digit is 0, then it is a multiple of 10.
5. No; counterexamples may vary. Sample: A giraffe is a large animal.
6. Yes; two planes intersect if and only if they form a line. 7. Yes; a number is an even number if and only if it ends in 0, 2, 4, 6, or 8.
8. Yes; a triangle is a three-sided figure if and only if the angle measures sum to 180°.

**Reteaching 2-3**
1. Law of Detachment; the police officer will give Darlene a ticket.
2. Law of Syllogism; if two planes do not have any points in common, then they are parallel.
3. Law of Detachment; Landon has a broken arm.
4. Not possible; a conclusion cannot be drawn from a conditional and its confirmed conclusion. (Brad may live in Peoria, Illinois.)
5. Law of Detachment; the circumference is π.

**Reteaching 2-4**
1. Addition Property of Equality
2. Symmetric Property of Equality
3. Distributive Property
4. Subtraction Property of Equality
5. Addition Property of Equality
6. Multiplication Property of Equality
7. Substitution
8. Transitive Property of Equality
9. 4, 5, 1, 3, 2

**Reteaching 2-5**
1. Addition Property of Equality
2. Subtraction Property of Equality
3. Addition Property of Equality
4. Subtraction Property of Equality
5. Addition Property of Equality
6. Subtraction Property of Equality
7. Addition Property of Equality
8. Subtraction Property of Equality
9. Addition Property of Equality
10. Subtraction Property of Equality
11. Addition Property of Equality
12. Subtraction Property of Equality

**Enrichment 2-3**
1. p: one is a student at Richmond High School; q: one takes geometry class; r: one studies logic.
2. If someone is a student at Richmond High School, then he or she takes geometry class.
3. If someone takes geometry class, then he or she studies logic.
4. All students at Richmond High School study logic.
5. Logic can be studied in classes other than geometry, such as algebra or trigonometry.
6. a; p: a figure is a square; q: a figure is a polygon; r: a figure is closed; if p → q and q → r then p → r.
7. b; p: a figure is a square; q: a figure is a rectangle; r: a figure has four right angles; if p → q and q → r, then p → r.
8. All mathematicians enjoy puzzles.
9. Triangles have an angle sum of 180°.
10. Mt. McKinley is 20,320 ft tall.
11. Nolan Ryan recorded 5714 strikeouts over his career.
12. Trapezoids are four-sided figures.

**Enrichment 2-4**

**Enrichment 2-5**
1. They are complementary angles; if the exterior sides of two adjacent acute angles are perpendicular, then the angles are complementary.
2. ∠ECD
3. They are equal;
because $\angle ACB$ and $\angle DCB$ have equal measures, you can use the definition of supplementary angles and the subtraction property to show that they are equal. 4. 90 5. 135 6. 45 7. NO bisects TP; a segment that is perpendicular to another segment at its midpoint is the perpendicular bisector of the segment.

Chapter Project
Check students’ work.

✓ Checkpoint Quiz 1
1. hypothesis: $x + 4 = 10$; conclusion: $x = 6$
2. hypothesis: if you want to get good grades in school; conclusion: you must study hard 3. Sample: It could be May. 4. Sample: Corn is a vegetable. 5. No; the lines must be in the same plane.
6. yes 7. not possible 8. $X$, $Y$, and $Z$ are coplanar. 9. If you ran a good race, then your coach is happy. 10. If the car is old, then it is not efficient.

✓ Checkpoint Quiz 2
1. $4x = 13$ 2. $\angle TQM \cong \angle LTS$ 3. Symmetric Property 4. Substitution Property 5. Division Property 6. Addition Property 7. $\angle 1 \cong \angle 3$, $\angle 2 \cong \angle 4$; Vertical Angles Theorem 8. $\angle 2 \cong \angle 3$, $\angle 1 \cong \angle 4$; Linear Pair and Transitive Property of Equality 9. $\angle 1 \cong \angle 4$, $\angle 2 \cong \angle 3$; Subtraction Property of Equality 10. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$; Vertical Angles Theorem

Chapter Test, Form A
1a. If a polygon has three sides, then it is a triangle. 1b. true 2a. If George lives in the United States, then he lives in Texas. 2b. false 3a. If two angles are congruent, then they are vertical angles. 3b. false 4. Addition Property of Equality 5. Transitive Property of Equality 6. Subtraction Property of Equality 7. Division Property of Equality 8. Reflexive Property of Equality 9. $90.9 b. 58 9 c. 148 9 d. 90 9 e. 122 9 f. 148 10. 17 11. 6 12. 14 13. 11 14. not possible 15. We win. 16. If the bus is late, then we will receive a tardy penalty. 17. The sum of the measures of $\angle A$ and $\angle B$ is 90. 18. If a quadrilateral is a rectangle, then it has four right angles. If a quadrilateral has four right angles, then it is a rectangle. 19. A rhombus has four congruent sides. 20. good definition 21. good definition 22. A bat has wings. 23a. Distributive Property 23b. Addition Property of Equality 23c. Division Property of Equality 24. 25 25. 19 26. 17 27. Answers may vary. Sample: (1, 2) 28. Answers may vary. Sample: (3, $-4$)

Chapter Test, Form B
1a. If a polygon has five sides, then it is a pentagon. 1b. true 2a. If Mary lives in Minnesota, then she lives in Minneapolis. 2b. false 3. Reflexive Property of Equality 4. Substitution Property of Equality 5. Division Property of Equality 6. Distributive Property 7a. 38 7b. 128 7c. 26 7d. 154 8. 50 9. 13.5 10. 4 11. Martina is quick. 12. If I don’t wear sunscreen while swimming, then I’ll be in pain. 13. If a quadrilateral is a parallelogram, then it has two pairs of opposite sides parallel. If a quadrilateral has two pairs of opposite sides parallel, then it is a parallelogram. 14. A rectangle has four right angles. 15. good definition 16. An octopus has eight legs. 17a. Division Property of Equality 17b. Subtraction Property of Equality 17c. Division Property of Equality 18. 17 19. 28 20. Answers may vary. There are two possibilities. Samples: (4, $-2$) or ($-2$, 5)

Alternative Assessment, Form C

TASK 1: Scoring Guide
a. Sample: If it is raining, then the lawn is wet.
b. Sample: If the temperature is below 32°F, then the temperature is below freezing.

3 Student gives conditionals that meet the criteria and provides clear and accurate explanations of why the conditionals meet the criteria.
2 Student gives examples and explanations that are generally clear but may contain errors.
1 Student makes significant errors in examples or explanations.
0 Student makes little or no attempt.

TASK 2: Scoring Guide
Sample: The definition cannot be written as a biconditional. A school is a place where people are educated.

3 Student gives an explanation and a definition that are accurate, clear, and complete.
2 Student gives an explanation and a definition that are generally clear but may contain errors.
1 Student makes significant errors in explanation or definition.
0 Student makes little or no attempt.

TASK 3: Scoring Guide
a. Sample: If two angles have sides that are opposite rays, then they are vertical angles. $\angle CEF$ and $\angle GED$ are angles whose sides are opposite rays. Therefore, $\angle CEF$ and $\angle GED$ are vertical angles.
b. Sample: $\angle CEF$ and $\angle GED$ are vertical angles. All vertical angles are congruent. Therefore, $\angle CEF \equiv \angle GED$.

3 Student gives accurate and complete examples and explanations of the Laws of Detachment and Syllogism.
2 Student gives examples and explanations that are generally correct but may be unclear.
1 Student gives examples or explanations that contain major errors.
0 Student makes little or no attempt.
Chapter 2 Answers (continued)

TASK 4: Scoring Guide

Sample:

a. $\angle TPR$ and $\angle RPV$; $\angle RPV$ and $\angle VPS$; $\angle VPS$ and $\angle SPT$; $\angle SPT$ and $\angle TPR$; $\angle RPT$ and $\angle VPS$; $\angle TPS$ and $\angle RPV$

b. $\angle RPT$ and $\angle VPS$; $\angle TPS$ and $\angle RPV$

c. $\angle RPT$ is a right angle because the lines are perpendicular, so $m\angle RPT = 90$. Therefore, if ray $PY$ bisects $\angle RPT$, $m\angle YPR = 45$.

3 Student draws a diagram and gives responses that indicate a thorough understanding of the definitions involved. Student shows logic in part c that is complete and accurate.

2 Student draws a diagram and gives responses that, while basically accurate, contain some minor flaws, inaccuracies, or omissions.

1 Student draws a diagram or gives responses that contain significant inaccuracies or omissions. Student displays flaws in logical argument in part c.

0 Student makes little or no attempt.

Cumulative Review


18. $BC$ 19. Substitution Property of Equality 20. $(-1, 2)$ 21. $(-2, 2)$ 22. $\sqrt{145}$ 23. 4 24. $60$ m$^2$ 25. $380.13$ cm$^2$ 26. Sample: A midpoint must be on the segment. Any point that lies on the perpendicular bisector of a segment is equidistant from the endpoints.
Chapter 3 Answers

Practice 3-1
1. corresponding angles 2. alternate interior angles
3. same-side interior angles 4. alternate interior angles
5. same-side interior angles 6. corresponding angles
7. \( \angle 1 \) and \( \angle 5 \), \( \angle 2 \) and \( \angle 6 \), \( \angle 3 \) and \( \angle 8 \), \( \angle 4 \) and \( \angle 7 \)
8. \( \angle 4 \) and \( \angle 6 \), \( \angle 3 \) and \( \angle 5 \)
9. \( \angle 4 \) and \( \angle 5 \), \( \angle 3 \) and \( \angle 6 \)
10. \( m \angle 1 = 100 \), alternate interior angles; \( m \angle 2 = 100 \), corresponding angles or vertical angles
11. \( m \angle 1 = 75 \), alternate interior angles; \( m \angle 2 = 75 \), vertical angles or corresponding angles
12. \( m \angle 1 = 135 \), corresponding angles
13. \( x = 103^\circ \), \( 103^\circ \)
14. \( x = 24; 12^\circ, 168^\circ \)
15. \( x = 30; 85^\circ, 85^\circ \)
16a. Alternate Interior Angles Theorem 16b. Vertical angles are congruent
16c. Transitive Property of Congruence

Practice 3-2
1a. same-side interior 1b. \( \overline{QR} \) 1c. \( \overline{TS} \)
1d. same-side interior 1e. Same-Side Interior Angles
1f. \( \overline{TS} \) 1g. 3-5 2. \( l \) and \( m \), Converse of Same-Side Interior Angles Theorem
3. none 4. \( \overline{BC} \) and \( \overline{AD} \),
Converse of Same-Side Interior Angles Theorem
5. \( \overline{RT} \) and \( \overline{HU} \), Converse of Corresponding Angles Postulate
6. \( \overline{BH} \) and \( \overline{CT} \), Converse of Corresponding Angles Postulate
7. \( a \) and \( b \), Converse of Same-Side Interior Angles Theorem
8. 43 9. 90 10. 38 11. 100 12. 70 13. 48

Practice 3-3
1. True. Every avenue will be parallel to Founders Avenue, and therefore every avenue will be perpendicular to any street that is parallel to Center City Boulevard.
2. Not necessarily true. No information has been given about the spacing of the streets.
3. True. The fact that one intersection is perpendicular, plus the fact that every street belongs to one of two groups of parallel streets, is enough to guarantee that all intersections are perpendicular.
4. True. Opposite sides of each block must be of the same type (avenue or boulevard), and adjacent sides must be of opposite type.
5. Not necessarily true. If there are more than three avenues and more than three boulevards, there will be some blocks bordered by neither Center City Boulevard nor Founders Avenue.
6. \( a \perp e \) 7. \( a \parallel e \) 8. \( a \parallel e \)
9. \( a \perp e \)
10. \( a \perp e \) 11. \( a \parallel e \) 12. If the number of \( \perp \) statements is even, then \( \ell_1 \parallel \ell_2 \).
If it is odd, then \( \ell_1 \perp \ell_2 \).
13. The proof can instead use alternate interior angles or alternate exterior angles (if the angles are congruent, the lines are parallel) or same-side interior or same-side exterior angles (if the angles are supplementary, the lines are parallel).
14. It is possible.

Practice 3-4
1. 125 2. 69 3. 143 4. 129 5. 140 6. 136
7. \( x = 35; y = 145; z = 25 \)
8. \( a = 55; b = 97; c = 83 \)
9. \( v = 118; w = 37; t = 62 \)
10. 50 11. 88
12. \( m\angle 3 = 22; m\angle 4 = 22; m\angle 5 = 88 \)
13. 136
14. 136
15. \( m\angle 1 = 33; m\angle 2 = 52 \)
16. isosceles
17. obtuse scalene 18. right scalene
19. obtuse
20. equiangular

Practice 3-5
1. \( x = 120; y = 60 \)
2. \( n = 51^\frac{1}{2} \)
3. \( a = 108; b = 72 \)
10. 30 11. 150 12. 6 13. 5 14. 8
15. \( \angle BEDC \) 16. \( \angle FAE \)
17. \( \angle FAE \) and \( \angle BAE \)
18. \( \angle ABCDE \)

Practice 3-6
1. \( y = \frac{1}{2}x - 7 \)
2. \( y = -2x + 12 \)
3. \( y = 7x - 18 \)
4. \( y = -\frac{1}{2}x - 3 \)
5. \( y = \frac{1}{3}x - \frac{3}{2} \)
6. \( y = \frac{4}{5}x - 2 \)
7. \( y = 4x - 13 \)
8. \( y = -x + 6 \)
9. \( y = -\frac{3}{4}x + 5 \)
10. \( y = \frac{1}{3}x - 1 \)
11. \( y = \frac{2}{3}x + 2 \)
12. \( y = \frac{2}{3}x - 2 \)
13. \( y = 2x - 7 \)
14. \( y = 2x - 3 \)
15. \( y = -\frac{3}{4}x + 5 \)
16. \( y = \frac{1}{3}x - 1 \)
Chapter 3 Answers (continued)

17. \[ y = 5x + 4 \]
18. \[ y = \frac{1}{2}x - 3 \]
19. \[ x = -2 \]
20. \[ y = -2x \]
21. \[ y = -5 \]
22. \[ y = x \]
23. \[ y = \frac{2}{3}x + 2 \]
24. \[ x = 2.5 \]
25. \[ y = -3x + 13 \]
26. \[ y = x + 4 \]
27. \[ y = \frac{1}{2}x - 3 \]
28. \[ y = -\frac{1}{2}x - \frac{1}{2} \]
29. \[ y = 2x + 4 \]
30. \[ y = \frac{1}{2}x + 4 \]
31. \[ y = -\frac{1}{3}x - \frac{2}{3} \]
32. \[ y = -6x + 45 \]
33. \[ x = 2; \]
34. \[ x = 0; y = 2 \]
35. \[ x = -4; y = -4 \]
36. \[ x = -1; y = 8 \]
37. \[ (4, 0) \]
38. \[ (-2, 0) \]
39. \[ (0, 6) \]
40. \[ (\frac{1}{2}, 0) \]
41. \[ (0, 4) \]
42. \[ (0, -4) \]
43. \[ (0, 12) \]
44. \[ (9, 0) \]
45. \[ (4, 0) \]
46. \[ (0, 12) \]

45a. \( m = \$0.10 \)  
45b. The amount of money the worker is paid for each box loaded onto the truck  
45c. \( b = \$3.90 \)  
45d. The base amount the worker is paid per hour  
46. \( y = -\frac{3}{2}x + 8 \)

Practice 3-7
1. neither; \( 3 \neq \frac{1}{3}, 3 \cdot \frac{1}{3} \neq -1 \)  
2. perpendicular; \( \frac{1}{2} \cdot -2 = -1 \)  
3. parallel; \( -\frac{2}{3} = -\frac{2}{3} \)  
4. parallel; \( -1 = -1 \)  
5. perpendicular; \( y = 2 \) is a horizontal line, \( x = 0 \) is a vertical line  
6. parallel; \( \frac{1}{2} = -\frac{1}{2} \)  
7. neither; \( 1 \neq \frac{1}{8}, 1 \cdot \frac{1}{8} \neq -1 \)  
8. parallel; \( -\frac{2}{3} = -\frac{2}{3} \)  
9. perpendicular; \( -1 \cdot 1 = -1 \)  
10. neither; \( \frac{1}{2} \neq -\frac{1}{2}, \frac{1}{2} \cdot -\frac{1}{2} \neq -1 \)  
11. neither; \( -\frac{5}{3} \neq -\frac{5}{3}, -\frac{5}{3} \cdot -\frac{5}{3} \neq -1 \)  
12. neither; \( 6 \neq -\frac{1}{5}, 6 \cdot -\frac{1}{5} \neq -1 \)  
13. neither; \( \frac{2}{5} \neq 4, \frac{2}{5} \cdot 4 \neq -1 \)  
14. parallel; \( \frac{1}{2} = \frac{1}{2} \)  
15. \( y = \frac{2}{3}x \)
16. \( y = -\frac{4}{3}x + 24 \)
17. \( y = -x - 3 \)
18. \( y = \frac{3}{2}x + 6 \)
19. \( y = 0 \)
20. \( y = 2x - 4 \)
21. \( y = 2x \)

Practice 3-8
1–3.
4–6.
Chapter 3 Answers (continued)

7.–9.

10. Sample:

11. Sample:

12.

13.

14. Sample:

15.

Reteaching 3-1
1a.–1b. Sample:

1c.–1d. Sample:

Reteaching 3-2
1. If \( l \parallel m \), then corresponding \( \angle s \) are \( \cong \).

Substitution

Definition of supplementary

\[ m\angle 3 = m\angle 2 = 180 \]

Angle Addition Postulate

2. 110  3. 70  4. 110  5. 110  6. 70  7. 70  8. 110
Chapter 3 Answers (continued)

2. \( \angle 2 \cong \angle 3 \)
   \( \angle 3 \cong \angle 1 \)

   Given

   Substitution  If \( \cong \) corresponding \( \angle \)s, then lines are parallel.

   Vertical \( \angle \)s are \( \cong \).

Reteaching 3-3

1. Draw a temporary line through \( A \) that is perpendicular to \( \ell \), then draw a second, permanent line through \( A \) that is perpendicular to the first. This line will be parallel to \( \ell \). Repeat for \( B \). The two permanent lines will be parallel not only to \( \ell \) but to each other (Theorem 3-9).

2. Draw two perpendicular lines through \( C \), then for each one draw a line perpendicular to it that runs through \( D \). The four lines will define a rectangle.

3. Draw a line that runs through both \( E \) and \( F \), and then a second line at an acute or obtuse angle that runs through \( E \). Draw a third line, parallel to the second and running through \( F \), using the same procedure as in the Example and in Exercise 1. In the same way, draw a fourth line parallel to the first (at any desired distance from it). The four lines will define a parallelogram.

Reteaching 3-4

1. \( \triangle ABD: m\angle ABD = 120, m\angle ADB = 30; \triangle CBE: m\angle CBE = 120, m\angle CEB = 30, m\angle BCE = 30; \triangle BDE: m\angle BDE = 60, m\angle DBE = 60, m\angle BED = 60 \)
2. \( \triangle DBE \) and \( \triangle ABC \) are acute, equiangular, and equilateral; \( \triangle ABD \) and \( \triangle CBE \) are isosceles and obtuse; \( \triangle ACE, \triangle ADE, \triangle CED, \) and \( \triangle CAD \) are right and scalene.
3. \( \triangle PQT; m\angle PQT = 45, m\angle PQT = 90; \triangle PQR: m\angle PQR = 90, m\angle QPR = 45, m\angle QRP = 45; \triangle RQS: m\angle RQS = 90, m\angle QSR = 45, m\angle STQ = 90, m\angle QST = 90 \)
4. \( PT = TS = RS = FR = 40 \) mm; \( PQ = QT = QR = QS = 28 \) mm
5. \( \triangle PQT, \triangle PQR, \triangle RQS, \triangle STQ, \triangle SQT, \triangle PTS, \triangle PRT, \) and \( \triangle RST \) are right and isosceles.

Reteaching 3-5

1. \( \angle 1 \) and \( \angle 2 \) are interior angles; \( \angle 3 \) and \( \angle 4 \) are exterior angles.
2. \( m\angle 1 = 135; m\angle 2 = 90; m\angle 3 = 45; m\angle 4 = 90 \)
3. \( \angle 1 \) is an interior angle; \( \angle 2 \) and \( \angle 4 \) are exterior angles; \( \angle 3 \) is neither.
4. \( m\angle 1 = 60; m\angle 2 = 120; m\angle 3 = 60; m\angle 4 = 120 \)

Reteaching 3-6

Check students’ graphs.

1. \( y = 2x - 6 \)
2. \( y = \frac{1}{3}x \)
3. \( y = -x - 3 \)
4. \( y = \frac{5}{6}x + 2 \)
5. \( y = -\frac{1}{2}x + 1 \)
6. \( y = 1 \)
7. \( y = -\frac{7}{2}x + 10 \)
8. \( y = -x + 1 \)
9. \( y = \frac{3}{2}x + 1 \)
10. \( y = 1 \)
11. \( y = -2x - 6 \)
12. \( x = -3 \)
13. \( y = -3x + 10 \)
14. \( y = 3x - 10 \)

15. \( y = \frac{1}{4}x + \frac{1}{2} \)
16. \( y = -\frac{3}{4}x + 4 \)
17. \( y = -x + 1 \)
18. \( y = 1 \)

Reteaching 3-7

1a. \( -2 \)
1b. \( \frac{1}{3} \)
2a. \( \frac{1}{2} \)
2b. \( -4 \)
3a. undefined
3b. \( 0 \)
4a. \( y = -2x + 4 \)
4b. \( y = \frac{1}{2}x + \frac{1}{2} \)
4c.

5a. \( y = -\frac{2}{3}x - 4 \)
5b. \( y = \frac{3}{2}x - 4 \)

6a. \( y = 3x + 7 \)
6b. \( y = -\frac{3}{2}x - 3 \)

7. \( m_{JK} = -1; m_{LM} = -1 \); parallel
8. \( m_{JK} = \frac{2}{3}; m_{LM} = \frac{2}{3} \); perpendicular
9. \( m_{JK} = -\frac{1}{3}; m_{LM} = \frac{1}{3} \); neither
10. \( m_{JK} = -\frac{3}{2}; m_{LM} = \frac{3}{2} \); neither
11. \( m_{JK} = 2; m_{LM} = \frac{1}{2} \); perpendicular
12. \( m_{JK} = \frac{7}{3}; m_{LM} = \frac{7}{3} \); neither
13. \( m_{JK} = \frac{2}{3}; m_{LM} = \frac{1}{3} \); parallel
14. \( m_{JK} \) undefined; \( m_{LM} = 0 \); perpendicular

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Chapter 3 Answers (continued)

Reteaching 3-8
1. Sample:

2–4. Check students’ work.

Enrichment 3-1
1. \(OE\) is \(\perp\) to \(AB\). 2. 3, 5, 1, 4, 2, 6 or 4, 5, 1, 2, 3; \(OE\) is \(\perp\) to \(AB\); if two angles are congruent and supplementary, then each measures 90°. 3. \(\angle 1 \equiv \angle 2\); Law of Reflection 4. \(\angle 2 \equiv \angle 3\); Alternate Interior Angles Theorem 5. \(\angle 3 \equiv \angle 4\); Law of Reflection 6. \(\angle 1 \equiv \angle 4\); Transitive Property of Congruence

Enrichment 3-2
1. \(x = 11\) 2. 106 3. 33 4. 41 5. Sample: Because \(m \angle BAC = 41, m \angle CAF = 180 - 41 = 139\); \(l \parallel m\) because a pair of alternate interior angles are congruent. 6. \(\angle D\) and \(\angle E\); \(\overline{AK}\) 7. Sample: \(\angle CAB\) and \(\angle IGH\) are corresponding angles related to parallel segments \(\overline{AB}\) and \(\overline{GH}\). \(\overline{AK}\) is the related transversal. 8. Sample: \(\angle A, \angle ADE,\) and \(\angle AED\) form a triangle, so \(180 - (43 + 76) = 61\), so \(m \angle AED = 61\). Because \(\angle AED\) and \(\angle C\) are congruent corresponding angles, \(\overline{DE} \parallel \overline{BC}\) by the Converse of the Corresponding Angles Postulate.

Enrichment 3-3
1. If the paper shifts during drawing, lines meant to be parallel will not be parallel, and lines meant to be perpendicular will not be perpendicular. 2. The rectangular shape gives the T-square two perpendicular guide edges on which to line up. 3. Theorem 3-10, which says that if two lines are perpendicular to the same line, they are parallel to each other. A line drawn with the T-square is perpendicular to the edge on which the T-square is lined up. Two lines drawn with the T-square lined up on the same edge will be parallel to each other. 4. \(a \perp d\). If the T-square is used to draw a line perpendicular to a vertical edge of the rectangular backing surface, and then is used to draw a line perpendicular to a horizontal edge of the backing surface, then the two lines will be perpendicular. 5. Figure A would be easy; all the lines are parallel or perpendicular. Figures B and C would be hard to draw, since the lines are not all parallel or perpendicular to each other. 6. They will all be parallel to \(\ell\). 7. A drafting machine can be used to draw slanted lines, not just vertical and horizontal ones, with precise control of the slant. 8. Figures A and B would be easy, because the lines have no more than two or three different slants. Figure C would be harder, because lots of different slants are used to achieve the perspective effect.

Enrichment 3-4
1. 48 2. 2880 3. 4320 4. Angles have measures of 20, 70, or 90; \(\overline{AC} \parallel \overline{MD} \parallel \overline{LE} \parallel \overline{KF} \parallel \overline{IG}; \overline{BH} \parallel \overline{CH} \parallel \overline{NK}; \overline{AH} \parallel \overline{PG} \parallel \overline{CM} \parallel \overline{DL} \parallel \overline{EK} \parallel \overline{TF} \parallel \overline{IG}.

Enrichment 3-5
1. 2 2. 5 3. 9 4. 14 5. 20 6. 27

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<tr>
<td>4</td>
<td>360</td>
<td>2</td>
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<tr>
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<tr>
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</tbody>
</table>

Enrichment 3-6
1–8.

Enrichment 3-7
1. \((-3, -4)\) 2. \((-2, -3)\) 3. \((3, 3)\) 4. \((-2, -2)\) 5. \((-1, -1)\) 6. \((4, -1)\) 7. \((1, 0)\) 8. \((2, -3)\) 9. \((-1, 2)\) 10. \((1, -4)\) 11. \((-3, -3)\) 12. \((2, 3)\) 13. \((3, -2)\) 14. \((5, 0)\) 15. \((0, 1)\) 16. \((4, 3)\)

RENE DESCARTES

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<td>x-axis</td>
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Chapter 3 Answers (continued)

Enrichment 3-8
1., 3., and 4.

2. scalene, acute triangle
5. No; refer to the answer to Exercises 1, 3, and 4.
6.

7. isosceles, obtuse triangle

Chapter Project
Activity 1: Paper Folding
5; all triangles are right isosceles; yes.

Activity 2: Exploring
triangle
quadrilaterals

pentagons
hexagons

Activity 3: Analyzing

activity 4: modeling

✓ Checkpoint Quiz 1
1. Converse of Corresponding Angles Postulate
2. Alternate Interior Angle Theorem
3. Same-Side Interior Angles Theorem
4. Corresponding Angles Postulate
5. Converse of Alternate Interior Angle Theorem
6. Vertical Angles Theorem
7. Converse of Corresponding Angles Postulate
8. Corresponding Angles Postulate
9. Converse of Same-Side Interior Angles Theorem
10. \( x = 50, y = 30, z = 65 \)

✓ Checkpoint Quiz 2
1. pentagon; 80
2. hexagon; \( x = 110; y = 98 \)
3. quadrilateral; \( x = 94; y = 105 \)
4.
5.
Chapter Test, Form A

1. true  
2. true  
3. false  
4. false  
5. true  
6. false  
7. true  
8. true  
9. Answers may vary. Sample: $m\angle 1 = 125$, Same-Side Interior Angles Theorem; $m\angle 2 = 55$, Alternate Interior Angles Theorem  
10. Answers may vary. Sample: $m\angle 1 = 60$, Corresponding Angles Postulate then Angle Addition Postulate; $m\angle 2 = 60$, Same-Side Interior Angles Theorem  
11. Answers may vary. Sample: $m\angle 1 = 85$, Alternate Interior Angles Theorem; $m\angle 2 = 95$, Same-Side Interior Angles Theorem  
12. Answers may vary. Sample: $m\angle 1 = 75$, Corresponding Angles Postulate; $m\angle 2 = 105$, Angle Addition Postulate  
13. Answers may vary. Sample: $m\angle 1 = 91$, Corresponding Angles Postulate and Same-Side Interior Angles Theorem; $m\angle 2 = 89$, Corresponding Angles Postulate  
14. Answers may vary. Sample: $m\angle 1 = 60$, Alternate Interior Angles Theorem; $m\angle 2 = 115$, Same-Side Interior Angles Theorem  
15.

16.

17. $WA$ and $XB$  
18. none  
19. $WZ$ and $AB$  
20. none  
21. $WZ$ and $AB$  
22. $WZ$ and $AB$; $AX$ and $BY$  
23. $x = 22; y = 120$  
24. $x = 70; y = 60$; $z = 120$  
25. $x = 35; y = 35; z = 55$  
26. $62; 62; 62$  
27. $18$  
28. perpendicular  
29. neither  
30. parallel  
31. $y = 6x + 23$  
32. $y = -\frac{1}{2}x + 3$  
33. $y = \frac{1}{2}x + 2$  

Chapter Test, Form B

1. true  
2. true  
3. false  
4. true  
5. false  
6. $m\angle 1 = 62, m\angle 2 = 62$, alternate interior angles  
7. $m\angle 1 = 85, m\angle 2 = 95$, same-side interior angles  
8. $m\angle 1 = 75, m\angle 2 = 75$, alternate interior angles  
9.  
10. $AD$ and $WZ$  
11. $AD$ and $WZ$  
12. $AD$ and $WZ$  
13. $AD$ and $WZ$  
14. $AW$ and $DZ$  
15. $x = 92, y = 88$  
16. $w = 31, x = 65, y = 65, z = 115$  
17. $1440$  
18. $45$  
19. neither  
20. perpendicular  
21. parallel  
22. $y = 3x - 1$  
23. $y = \frac{1}{3}x - 3$  
24. $y = -2x - 2$  

Alternative Assessment, Form C

TASK 1: Scoring Guide

a.

b. Student makes little or no attempt.  
2. Student draws a figure or gives answers that contain minor errors.  
3. Student makes little or no attempt.  
4. Student draws a figure or gives answers that contain significant errors or omissions.  
5. Student draws a figure or gives answers that contain a complete and accurate flow proof.

Answers
Chapter 3 Answers (continued)

TASK 2: Scoring Guide
Sample:

3 Student constructs an accurate figure.
2 Student constructs a figure that contains minor errors or omissions.
1 Student constructs a figure that contains significant errors or omissions.
0 Student makes little or no attempt.

TASK 3: Scoring Guide
Sample:

a. For the figure given, $y = x$ and $y = x + 3$. The lines are parallel because both lines have a slope of 1.

b. For $\triangle ABC$, the sum of the three angles is $\approx 180^\circ$. Minor discrepancies are the result of measurement error and rounding error.

c. For the figure given, $ABCD$ is not regular. By the distance formula, the sides are not congruent.

3 Student draws the figure accurately, writes correct equations, and reasons logically.
2 Student draws a figure, gives arguments, and writes equations that are mainly correct but may contain minor errors.
1 Student presents work with significant errors.
0 Student makes little or no attempt.

TASK 4: Scoring Guide
Sample:

3 Student draws an accurate diagram.
2 Student draws a diagram that contains minor errors or omissions.
1 Student draws a diagram that contains significant errors or omissions.
0 Student makes little or no attempt.

Cumulative Review
Chapter 4 Answers

Practice 4-1
1. $m\angle 1 = 110^\circ; m\angle 2 = 120^\circ$  2. $m\angle 3 = 90^\circ; m\angle 4 = 135^\circ$  3. $m\angle 5 = 140^\circ; m\angle 6 = 90^\circ; m\angle 7 = 40^\circ; m\angle 8 = 90^\circ$
4. $\triangle A \cong \triangle F$, $\angle A \cong \angle S$, $\angle T \cong \angle D$  5. $\angle C \cong \angle I$, $\angle A \cong \angle S$, $\angle T \cong \angle D$  6. $\angle W \cong \angle J$, $\angle M \cong \angle K$, $\angle X \cong \angle L$, $\angle Y \cong \angle M$  7. $\angle W \cong \angle J$, $\angle X \cong \angle K$, $\angle Y \cong \angle L$, $\angle Z \cong \angle M$  8. Yes; $\angle GHJ \cong \angle IHJ$ by Theorem 4-1 and by the Reflexive Property of $\cong$. Therefore, $\triangle GHI \cong \triangle IHJ$ by the definition of $\cong$ triangles.  9. No; $\angle QSR \cong \angle TS$ because vertical angles are congruent, and $\angle QRS \cong \angle TVS$ by Theorem 4-1, but none of the sides are necessarily congruent.  10a. Given  10b. Vertical angles are $\cong$.  10c. Theorem 4-1  10d. Given  10e. Definition of $\cong$ triangles

Practice 4-2
1. $\triangle ADB \cong \triangle CDB$ by SAS  2. not possible  3. not possible  4. $\triangle TUS \cong \triangle XWV$ by SSS  5. not possible  6. $\triangle DEC \cong \triangle GHF$ by SAS  7. $\triangle MKL \cong \triangle KMP$ by SAS  8. $\triangle PRN \cong \triangle PRQ$ by SSS  9. not possible  10. $\angle C$  11. $\overline{AB}$ and $\overline{BC}$  12. $\angle A$ and $\angle B$  13. $\angle AC \cong \angle DB$  14a. Given  14b. Reflexive Property of Congruence  14c. SAS Postulate

Practice 4-3
1. not possible  2. ASA Postulate  3. AAS Theorem  4. ASA Postulate  5. not possible  6. not possible
7. ASA Postulate  8. not possible  9. AAS Theorem

Practice 4-4
1. $\angle A$  2. not possible  3. not possible  4. $\triangle ADB \cong \triangle CDB$ by SAS, and $\angle A \cong \angle C$ by CPCTC  5. $\triangle FHE \cong \triangle HFG$ by ASA, and $\angle HE \cong \angle FG$ by CPCTC

Practice 4-5
1. $x = 35; y = 35$  2. $x = 80; y = 90$  3. $t = 150$
4. $x = 45; y = 45$  5. $x = 55; y = 70; z = 125$
6. $a = 132; b = 36; c = 60$  7. $x = 6$
8. $a = 30; b = 30; c = 75$  9. $z = 120$
10. $\angle ADB \cong \angle D$  11. $\angle GAC \cong \angle AGC$
12. $\angle KLM \cong \angle KMP$  13. $\angle DCE \cong \angle CED$
14. $\angle BAC \cong \angle BAC$
15. $\angle BCD \cong \angle BCD$
16. $130$  17. $65$  18. $130$  19. $90$
20. $x = 70; y = 55$  21. $x = 70; y = 20$
22. $x = 45; y = 45$

Practice 4-6
1. $\angle A \perp \angle B$, $\angle D \perp \angle E$  2. $\angle A \perp \angle B$, $\angle D \perp \angle E$
3. $\angle A \perp \angle B$, $\angle D \perp \angle E$  4. $\angle A \perp \angle B$, $\angle D \perp \angle E$
5. $\triangle MN \cong \triangle MKL$  6. $\angle A \perp \angle B$, $\angle D \perp \angle E$
7. $\angle A \perp \angle B$, $\angle D \perp \angle E$
8. $\triangle PQS \cong \triangle RQS$
9. $\triangle MN \cong \triangle MKL$
10. $\angle A \perp \angle B$, $\angle D \perp \angle E$  11. $\angle A \perp \angle B$, $\angle D \perp \angle E$
12. $\angle A \perp \angle B$, $\angle D \perp \angle E$  13. $\angle A \perp \angle B$, $\angle D \perp \angle E$
14. $\angle A \perp \angle B$, $\angle D \perp \angle E$
15. $\angle A \perp \angle B$, $\angle D \perp \angle E$
16. $\triangle MN \cong \triangle MKL$
17. $\angle A \perp \angle B$, $\angle D \perp \angle E$
18. $\angle A \perp \angle B$, $\angle D \perp \angle E$
19. $\angle A \perp \angle B$, $\angle D \perp \angle E$
20. $\angle A \perp \angle B$, $\angle D \perp \angle E$
21. $\angle A \perp \angle B$, $\angle D \perp \angle E$
22. $\angle A \perp \angle B$, $\angle D \perp \angle E$
Chapter 4 Answers (continued)

4. \[ GI \cong TI \]
   Given
   Reflexive Property of \(\cong\)
   \[ \triangle IHG \cong \triangle JHI \]
   HL Theorem

\[ \angle GHI \cong \angle JHI \]
Given
\[ \angle GHI \text{ and } \angle JHI \text{ are right } \angle s. \]

\[ m\angle GHI + m\angle JHI = 180 \]
Angle Addition Postulate

5. \( RS \cong VW \)
6. none
7. \( m\angle C \text{ and } m\angle F = 90 \)
8. \( GH \cong JH \)
9. \( LN \cong PR \)
10. \( ST \cong UV \text{ or } SV \cong UT \)
11. \( m\angle A \text{ and } m\angle X = 90 \)
12. \( m\angle F \text{ and } m\angle D = 90 \)
13. \( GI \perp JH \)

Practice 4-7
1. \( \triangle ZWX \cong \triangle YXW; \text{ SAS} \)
2. \( \triangle ABC \cong \triangle DCB; \text{ ASA} \)
3. \( \triangle EJG \cong \triangle FKH; \text{ ASA} \)
4. \( \triangle LNP \cong \triangle LMO; \text{ SAS} \)
5. \( \triangle ADF \cong \triangle BGE; \text{ SAS} \)
6. \( \triangle UVY \cong \triangle VUX; \text{ ASA} \)
7. \( \triangle UVY \cong \triangle VUX; \text{ ASA} \)
8. \( \triangle UVY \cong \triangle VUX; \text{ ASA} \)
9. \( \text{common side: } BC \)
10. \( \text{common side: } FG \)
11. \( \text{common angle: } \angle L \)

Statements
1. \( AX \cong \overline{AY} \)
   Given
2. \( CX \perp AB; \text{ or } AB \perp AC \)
   \( m\angle BYA = 90 \)
3. \( \angle A \cong \angle A \)
   Reflexive Property of \(\cong\)
4. \( \triangle BYA \cong \triangle CXA \)
   ASA Postulate

11. Sample: Because \( FG \parallel GE, \angle HFG \parallel \angle EGF \), and \( FG \parallel GE \), then \( \triangle FGE \cong \triangle GHF \) by SAS. Thus, \( FE \parallel GH \) by CPCTC and \( EH \parallel HE \), then \( \triangle GEH \cong \triangle HFE \) by SSS.

Reteaching 4-1
1. b 2. c 3. a 4. 117  5. 119

Reteaching 4-2
1. – 2. Check students’ work.
3. \( \triangle AEB \cong \triangle CDB \)
4. \( \triangle MNQ \cong \triangle ONP \)
5. \( \triangle PRQ \cong \triangle VUT \)
6. \( \triangle JMK \cong \triangle LMK \)
7. \( \triangle QSP \cong \triangle QSR \)
8. \( \triangle YTX \cong \triangle WXT \)

Reteaching 4-3
1. \( \triangle \)
2. \( \triangle \)
3. Check students’ work.
4. \( \angle ABD \cong \angle CBD \)
5. \( \triangle JMK \cong \triangle LKM \text{ or } \triangle LMK \cong \triangle JMK \)
6. \( \triangle UZ \cong \triangleYZ \)
7. \( \triangle DY \cong \triangle DO \)
8. \( \angle P \cong \angle A \)
9. \( \angle CYL \cong \angle ALY \)

Reteaching 4-4
1a. \( \triangle QK \cong \triangle OA; \overline{OQ} \text{ bisects } \angle KQA \)
1b. definition of bisector
1c. \( \overline{BQ} \cong \overline{BQ} \)
1d. SAS Postulate
1e. CPCTC
2. \( \triangle MN \cong \triangle MP, \text{ ASA Postulate} \)
2. \( \overline{NO} \cong \overline{PO} \)
3. \( \overline{MO} \cong \overline{MO} \)
3. \( \triangle MPO \cong \triangle MNO \)
4. \( \angle N \cong \angle P \)
4. \( \angle N \cong \angle P \)
5. \( \angle JON \cong \angle JON \)
5. \( \triangle JON \cong \triangle JON \)
6. \( \triangle JN \cong \triangle JN \)
6. \( \triangle JN \cong \triangle JN \)

Reteaching 4-5
1. Each angle is 60°.
2. 120
3. 120
4. 50
5. 70
6. 60
7. 65
8. 115
9. 55
10. 120
11. 60

Reteaching 4-6
1. Sample: \( RS = 1.3 \text{ cm}, \overline{ST} = 1.6 \text{ cm}, \overline{QT} = 2.5 \text{ cm}, \overline{QR} = 2.3 \text{ cm}; \) not congruent
2. Sample: \( NT = 2.3 \text{ cm}, \overline{TG} = 2.3 \text{ cm}, \overline{AT} = 1.9 \text{ cm}; \angle NAT \cong \angle GAT \)
3. Sample: \( TO = 3.3 \text{ cm}, \overline{TR} = 2.8 \text{ cm}, \overline{MO} = 2.8 \text{ cm}; \angle TOM \cong \angle OTR \)
4. HL Theorem can be applied; \( \triangle BDA \cong \triangle CAD \)
5. HL Theorem cannot be applied.
6. HL Theorem can be applied; \( \triangle MUN \cong \triangle MLN \)
7. HL Theorem can be applied; \( \triangle THF \cong \triangle FET \text{ or } \triangle THF \cong \triangle TEF \)
8. HL Theorem can be applied; \( \triangle OKR \cong \triangle AHR \)
9. HL Theorem cannot be applied.
Chapter 4 Answers (continued)

Reteaching 4-7

1. **Statements**
   
   1. \( \triangle \text{PSR} \) and \( \triangle \text{PQR} \) are right \( \triangle \)s; \( \angle \text{QPR} \) and \( \angle \text{SRP} \)
   
   2. \( \angle \text{PSR} \) and \( \angle \text{PQR} \)
   
   3. \( \angle \text{STR} \) congruent to \( \angle \text{QTP} \)
   
   4. \( \triangle \text{QPR} \) and \( \triangle \text{SRP} \)
   
   5. \( \angle \text{STR} \) congruent to \( \angle \text{QTP} \)
   
   6. \( \angle \text{STR} \) congruent to \( \angle \text{QTP} \)
   
   **Reasons**
   
   1. Given
   
   2. Right \( \triangle \)s are congruent.
   
   3. Reflexive Property of \( \cong \)
   
   4. AAS Theorem
   
   5. Vertical \( \angle \)s are \( \cong \).
   
   6. CPCTC
   
   7. AAS Theorem

2. Sample: Prove \( \triangle \text{MLP} \) \( \cong \) \( \triangle \text{QPL} \) by the AAS Theorem. Then use CPCTC and vertical angles to show \( \triangle \text{MLN} \) \( \cong \) \( \triangle \text{QPN} \) by the AAS Theorem.

**Enrichment 4-1**

Check students’ work. Samples shown.

1. 

2. 

3. 

4. 

5. 

**Enrichment 4-2**

1a. Definition of perpendicular lines 1b. \( \angle \text{AKF} \) \( \cong \) \( \angle \text{GEL} \) 1c. SAS 2a. Segment Addition Postulate 2b. \( \text{LR} + \text{RG} = \text{TF} + \text{TA} \) 2c. \( \text{RG} \equiv \text{TA} \) 2d. Alternate Interior Angles 2e. Corresponding Angles 2f. \( \angle \text{DAT} \) \( \cong \) \( \angle \text{JGR} \) 2g. SAS

**Enrichment 4-3**

1–11. Check students’ work. 2a. ASA 2b. The top \( \angle \)s are congruent because the fold bisected the right \( \angle \)s formed by the folds in steps 1 and 3. The corners of the paper are right \( \angle \)s; therefore, those \( \angle \)s are congruent. The included sides are congruent because the fold in step 1 found the midpoint of the width of the paper, thus creating two equal segments.

3a. ASA 3b. The top \( \angle \)s are congruent because the fold bisected the right \( \angle \)s formed by the folds in steps 1 and 2. The upper corners that became inside \( \angle \)s along the center line are right \( \angle \)s; therefore, those \( \angle \)s are congruent. The included sides are congruent because the fold in step 1 found the midpoint of the width of the paper, thus creating two equal segments.

4a. The top \( \angle \)s are congruent because the fold bisected the right \( \angle \)s formed by the fold in step 1. The inside \( \angle \)s along the center line are right \( \angle \)s because the horizontal fold that formed them is perpendicular to the original fold in step 1. 4b. The included sides are congruent because the fold in step 1 found the midpoint of the width of the paper, thus creating two equal segments.

4a. ASA 4b. The top \( \angle \)s are congruent because the fold bisected the right \( \angle \)s formed by the fold in step 7. The inside \( \angle \)s along the center line are congruent because of the Angle Addition Postulate. The included sides are congruent because the fold in step 7 found the midpoint of the width of the paper, thus creating two equal segments.

**Enrichment 4-4**

1. \( \text{ABT} \) 2. \( \text{ACT} \) 3. \( 45; 45; \text{ABT}; \text{ACT} \) 4. \( 30; 30; \text{ATB}; \text{ATC} \) 5. Reflexive Property of \( \cong \) 6. AAS Theorem 7. CPCTC 8. Definition of \( \cong \) segments 9. Definition of \( \cong \) segments 10. Definition of \( \cong \) segments 11. SSS Postulate 12. CPCTC 13. 60

**Enrichment 4-5**

1. 60 2. 60 3. 60 4. 70 5. 70 6. 40 7. 72 8. 72 9. 36 10. 30 11. 30 12. 120 13. 80 14. 80 15. 20 16. 80 17. 80 18. 20 19. 41 20. 23 21. 116 22. 23 23. 41 24. 116 25. 80 26. 80 27. 20 28. 82 29. 82 30. 16 31. 78.5 32. 78.5 33. 37.5 34. 40 35. 40 36. 100

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Chapter 4 Answers (continued)

Enrichment 4-6

15. Check students' work.

4. Sample:

8. ...

7. ...

5. Common side:

3. Common angle:

2. 1.

Enrichment 4-7

1. Sample: $\triangle ABD \cong \triangle AEC$

2. Sample: $\triangle DEC \cong \triangle CBD$

3. Sample: $\triangle GNI \cong \triangle KLH$

4. Sample: $\triangle HIM \cong \triangle JIM$

5. 8

6. Sample: $\triangle JIM \cong \triangle JIM$

7. 11. 4 12. $\triangle PUV \cong \triangle TWV$ by AAS; $\triangle PSV \cong \triangle TVU$ by AAS; $\triangle PVW \cong \triangle TVW$ by SSS; $\triangle WQV \cong \triangle USV$ by AAS.

13. 14 15. Check students' work.

Chapter Project

Activity 1: Modeling
Yes; the brace makes two rigid triangles.

Activity 2: Observing
Check students' work.

Activity 3: Investigating
tetrahedron
Sample: You could add in diagonals of the cube.
Check students' work.

Finishing the Project
Check students' work.

✓ Checkpoint Quiz 1
1. AAS 2. SAS 3. SSS 4. not possible
5. AAS 6. not possible 7. $\triangle LM \cong \triangle TR, \triangle MN \cong \angle QR, \triangle LN \cong \triangle TR, \angle M = \angle Q, \angle N = \angle R$

✓ Checkpoint Quiz 2
1. $\triangle ABC, \triangle ABD$ 2. Hypotenuse-Leg Theorem
3. CPCTC 4. $\angle S = \angle Q, \angle R = \angle T, \angle M = \angle Q$, $\angle N = \angle R$
5a. definition of a bisector 5b. Reflexive 5c. ASA
5d. CPCTC 5e. definition

Chapter Test, Form A
1. $x = 50; y = 65$ 2. $a = 118; b = 62; c = 59$
3. HL 4. not possible 5. SAS 6. AAS
7. ASA 8. SSS 9. not possible 10. SSS
11. not possible 12. Check students' work: $\angle J = \angle P$, $\angle K = \angle Q, \angle L = \angle R$, $\angle J = \angle P$, $\angle K = \angle Q$, $\angle L = \angle R$

16d. SAS Postulate 16e. CPCTC 16f. Isosceles Triangle Theorem

17. Sample:

\begin{align*}
\text{Statements} & \\
1. BD \bot AC; D & \text{ midpoint of } AC \\
2. \angle BDC & \cong \angle BDA \\
3. \triangle ABD & \cong \triangle ABC \\
4. \triangle BD & \cong \triangle BD \\
5. \triangle BAD & \cong \triangle BCD \\
6. BC & \cong BA \\
7. \triangle BAD & \cong \triangle BCD \\
8. \triangle BDC & \cong \triangle BD \\
9. \triangle BD & \cong \triangle BD \\
10. \triangle ABD & \cong \triangle ABC \\
11. BD & \bot AC \\
12. \triangle PUV & \cong \triangle TWV \\
13. \triangle PSV & \cong \triangle TVU \\
14. \triangle PVW & \cong \triangle TVW \\
15. \triangle WQV & \cong \triangle USV \\
16. \angle AXB & \cong \angle DXC \\
17. \angle AXC & \cong \angle CX \\
18. \angle AXB & \cong \angle DXC \\
\end{align*}

19. Sample: Given that $X$ is the midpoint of $\overline{AB}$ and $\overline{BC}$, $\overline{AX}$ and $\overline{BX}$ by the definition of midpoint. $\triangle AXY \cong \triangle DXC$ because all vertical angles are congruent. $\triangle AXY \cong \triangle DXC$ by the SAS Postulate, and therefore $\overline{AB} \cong \overline{DC}$ by CPCTC.
Chapter 4 Answers (continued)

Chapter Test, Form B
1. \(x = 58, y = 64\)\)  2. \(a = 40, b = 70, c = 70\)  3. SSS  4. SAS  5. HL  6. AAS  7. not possible  8. not possible  9. SSS or SAS  10. SSS or SAS  11. ASA

\[ \triangle ABC \cong \triangle DEF \]
\[ \angle A \cong \angle D; \angle B \cong \angle E; \angle C \cong \angle F; \ AB \cong DE; \]
\[ AC \cong DF; BC \cong EF \]
\[ \triangle ABE \cong \triangle ACD \]
\[ \triangle POS \cong \triangle POR \]

Alternative Assessment, Form C

**TASK 1: Scoring Guide**
Sample:
SSS:

\[ \triangle MNO \cong \triangle OPM \]

ASA:

\[ \triangle ABE \cong \triangle ACD \]

SAS:

\[ \triangle POS \cong \triangle POR \]

**TASK 2: Scoring Guide**

a. Sample:

\[ \begin{array}{c}
\triangle LKMN \\
\triangle OPNM
\end{array} \]

b. Sample: Using the Pythagorean theorem, show that \(KO = MO\). Then \(\triangle KOL \cong \triangle MON\) by SAS Postulate or SSS Postulate.

**TASK 3: Scoring Guide**

Sample:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{AE} \cong \overline{AD}, \angle B \cong \angle C )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle A \cong \angle A )</td>
<td>2. Reflexive Property of ( \cong )</td>
</tr>
<tr>
<td>3. ( \triangle ABD \cong \triangle ACE )</td>
<td>3. AAS Theorem</td>
</tr>
<tr>
<td>4. ( \overline{AB} \cong \overline{AC} )</td>
<td>4. CPCTC</td>
</tr>
<tr>
<td>5. ( \overline{EB} \cong \overline{DC} )</td>
<td>5. Segment Addition Postulate</td>
</tr>
</tbody>
</table>

**TASK 4: Scoring Guide**
Sample: The SSS, ASA, and SAS Postulates are statements that are accepted as true without proof. The HL and AAS Theorems, on the other hand, can be proved true, using postulates, definitions, and previously proved theorems.

3 Student gives an explanation that is thorough and correct.
2 Student gives an explanation that is partially correct.
1 Student gives an explanation that lacks demonstrated understanding of the difference between a theorem and a postulate.
0 Student makes little or no attempt.
Chapter 4 Answers (continued)

Cumulative Review


13. 

14. $x = 102; y = 102$

15. c, e, a, b, d or e, c, a, b, d

16. \[
\begin{align*}
\triangle AED & \cong \triangle CDE \\
\triangle AED & \cong \triangle CDE \\
\angle CED & \cong \angle ADE
\end{align*}
\]

Reflexive Property

17. Sample: The alarm sounds if and only if there is smoke. If the alarm sounds, then there is smoke. If there is smoke, then the alarm sounds.

18. AAA, SSA
Chapter 5 Answers

Practice 5-1
1. a. 8 cm b. 16 cm c. 14 cm 2a. 22.5 in.
2. b. 15.5 in. c. 15.5 in. 3a. 9.5 cm 3b. 17.5 cm
3. c. 14.5 cm 4. 17 5. 20.5 6. 7 7. 128
8. 42 9. 16.5 10a. 18 10b. 61 11. $GH \parallel AC, HI \parallel BA, GI \parallel BC$
12. $PR \parallel YZ, PQ \parallel XZ, XY \parallel RQ$

Practice 5-2
1. $\overline{WY}$ is the perpendicular bisector of $XY$. 2. 4
3. 7.5 4. 9 5. right triangle 6. 5 7. 17
8. 17 9. equidistant 10. isosceles triangle
11. 3.5 12. 21 13. 21 14. right triangle
15. $\overline{FP}$ is the bisector of $\angle JLN$. 16. 9 17. 45
18. 45 19. 14 20. Sample: Point $M$ lies on $\overline{FP}$.
21. right isosceles triangle

Practice 5-3
1. (-2, 2) 2. (4, 0) 3. (2, 1)
4. Check students' work. The final result of the construction is shown.

5. altitude 6. median 7. none of these
8. perpendicular bisector 9. angle bisector
10. altitude 11a. (2, 0) 11b. (-2, -2)
12a. (0, 0) 12b. (3, -4) 13a. (0, 0) 13b. (0, 3)

Practice 5-4
1. I and III 2. I and II 3. The angle measure is not 65.
4. Tina does not have her driver's license.
5. The figure does not have eight sides.
6. The restaurant is open on Sunday.
7. $\triangle ABC$ is congruent to $\triangle XYZ$.
8. $m\angle Y \leq 50$ 9a. If two triangles are not congruent, then their corresponding angles are not congruent; false.
9b. If corresponding angles are not congruent, then the triangles are not congruent; true.
10a. If you do not live in Toronto, then you do not live in Canada; false.
10b. If you do not live in Canada, then you do not live in Toronto; true.
11. Assume that $m\angle A \neq m\angle B$. 12. Assume that $TUVW$ is not a trapezoid.
13. Assume that $LM$ does not intersect $\overline{NO}$. 14. Assume that $\triangle FGH$ is not equilateral.
15. Assume that it is not sunny outside.
16. Assume that $\angle D$ is obtuse. 17. Assume that $m\angle A \geq 90$. This means that $m\angle A + m\angle C \geq 180$.
This, in turn, means that the sum of the angles of $\triangle ABC$ exceeds $180$, which contradicts the Triangle Angle-Sum Theorem. So the assumption that $m\angle A \geq 90$ must be incorrect. Therefore, $m\angle A < 90$.

Practice 5-5
1. $\angle M, \angle N$ 2. $\angle C, \angle D$ 3. $\angle S, \angle Q$
4. $\angle R, \angle P$ 5. $\angle A, \angle T$ 6. $\angle S, \angle A$
7. yes; 4 + 7 > 8, 7 + 8 > 4, 8 + 4 > 7
8. no; 6 + 10 > 17 9. yes; 4 + 4 > 7
10. yes; 1 + 9 > 9, 9 + 9 > 1, 9 + 1 > 9
11. yes; 11 + 12 > 13, 12 + 13 > 11, 13 + 11 > 12
12. no; 18 + 20 > 40 13. no; 1.2 + 2.6 > 4.9
14. no; 8.4 + 9.4 > 18 15. no; 2.5 + 3.5 > 6
16. $\overline{BC}, \overline{AB}, \overline{AC}$ 17. $\overline{BO}, \overline{BL}, \overline{LO}$ 18. $\overline{RS}, \overline{ST}, \overline{RT}$
19. $\angle D, \angle S, \angle A$ 20. $\angle N, \angle S, \angle J$ 21. $\angle R, \angle O, \angle P$
22. 3 < $x < 11$ 23. 8 < $x < 26$ 24. 0 < $x < 10$
25. 9 < $x < 31$ 26. 2 < $x < 14$ 27. 13 < $x < 61$

Reteaching 5-1
Triangles will vary.
1a. $PQ = 8$ 1b. $MN = 16$ 1c. $YZ = 32$
2a. $NO = 2.5$ 2b. $ST = 10$ 2c. $UV = 20$
3a. $QR = 4$ 3b. $ST = 8$ 3c. $UV = 16$

Reteaching 5-2
1. no 2. yes 3. yes 4. no 5. B 6. C

Reteaching 5-3
1. Sample:

2. Sample:

Reteaching 5-4
1. Step 1: B; Step 2: D; Step 3: F; Step 4: E; Step 5: A; Step 6: C
2. Step 1: Assume $EF \cong DE$. Step 2: If $EF \cong DE$, then by the Isosceles Triangle Theorem, $\angle D \cong \angle F$. Step 3: But $\angle D \neq \angle F$. Step 4: Therefore, $EF \neq DE$.

Reteaching 5-5
1. Check students' work. The longest side will be opposite the largest angle. The shortest side will be opposite the smallest angle.
2. largest: $\angle DEF$; smallest: $\angle DFE$
3. largest: $\angle PQR$; smallest: $\angle PRQ$
4. largest: $\angle ACB$; smallest: $\angle CBA$
5. longest: $\overline{DF}$; shortest: $\overline{FE}$
6. longest: $\overline{PQ}$; shortest: $\overline{RQ}$
7. longest: $\overline{SV}$; shortest: $\overline{ST}$
Chapter 5 Answers (continued)

**Enrichment 5-1**

1. 5.

2. \(D(0,3)\)  \(E(2.5,0)\)  \(F(2.5,3)\)

3. 6. 3.75 square units  7. They are equal.  8. 1 : 4


10. \(T(3.5,0.5)\)  11. \(U(9,-3)\)  12. \(V(6.5,0.5)\)

14. 21 square units  15. 5.25 square units

**Enrichment 5-2**

1.–10. Check students’ work.

11. Check students’ work. Give hint: \(\angle AOB = 45°\).

**Enrichment 5-3**

1.–2.

3. centroid  4. interior  5. It is not possible for the centroid to be on the exterior because all the medians are in the interior of the triangle.

6.

7. orthocenter  8. It is possible for the orthocenter to be in the interior, the exterior, or on the triangle itself. Explanations may vary.

9.

10. Check students’ work.  11. centroid

**Enrichment 5-4**

1. If a fish is a salmon, then it swims upstream.  2. If a fish swims upstream, then it is a salmon.  3. If a fish is not a salmon, then it does not swim upstream.  4. If a fish does not swim upstream, then it is not a salmon.  5. Exercises 1 and 4  6. the salmon  7. the fish that swim upstream
Chapter 5 Answers (continued)

12. If a quadrilateral is a rectangle, then the angles of the quadrilateral are congruent. 13. If the angles of a quadrilateral are not congruent, then the quadrilateral is not a rectangle. 14. If a quadrilateral is not a rectangle, then the angles of the quadrilateral are not congruent. 15. All are true (12, 13, and 14). 16. The contrapositive

Enrichment 5-5


The name of the mathematician is THALES (θαλῆς).

Chapter Project

Activity 1: Creating
isocline, median, angle bisector, perpendicular bisector

Activity 2: Experimenting
Check students’ work.

Activity 3: Designing
Check students’ work.

✓ Checkpoint Quiz 1
1. angle bisector, since \( \overline{TM} \cong \overline{NM} \)
2. congruent, since \( \overline{QM} \) is the angle bisector
3. 20, since \( \overline{TM} \cong \overline{NM} \)
4. 6 5. 4 6a. 6b. 42 7. altitude

✓ Checkpoint Quiz 2
1. Inv: If you don’t eat bananas, then you will not be healthy. Contra: If you are not healthy, then you don’t eat bananas.
2. Inv: If a figure is not a triangle, then it does not have three sides. Contra: If a figure does not have three sides, then it is not a triangle.
3. Suppose that there are two obtuse angles in a triangle. This would mean that the sum of two angles of a triangle would exceed 180°. This contradicts the Triangle-Sum Theorem. Therefore, there can be only one obtuse angle in a triangle.
4. Suppose that the temperature is below 32°F and it is raining. Because 32°F is the freezing point of water, any precipitation will be in the form of sleet, snow, or freezing rain. This contradicts the original statement. Therefore, the temperature must be above 32°F for it to rain.
5. yes; 6 + 4 > 8, 8 + 6 > 4 6. no; 2.6 + 4.1 > 6.7
7. \( \angle B, \angle C, \angle A \)
8. \( \angle D, \angle F, \angle E \)

Chapter Test, Form A

1a. If a triangle does not have three congruent sides, then it is not equiangular; true. 1b. If a triangle is not equiangular, then it does not have three congruent sides; true. 2a. If an isosceles triangle is not obtuse, then the vertex angle is not obtuse; true. 2b. If the vertex angle of an isosceles triangle is not obtuse, then the triangle is not obtuse; true.

Alternative Assessment, Form C

TASK 1: Scoring Guide
Sample: Assume that Harold can create a triangular garden plot with sides of 6 ft, 6 ft, and 13 ft. Then a triangle is possible with sides 6, 6, and 13. But 6 + 6 ≠ 13, so by the Triangle Inequality Theorem, no such triangle exists. Therefore, the plan is impossible.

3 Student gives a clear and accurate explanation.
2 Student gives an explanation that contains minor inaccuracies.
1 Student gives an explanation that contains significant flaws in logical reasoning.
0 Student makes little or no attempt.

TASK 2: Scoring Guide
Sample: The locus of points in a plane equidistant from a single point forms a circle, as shown in the first figure. The locus of points in a plane equidistant from the endpoints of a segment forms a perpendicular bisector of the segment, as shown in the second figure.
Chapter 5 Answers (continued)

3 Student gives an accurate explanation and drawings.
2 Student gives a drawing or an explanation that contains minor errors.
1 Student gives an explanation or a drawing that contains significant errors.
0 Student makes little or no attempt.

TASK 3: Scoring Guide
Expressed as a conditional: If $\triangle ABC$ is an equilateral triangle with midpoints $D$, $E$, and $F$ on sides $AB$, $BC$, and $AC$ respectively, then $\triangle DEF$ forms an equilateral triangle.
Sample: $\triangle DEF$ is an equilateral triangle. Each of $\triangle DEF$’s sides is exactly $\frac{1}{2}$ the corresponding side of $\triangle ABC$. If $\triangle ABC$’s sides are all the same length, then all of $\triangle DEF$’s sides are all the same length as well.

3 Student accurately states the conditional and the proof.
2 Student has minor errors in the statement of the conditional or in the proof.
1 Student gives a statement of conditional or a proof that contains significant errors.
0 Student makes little or no attempt.

TASK 4: Scoring Guide
Sample:

$\triangle ADC$ is isosceles. $\angle BAC = \angle BCA$ because $\triangle ABC$ is an isosceles triangle with $AB = BC$. The angle bisectors create two new angles at each vertex of $\triangle ABC$, each of which has a measure $\frac{1}{2}$ that of the angle bisected. Thus $m\angle DAC = \frac{1}{2}m\angle BAC$ and $m\angle DCA = \frac{1}{2}m\angle BCA$.
Because $m\angle BAC = m\angle BCA$, $m\angle DAC = m\angle DCA$, which makes $\triangle ADC$ an isosceles triangle.

3 Student gives accurate answers and justification.
2 Student gives answers containing minor errors.
1 Student gives a justification containing major logical gaps.
0 Student makes little or no effort.

Cumulative Review
18. Suppose that an equilateral triangle has an obtuse angle. Then one angle in the triangle has a measure greater than 90. But this contradicts the fact that in an equilateral triangle, the measure of each angle is 60. Therefore, the equilateral triangle does not have an obtuse angle.
19. Sample:

20. $(-1, 2)$

22. $\angle C, \angle B, \angle A$
Chapter 6 Answers

Practice 6-1
1. parallelogram 2. rectangle 3. quadrilateral 4. parallelogram, quadrilateral 5. kite, quadrilateral 6. rectangle, parallelogram, quadrilateral 7. trapezoid, isosceles trapezoid, quadrilateral 8. square, rectangle, parallelogram, rhombus, quadrilateral 9. \( x = 7 \); \( AB = BD = DC = CA = 11 \) 10. \( m = 9; x = 42 \); \( ON = LM = 26; OL = MN = 43 \) 11. \( f = 5; g = 11 \); \( FG = GH = HI = IF = 17 \) 12. parallelogram 13. rectangle 14. kite 15. parallelogram

Practice 6-2
1. 15 2. 32 3. 7 4. 8 5. 12 6. 9 7. 8 8. 3\( \frac{1}{2} \) 9. 54 10. 34 11. 54 12. 34 13. 100 14. 40; 140; 40 15. 70; 110; 70 16. 113; 45; 22 17. 115; 15; 50 18. 55; 105; 55 19. 61; 72; 108; 32 20. 32; 98; 50 21. 16 22. 35 23. 28 24. 4

Practice 6-3
1. no 2. yes 3. yes 4. no 5. yes 6. yes 7. \( x = 2; y = 3 \) 8. \( x = 6; y = 3 \) 9. \( x = 64 \); \( y = 10 \) 10. \( x = 8 \); the figure is a \( \square \) because both pairs of opposite sides are congruent. 11. \( x = 40 \); the figure is not a \( \square \) because one pair of opposite angles is not congruent. 12. \( x = 25 \); the figure is a \( \square \) because the congruent opposite sides are \( || \) by the Converse of the Alternate Interior Angles Theorem. 13. Yes; the diagonals bisect each other. 14. No; the congruent opposite sides do not have to be \( || \). 15. No; the figure could be a trapezoid. 16. Yes; both pairs of opposite sides are congruent. 17. Yes; both pairs of opposite sides are \( || \) by the converse of the Alternate Interior Angles Theorem. 18. No; only one pair of opposite sides is congruent. 19. Yes; one pair of opposite sides is both congruent and \( || \). 20. No; only one pair of opposite sides is congruent.

Practice 6-4
1a. rhombus 1b. 72; 54; 72 2a. rectangle 2b. 72; 36; 18 3a. rectangle 3b. 37; 53; 106; 74 4a. rhombus 4b. 59; 90; 59 5a. rectangle 5b. 60; 30; 60 6a. rhombus 6b. 22; 68; 68; 90 7. Yes; the parallelogram is a rhombus. 8. Possible; opposite angles are congruent in a parallelogram. 9. Impossible; if the diagonals are perpendicular, then the parallelogram should be a rhombus, but the sides are not of equal lengths. 10. \( x = 7 \); \( HJ = 7 \); \( IK = 7 \) 11. \( x = 7; HJ = 26; IK = 26 \) 12. \( x = 6 \); \( HJ = 25; IK = 25 \) 13. \( x = -3; HJ = 13; IK = 13 \) 14a. 90; 90; 29; 29 14b. 288 cm\(^2 \) 15a. 70; 90; 70 15b. 88 in\(^2 \) 16a. 38; 90; 38 16b. 260 m\(^2 \) 17. possible 18. Impossible; because opposite angles are congruent and supplementary, for the figure to be a parallelogram they must measure 90, the figure therefore must be a rectangle.

Practice 6-5
1. 118; 62 2. 99; 81 3. 59; 121 4. 96; 84 5. 101; 79 6. 67; 113 7. \( x = 4 \) 8. \( x = 16 \); \( y = 116 \) 9. \( x = 1 \) 10. 105.5; 105.5 11. 90; 25 12. 118; 118 13. 90; 63; 63 14. 107; 107 15. 90; 51; 39 16. \( x = 8 \) 17. \( x = 7 \) 18. \( x = 28; y = 32 \)

Practice 6-6
1. \( (1.5a, 2b); a \) 2. \( (1.5a, b); \sqrt{a^2 + 4b^2} \) 3. \( (0.5a, 0); a \) 4. \( (0.5a, b); \sqrt{a^2 + 4b^2} \) 5. 0 6. 1 7. \(-\frac{1}{2} \) 8. 2 \( 9. \frac{2b}{3a} \) 10. \(-\frac{2b}{3a} \) 12. \(-\frac{2b}{3a} \) 13. \( E(a, 3b); I(4a, 0) \) 14. \( O(3a, 2b); M(3a, -2b); E(-3a, -2b) \) 15. \( D(4a, b); I(3a, 0) \) 16. \( T(0, 2b); A(a, 4b); L(2a, 2b) \) 17. \((-4a, b) \) 18. \((-b, 0) \)

Practice 6-7
1a. \( \frac{p}{q} \) 1b. \( y = mx + b; q = \frac{p}{q}(p) + b; b + q = \frac{p^2}{q^2}; y = \frac{p^2}{q^2} + \frac{p}{q} + q - \frac{p^2}{q^2} \) 1c. \( x = r + p \) 1d. \( y = \frac{p}{q}(r + p) + q - \frac{p^2}{q^2}; y = \frac{p^2}{q^2} + \frac{p}{q} + q \) 1e. \( \frac{p}{q} \) 1f. \( (r, q) \) 1g. \( y = mx + b; q = \frac{p}{q}(r + p); b = q - \frac{r^2}{q^2}; y = \frac{p^2}{q^2} + \frac{p}{q} + q \) 1h. \( y = \frac{p}{q}(r + p) + q - \frac{r^2}{q^2}; y = \frac{p^2}{q^2} + \frac{p}{q} + q \) 1i. \( y = \frac{p}{q}(r + p) + q - \frac{r^2}{q^2}; y = \frac{p^2}{q^2} + \frac{p}{q} + q \) 1j. \( y = \frac{p}{q}(r + p) + q - \frac{r^2}{q^2}; y = \frac{p^2}{q^2} + \frac{p}{q} + q \) 1k. \( y = \frac{p}{q}(r + p) + q - \frac{r^2}{q^2}; y = \frac{p^2}{q^2} + \frac{p}{q} + q \) 1l. \( y = \frac{p}{q}(r + p) + q - \frac{r^2}{q^2}; y = \frac{p^2}{q^2} + \frac{p}{q} + q \) 2a. \((2a, 0)\) 2b. \((-a, b) \) 2c. \((-\frac{3a}{2}, \frac{b}{2}) \) 2d. \( \frac{b}{2} \) 3a. \((-4a, 0) \) 3b. \((-2a, 3a) \) 3c. \(\frac{3}{2} \) 3d. \((2a, -a)\)

Reteaching 6-1
1. Samples:

\[ \text{trapezoid} \]

\[ \text{parallelogram} \]

\[ \text{rectangle} \]
4. rhombus


Reteaching 6-2

1. **Statements**
   1. Parallelogram $ABCD$
   2. $AB \cong CD$, $BC \cong DA$
   3. $BD \cong DB$
   4. $\triangle ABD \cong \triangle CBD$
   5. $\angle A \cong \angle C$

   **Reasons**
   1. Given
   2. Opposite sides of a parallelogram are congruent.
   3. Reflexive Prop. of $\cong$
   4. SSS
   5. CPCTC

2. **Statements**
   1. Parallelogram $ACDE$; $\angle DCE \cong \angle ECD$
   2. $AE \cong FE$  

   **Reasons**
   1. Given
   2. Opposite angles of a parallelogram are $\cong$
   3. Isosceles Triangle Theorem
   4. Substitution

3. **Statements**
   1. Parallelogram $ACDE$; $\angle DCE \cong \angle ECD$
   2. $AE \cong CE$

   **Reasons**
   1. Given
   2. Opposite sides of a parallelogram are $\cong$
   3. Substitution
   4. Isosceles Triangle Theorem

4. **Statements**
   1. Parallelogram $ACDE$; $\angle DCE \cong \angle ECD$
   2. $E \cong E$

   **Reasons**
   1. Given
   2. Opposite angles of a parallelogram are $\cong$
   3. Substitution
   4. If 2 $\angle$s of a $\triangle$ are $\cong$, sides opposite them are $\cong$.  

5. **Statements**
   1. Parallelogram $ACDE$; $\angle DCE \cong \angle ECD$
   2. $\angle BCD \cong \angle ECD$

   **Reasons**
   1. Given
   2. Opposite angles of a parallelogram are $\cong$
   3. Substitution

**Reteaching 6-3**

1. yes 2. no 3. no 4. yes

5. **Statements**
   1. $BD \cong CD$, $AE \cong BD$, $AE \parallel CD$
   2. $AE \cong CD$
   3. $\triangle ACD$ is a parallelogram.

   **Reasons**
   1. Given
   2. Substitution
   3. If one pair of opposite sides is both congruent and parallel, then the quadrilateral is a parallelogram.

**Reteaching 6-4**

1. $m\angle 1 = 60; m\angle 2 = 30; m\angle 3 = 90$  
2. $m\angle 1 = 80; m\angle 2 = 100; m\angle 3 = 50; m\angle 4 = 40$  
3. $m\angle 1 = 80; m\angle 2 = 75; m\angle 3 = 15; m\angle 4 = 90$  
6. $m\angle 1 = 45; m\angle 2 = 45$

**Reteaching 6-5**

7. 90 2. 52 3. 90 4. 38 5. 52 6. 90

10. Sample:

   **Statements**
   1. $\angle LP \cong \angle MN$
   2. $\angle LNP \cong \angle MNP$
   3. $\angle LPN \cong \angle MPN$
   4. $\angle LPM \cong \angle NML$
   5. $\angle LPM \cong \angle MNP$
   6. $\triangle LPQ \cong \triangle MQN$
   7. $\triangle LM = \triangle LM$
   8. $\triangle LNP \cong \triangle MNQ$
   9. $\triangle LOP \cong \triangle MQN$
   10. $\triangle LOP \cong \triangle MQN$

   **Reasons**
   1. Given
   2. Theorem 6-15
   3. Reflexive Prop. of $\cong$
   4. SAS Postulate
   5. CPCTC
   6. Reflexive Prop. of $\cong$
   7. SSS Postulate
   8. CPCTC
   9. Vertical angles are $\cong$
   10. AAS Theorem

11. 87 12. 48 13. 45
14. 48 15. 24
16. 24 17. 132 18. 24
19. 132 20. Sample: Both $\triangle LMQ$ and $\triangle PNQ$ have the same angle measures, but their sides have different lengths.

**Reteaching 6-6**

1. $Q(x + k, m)$ 2. $X(-a, 0); W(0, -b)$
3. $S(a, -a)$
4. Each side has length $a\sqrt{2}$, so it is a rhombus.

One pair of opposite sides has slope of 1, and the other pair has slope of $-1$. Therefore, because $(1)(-1) = -1$, the rhombus has four right angles and is a square. 5. Each side
Chapter 6 Answers (continued)

has length of $\sqrt{2a^2 + 2a + 1}$. Therefore, the figure is a rhombus. 6. $C(x - k, m)$

**Reteaching 6-7**

1. Each diagonal has length $\sqrt{c^2 + (b + a)^2}$. 2. The midpoints are $(\frac{a}{2}, \frac{b}{2})$ and $(\frac{b}{2}, \frac{a}{2})$. The line connecting the midpoints has slope 0 and is therefore parallel to the third side. 3. The midpoints are $(\frac{a}{2}, 0), (a, \frac{b}{2}), (\frac{a}{2}, b)$ and $(0, \frac{a}{2})$. The segments joining the midpoints each have length $a^2 + b^2$. 4. The midpoints are $(\frac{a}{2}, 0), (-\frac{a}{2}, \frac{b}{2}), (-\frac{a}{2}, -\frac{b}{2})$, and $(\frac{a}{2}, -\frac{b}{2})$. The quadrilateral formed by these points has sides with slopes of 0, 0, undefined, and undefined. Therefore, the sides are vertical and horizontal, and consecutive sides are perpendicular. 5. The median meets the base at (0, 0), the midpoint of the base. Therefore, the median has undefined slope; i.e., it is vertical. Because the base is a horizontal segment, the median is perpendicular to the base. 6. The midpoints are $(\frac{b}{2}, 0), (a, \frac{c}{2}), (\frac{c}{2}, c), (b, \frac{c}{2})$, and $(\frac{c}{2}, 0)$. One pair of opposite sides has slope of $\frac{c}{b}$ and the other pair of opposite sides has slope of $\frac{c}{b}$. Therefore, the figure is a parallelogram because opposite sides are parallel.

**Enrichment 6-1**


**Enrichment 6-2**

1. ABED, BCFE, DEHG, EFIX, ACIG, DBFH 2. ABED, BCFE, DEHG, EFIH, ACIG, DBFH, ACFD, DFIG, ABHG, BCHI 3. ABED, BCFE, DEHG, EFIH, ACIG, DBFH, ACFD, DFIG, ABHG, BCHI, AEHD, DEBG, BFIG, BFCA, BCFD, BFDH, DEIH 4. DBHH, GCHI, ADEG, EACF, ABHD, BCIE, HDFH, GDFH, BCGD, GCDF, DAIF, ABFI 5. pentagon, scalene triangle, two rectangles, two trapezoids, two isosceles right triangles

**Enrichment 6-3**

1a. Given 1b. Definition of a regular hexagon 1c. SAS 1d. CPCTC 1e. Given any two distinct points, there is a unique line segment with these points as endpoints 1f. Definition of a diagonal 1g. Definition of a regular hexagon 1h. Reflexive Property of Congruence 1i. SSS 1j. CPCTC 1k. If two lines and a transversal form alternate interior angles that are congruent, then the two lines are parallel. 1l. Definition of a regular hexagon 1m. SAS 1n. CPCTC 1o. Definition of a regular hexagon 1p. SSS 1q. CPCTC 1r. If two lines and a transversal form alternate interior angles that are congruent, then the two lines are parallel. 1s. Definition of a parallelogram 2. A parallelogram can be constructed in an octagon by drawing the diagonals as shown. Other answers are possible.
Chapter 6 Answers (continued)

Enrichment 6-6
1. (5, 60°)  10. (1; 300°)  11. (3.5, 225°)
12. (6; 150°)  13. (4; 315°)  14. (5; 120°)
16. (2; 90°)  17. (5; 255°)  18. (1; 180°)

Enrichment 6-7
1. (0, 7)  2. (0, 1)  3. (5, 1)  4. (5, 7)
5. The x-coordinate of each point decreased by 4.
6. (7, 0)  7. (−3, −5)  8. (−1.5, −2.5)
9. Add 3 to the y-coordinate.  10. The y-coordinate of each point decreased by 3.
11. The y-coordinate decreased by 3.
12. (3, 4)  13. M(10, 6), N(2, 6)  14. rectangle
15. Either all the x-coordinates or all the y-coordinates change by a constant amount.

Chapter Project
Activity 1: Doing
Check students’ work.

Activity 2: Analyzing
1. The effective area is rectangular.
2. The effective area is rectangular. The effective area is larger in Figure 2 than in Figure 1 because the diagonal is longer than the face.
3. You should tie the string to a vertical stick of the kite.
4. If the faces of the kite were unchanged, one diagonal of the rhombus is longer than the diagonals of the square, so the effective area would increase.

Activity 3: Researching
Check students’ work.

✓ Checkpoint Quiz 1
1. 70, 110, 70  2. 53, 53, 52  3. 96, 84, 46  4. square
5. x = 20, y = 3  6. x = 2, y = 5  7. 30.5

✓ Checkpoint Quiz 2
1. x = 66, y = 57  2. x = 35, y = 35  3. x = 3, y = 4  4. True; they are the only quadrilaterals that possess these properties.
5. False; only two triangles at a time are congruent.  6. (n + 1, m)  7. (k, 0)

Chapter Test, Form A
1. trapezoid  
2. rhombus  
3. rectangle  
4. kite
5. 5 cm  6. 3 in.  7. 4 m  8. 20  9. x = 33; y = 81  10. 10  11. 20  12. x = 30; y = 30
13. 12  14. D(a, 0); E(b, c); (a + b, c)  15. D(−c, 0);
E(0, b); (−c, b)  16. D(0, b); E(a, 0); (a, b)  17. 28; 28
18. 105; 75  19. 90; 48  20. 55; 90  21. 22; 68
22. 53; 37  23. The lengths of segments AB, BC, and AC are:
AB = √j^2 + k^2, BC = √l^2 + j^2, AC = l + j. Thus,
Chapter 6 Answers (continued)

the perimeter of \( \triangle ABC \) is \( l + j + \sqrt{j^2 + k^2} + \sqrt{k^2 + l^2} \).
The midpoints of segments \( AB, BC, \) and \( AC \) are: \( M(\frac{l-j}{2}, \frac{k-j}{2}) \),
\( N(\frac{l-j}{2}, 0) \), \( O(\frac{l-j}{2}, 0) \). The lengths of segments \( MN, NO, \) and \( MO \) are:
\( MN = \frac{1}{2}(l+j), NO = \frac{1}{2}\sqrt{j^2 + k^2}, \)
\( MO = \frac{1}{2}\sqrt{k^2 + l^2} \). Thus, the perimeter of \( \triangle MNO \) is
\( \frac{1}{2}(l+j+\sqrt{j^2+k^2}+\sqrt{k^2+l^2}) \), which is half the
perimeter of \( \triangle ABC \).

24. parallelogram  25. kite  26. rectangle  27. parallelogram  28. square
29. rhombus  30. isosceles trapezoid

Chapter Test, Form B

1. square

2. parallelogram

3. kite

4. trapezoid

5. 7 in.  6. 6 cm  7. 22 m  8. \( x = 3.5 \)  9. \( x = 19; y = 123 \)
10. \( x = 6 \)
11. 78; 102  12. 90; 61  13. 64; 128  14. 90; 63; 27
15. 90; 45; 45  16. 71; 71; 38  17. parallelogram  18. rhombus
19. trapezoid  20. square

Alternative Assessment, Form C

TASK 1: Scoring Guide
Samples:
a. \( \overline{AB} \cong \overline{CD}, \overline{BC} \cong \overline{AD}, \overline{AB} \parallel \overline{CD}, \overline{BC} \parallel \overline{AD}, \)
\( \angle ABD \cong \angle BDC, \angle ACD \cong \angle BAC, \angle CBD \cong \angle BDA, \)
\( \angle CAD \cong \angle BCA, \overline{BE} = \overline{ED}, \overline{AE} = \overline{EC}, \)
\( \overline{ABC} \cong \overline{BCD} \cong \overline{BAD}, \)
b. C, E, F

3 Student lists all statements accurately in part a and gives the
   correct answers in part b.
2 Student gives mostly correct answers but with some errors.
1 Student gives answers that fail to demonstrate
   understanding of the properties of parallelograms.
0 Student makes little or no effort.

TASK 2: Scoring Guide

3 Student gives accurate and complete answers and diagram.
2 Student gives answers and a diagram that are mostly accurate.
1 Student gives answers or a diagram containing significant
   errors.
0 Student makes little or no effort.

TASK 3: Scoring Guide
\( x = 90 \) (Diagonals of a kite are \( \perp \).
\( y = 5 \) (Def. of isos. trapezoid)
\( z = 75 \) (Base angles of isos. trap. are \( \cong \).

3 Student gives correct answers and reasons.
2 Student gives mostly correct answers and reasons.
1 Student gives mostly incorrect answers and reasons.
0 Student makes little or no effort.

TASK 4: Scoring Guide
a. \( Q = (5-a, 5); S = (5, 5-a) \)  b. Slope of
\( \overline{QR} = \frac{5-5}{5-0} = 1 \). Slope of \( \overline{QS} = \frac{5-5}{(5-a)-(5-a)} = -1. \)
Because the product of their slopes is \(-1\), \( \overline{QR} \perp \overline{QS} \).

3 Student gives correct coordinates and a valid proof.
2 Student gives answers or a proof that contains minor errors.
1 Student gives incorrect coordinates in part a or a poorly
   constructed proof in part b.
0 Student makes little or no effort.
Chapter 6 Answers (continued)

Cumulative Review

16. Proof: $AB \cong BC \cong DC \cong AD$ by the definition of a rhombus. Also, $AC \cong AC$. Therefore, $\triangle ABC \cong \triangle CDA$ by the SSS Theorem.

17. 

18. 

19. Sample: The construction of the undercarriage of a bridge; it is a combination of triangles, which are the strongest geometric polygon.
Chapter 7 Answers

Practice 7-1
1. 1 : 278 2. 18 ft by 10 ft 3. 18 ft by 14 ft
4. 18 ft by 16 ft 5. 8 ft by 8 ft 6. 3 ft by 10 ft
7. true 8. false 9. true 10. true
11. false 12. true 13. true 14. false
15. false 16. 12 17. 12 18. 33 19. 21
20. ±8 21. y = 15
22. 6 23. ±7 24. ±7
25. 14 : 5 26. 3 : 2 27. 12 : 7 28. 5 29. 7

Practice 7-2
1. \(\triangle ABC \sim \triangle XYZ\), with similarity ratio 2 : 1
2. \(\triangle QMN \sim \triangle RST\), with similarity ratio 5 : 3
3. Not similar; corresponding sides are not proportional.
4. Not similar; corresponding angles are not congruent.
5. \(\triangle ABC \sim \triangle KMN\), with similarity ratio 4 : 7
6. Not similar; corresponding sides are not proportional.
7. \(\triangle I\) 8. \(\angle O\) 9. \(\angle J\) 10. NO 11. LO
12. LO 13. 3.96 ft 14. 4.8 in. 15. 3.75 cm
16. 10 m 17. \(\frac{2}{3}\) 18. 53 19. \(\frac{7}{2}\) 20. \(\frac{4}{2}\)
21. 53 22. 37 23. 5

Practice 7-3
1. \(\angle AXB \equiv \angle RXQ\) because vertical angles are \(\angle A \equiv \angle R\) (Given). Therefore \(\triangle AXB \sim \triangle RXQ\) by the AA ~ Postulate. 2. Because \(\frac{PM}{WM} = \frac{PX}{WX} = \frac{XY}{XZ}\), \(\triangle MPX \sim \triangle LWA\) by the SSS ~ Theorem.
3. \(\angle QMP \equiv \angle AMB\) because vertical \(s\) are \(\equiv\). Then, because \(\frac{QM}{AM} = \frac{PM}{BM} = \frac{2}{1}\), \(\triangle QMP \sim \triangle AMB\) by the SAS ~ Theorem. 4. \(\angle M \equiv \angle A\) (Given). Because there are 180° in a triangle, \(m \angle M = 130\), and \(\angle J \equiv \angle C\). So \(\triangle M J N \sim \triangle A C B\) by the AA ~ Postulate. 5. Because \(AX = BX\) and \(CX = RX\), \(\angle AXB \equiv \angle CXR\) because vertical angles are \(\equiv\). Therefore \(\triangle AXB \sim \triangle CXR\) by the SAS ~ Theorem. 6. Because \(AB = BC = CA\) and \(XY = YZ\), \(\frac{AX}{XY} = \frac{BY}{BZ} = \frac{CA}{AZ}\). Then \(\triangle ABC \sim \triangle XYZ\) by the SSS ~ Theorem.
7. \(\frac{15}{3}\) 8. \(\frac{45}{1}\) 9. \(\frac{39}{3}\)
10. \(\frac{85}{3}\) 11. \(\frac{40}{3}\) 12. 36 13. 33 ft

Practice 7-4
1. 16 2. 8 3. \(\sqrt{77}\) 4. \(2\sqrt{11}\) 5. \(10\sqrt{2}\)
6. \(6\sqrt{5}\) 7. h 8. y 9. x 10. a 11. b
12. c 13. \(x = 6; y = 6\sqrt{3}\) 14. \(x = 8\sqrt{3}; y = 4\sqrt{3}\)
15. \(\frac{9}{7}\) 16. \(x = 4\sqrt{5}; y = \sqrt{55}\) 17. \(x = \sqrt{3}; y = \sqrt{6}\)
18. \(x = 8; y = 2\sqrt{2}; z = 6\sqrt{2}\)
19. \(2\sqrt{15}\) in.

Practice 7-5
1. \(BE\) 2. \(EH\) 3. \(BC\) 4. \(JD\) 5. \(\frac{12}{17}\)
6. \(BE\) 7. \(\frac{16}{3}\) 8. 4 9. 4 10. \(\frac{25}{7}\) 11. \(x = \frac{25}{9}\)
12. \(\frac{15}{7}\) 13. \(x = 6; y = 6\) 14. \(x = \frac{189}{5}\)
15. \(y = \frac{198}{5}\)
16. 2 17. 4

Reteaching 7-1
1. 534 apples 2. 768 gallons 3. 4627 employees
4. about 4800 picture frames 5. 90 6. 2450 7. 85
13. 2500

Reteaching 7-2
1. \(\triangle KLM \sim \triangle QRS; 5 : 3\) 2. \(\triangle ABC \sim \triangle XYZ; 10 : 7\)
3. not similar 4. not similar 5. \(\triangle LM P \sim \triangle Q C I K; 1 : 1\)
6. not similar 7. similar with ratio 10 : 11 8. not similar
9. similar with ratio 4 : 9

Reteaching 7-3
1. \(\triangle ABC \sim \triangle Z Y X\) by AA ~ 2. not similar
3. \(\triangle Q E U \sim \triangle S I O\) by SSS ~ 4. \(\triangle ABC \sim \triangle R S T\)
by AA ~ 5. not similar 6. \(\triangle B A C \sim \triangle X Q R\) by SAS ~ 7. yes; by SSS ~ or by AA ~ 8. No; the vertex angles may differ in measure. 9. All congruent triangles are similar with ratio 1 : 1. Similar triangles are not necessarily congruent.

Reteaching 7-4
1. \(x = \frac{25}{13}; y = \frac{144}{13}; z = \frac{60}{13}\) 2. \(x = \frac{400}{29}; y = \frac{441}{29}\)
3. \(x = \frac{240}{29}\) 4. \(x = 4.8; y = 8\) 5. \(x = 4\) 6. \(x = 2\sqrt{10}\)
7. \(\frac{120}{13}\)

Reteaching 7-5
1. \(x = 4.5\) 2. \(x = \frac{8}{3}\) 3. \(x = 7.2; y = 4.8\)
4. \(x = \frac{15}{8}; y = \frac{25}{8}\) 5. \(x = 9\) 6. \(x = \frac{3}{2}; y = \frac{5}{2}\)
7. \(BN = \frac{16}{5}; CN = \frac{24}{5}\) 8. \(BY = \frac{3}{2}\)

Enrichment 7-1
1. 10 lb 2. $250; costs have not changed during the past year.
3. $240,000 4. $168,750 5. $6785
6. $106 7. 560 yd 8. 40 9. 42 10. 63

Enrichment 7-2
1–4. Check students' work.
Chapter 7 Answers (continued)

Enrichment 7-3

1. **Statements**
   1. $ABCD$ is a trapezoid
   2. $\overline{AD} \parallel \overline{BC}$
   3. $\angle AED \cong \angle CEB$
   4. $\angle EBC \cong \angle EDA$
   5. $\angle AED \sim \angle CEB$

   **Reasons**
   1. Given
   2. Def. of a trapezoid
   3. Vertical angles are $\cong$.
   4. Alt. int. angles are $\cong$.
   5. AA Similarity Post.

2. **Statements**
   1. $T, U,$ and $V$ are mdpts
   2. $TU \parallel RV, UV \parallel TR$, and $TV \parallel US$
   3. $TUVR$ and $TUSV$ are parallelograms.
   4. $\angle TUV \cong \angle R$
   5. $\angle UTV \cong \angle S$
   6. $\triangle QRS \sim \triangle VUT$

   **Reasons**
   1. Given
   2. Triangle Midsegment Theorem
   3. Def. of parallelogram
   4. Opposite $\angle$s of a parallelogram are $\cong$.
   5. Opposite $\angle$s of a parallelogram are $\cong$.
   6. AA Similarity Post.

3. Using the distance formula, $AB = \sqrt{180}$ or $6\sqrt{5}$.
   
   $BC = \sqrt{45}$ or $3\sqrt{5}$, $CA = 15$, $ST = 10$, $TB = \sqrt{80}$ or $4\sqrt{5}$, and $BS = \sqrt{20}$ or $2\sqrt{5}$. If the two triangles are similar, then corresponding $\angle$s are proportional. Therefore,
   
   $\frac{CA}{ST} = \frac{15}{10} = \frac{3}{2} = \frac{AB}{TB} = \frac{6\sqrt{5}}{4\sqrt{5}} = \frac{3}{2} = \frac{BC}{BS} = \frac{3\sqrt{5}}{2\sqrt{5}} = \frac{\sqrt{5}}{2}$.

   Because $\frac{\overline{CA}}{\overline{ST}} = \frac{\overline{AB}}{\overline{TB}} = \frac{\overline{BC}}{\overline{BS}}$, $\triangle ABC \sim \triangle TBS$ by the SSS Similarity Postulate.

Enrichment 7-4

1. The triangles have two congruent angles; AA Similarity Postulate.
   2. $\frac{AB + BD}{DE} = \frac{AB}{BC}$
   3. $BD = \frac{(DE - BC)AB}{BC}$

2. 800 yd
   5. 4500 ft
   6. 43 ft
   7. 9 m

Enrichment 7-5

1. 18
   2. 1
   3. 19
   4. 3
   5. 8
   6. 7
   7. 22
   8. 9
   9. 13
   10. 4
   11. 5
   12. 6
   13. 12
   14. 17
   15. 2
   16. 10
   17. 21
   18. 15
   19. 20
   20. 16

Chapter 7 Answers (continued)

Chapter 7 Test, Form A

1. 28
   2. $\frac{25}{15}$
   3. 45
   4. $\frac{4}{5}$
   5. $\triangle ABC \sim \triangle XYZ$ by the AA ~ Postulate
   6. not similar
   7. $\triangle KPM \sim \triangle RPQ$ by the SAS ~ Theorem
   8. 30 ft
   9. 9
   10. $6\sqrt{5}$
   11. B
   12. 12
   13. $\frac{3}{2}$
   14. 6
   15. $\frac{18}{7}$
   16. Check students’ work.
   17. If two parallel lines are cut by a transversal, alternate interior angles are congruent. Therefore, $\triangle ABC \sim \triangle EDC$ by the AA ~ Postulate.
   18. $x = 9; y = 6\sqrt{3}; z = 3\sqrt{3}$
   19. $x = \frac{10}{\sqrt{2}}; y = \frac{8}{3}
   20. C

Chapter 7 Test, Form B

1. 21
   2. 3.5
   3. 12.5
   4. $\triangle SLQ \sim \triangle NTR$ by SSS ~ Theorem
   5. $\triangle MBD \sim \triangle FWY$ by AA ~ Postulate
   6. $\triangle PRG \sim \triangle KRN$ by SAS ~ Theorem
   7. not similar
   8. 9.6 m
   9. 15
   10. B
   11. 9
   12. 6.4
   13. 3 : 1
   14. 2 : 3
   15. $x = 27$
   16. $x = 10; y = 4.5$
   17. 4.8

Alternative Assessment, Form C

**TASK 1: Scoring Guide**
AA Similarity Postulate and SAS and SSS Similarity Theorems; check students’ work.

- 3 Student gives correct answers and accurate drawings and explanations.
- 2 Student gives mostly correct answers, drawings, and explanations.
- 1 Student gives answers, drawings, and explanations that contain significant errors.
- 0 Student makes little or no effort.
Chapter 7 Answers (continued)

TASK 2: Scoring Guide
Claim 1 is true, and Claim 2 is false. Claim 3 is partly true; the triangles are similar, but the ratio of the areas is 1 : 4, not 1 : 2.

3 Student gives correct answers and provides logical reasons to support the answers.
2 Student gives mostly correct answers, but the work may contain minor errors or omissions.
1 Student gives answers that contain significant errors.
0 Student makes little or no effort.

3 Student gives a mathematically sound analysis.
2 Student gives an explanation that contains minor errors or omissions.
1 Student gives an explanation containing significant errors or omissions.
0 Student makes little or no effort.

TASK 3: Scoring Guide
One method is to pick any point $A$ on land and measure $AB$ and $AC$. Then select a fraction (say, $\frac{1}{2}$), and mark $X$ and $Y$ halfway from $A$ to $B$ and $C$, respectively. Then measure $XY$. Then $BC = 2 \cdot XY$.

3 Student gives a mathematically correct procedure and explanation.
2 Student gives a procedure and explanation that may contain minor errors.
1 Student gives an invalid procedure or a valid procedure with little or no explanation.
0 Student makes little or no effort.

TASK 4: Scoring Guide
Comparing the adult to the baby, we have:
- ratio of heights $\approx 3.5 : 1$
- ratio of head diameter $\approx 1.7 : 1$

Because the ratio of heights is not close to the ratio of head diameter, the baby and the adult are not similar. (The baby’s head will grow more slowly than the rest of the body.)

Cumulative Review
17. Sample:

18. If cows do not have six legs, then pigs cannot fly.
Chapter 8 Answers

Practice 8-1
1. \(\sqrt{\frac{51}{1}}\) 2. \(4\sqrt{7}\) 3. \(2\sqrt{65}\) 4. \(\sqrt{7}\)
5. \(2\sqrt{21}\) 6. \(18\sqrt{2}\) 7. 46 in. 8. 78 ft
9. 279 cm 10. 19 m 11. acute 12. right
13. obtuse 14. right 15. obtuse 16. acute

Practice 8-2
1. \(x = 2; y = \sqrt{3}\) 2. \(a = 4.5; b = 4.5\sqrt{3}\)
3. \(c = \frac{50\sqrt{5}}{3}; d = \frac{25\sqrt{5}}{3}\) 4. \(8\sqrt{2}\) 5. \(14\sqrt{2}\)
6. \(\frac{28\sqrt{3}}{3}\) 7. 2 8. \(x = 15; y = 15\sqrt{3}\) 9. \(3\sqrt{2}\)
10. 42 cm 11. 5.9 in. 12. 10.4 ft, 12 ft 13. 170 in.²
14. \(w = \frac{10\sqrt{3}}{3}; x = 5; y = 5\sqrt{2}; z = \frac{5\sqrt{3}}{3}\) 15. \(a = 4; b = 3\)
16. \(p = 4\sqrt{3}; q = 4\sqrt{3}; r = 8; s = 4\sqrt{6}\)

Practice 8-3
1. \(\tan E = \frac{3}{4}; \tan F = \frac{4}{3}\) 2. \(\tan E = 1; \tan F = 1\)
3. \(\tan E = \frac{2}{3}; \tan F = \frac{3}{2}\) 4. 12.4 5. 31.0 6. 14.1
7. 7.1 8. 2.3 9. 6.4 10. 78.7 11. 26.6
12. 71.6 13. 39 14. 72 15. 27 16. 68
17. 39 18. 54

Practice 8-4
1. \(\sin P = \frac{2\sqrt{10}}{5}; \cos P = \frac{3}{5}\) 2. \(\sin P = \frac{4}{5}; \cos P = \frac{3}{5}\)
3. \(\sin P = \frac{12}{13}; \cos P = \frac{5}{13}\) 4. \(\sin P = \frac{\sqrt{11}}{6}; \cos P = \frac{5}{6}\)
5. \(\sin P = \frac{\sqrt{2}}{2}; \cos P = \frac{\sqrt{2}}{2}\) 6. \(\sin P = \frac{15}{17}; \cos P = \frac{8}{17}\)
7. 64 8. 11.0 9. 7.0 10. 42 11. 7.8 12. 53
13. 6.6 14. 37 15. 11.0 16. 56 17. 11.5
18. 9.8

Practice 8-5
1a. angle of depression from the birds to the ship
1b. angle of elevation from the ship to the birds
1c. angle of depression from the ship to the submarine
1d. angle of elevation from the submarine to the ship
2a. angle of depression from the plane to the person
2b. angle of elevation from the person to the plane
2c. angle of depression from the person to the sailboat
2d. angle of elevation from the sailboat to the person
3. 116.6 ft 4. 84.8 ft 5. 46.7 ft 6. 31.2 yd
7. 127.8 m 8. 323.6 m
9a. 35° 30 ft
9b. 26 ft

Practice 8-6
1. (46.0, 46.0) 2. (−37.1, −19.7) 3. (89.2, −80.3)
4. 38.6 mi/h; 31.2° north of east 5. 134.5 m; 42.0° south of west
6. 1.3 m/sec; 38.7° south of east 7. 55° north of east
8. 20° west of south 9. 33° west of north
10a. (1.5, 5) 10b.

11a. (1, −1) 11b.
12a. (−7, −1) 12b.

13. Sample:

14. Sample:
Reteaching 8-1

1.–2. Sample:

3. Sample: 9, 16, and 25

4. They are equal.

5. The sum of the squares of the two legs equals the square of the hypotenuse.

Reteaching 8-2

1.–2. Sample:

3. Sample: 3.5 and 7

4. Sample: 3.5 and 7

5. Sample: They are approximately equal.

6. Check students' work.

7. $d = 2 \sqrt{3}; e = 4$

8. $f = \frac{4 \sqrt{3}}{3}, g = \frac{8 \sqrt{3}}{3}$

9. $x = 3 \sqrt{3}; y = 3$

10. $u = 6 \sqrt{3}; v = 12$

Reteaching 8-3

1. 26.6

2. 76.0

3. 80.5

4. 26.6

5. 8.1

6. 0.8

7. $m \angle A = 53.1; m \angle B = 36.9$

8. $m \angle A = 67.4; m \angle B = 22.6$

9. $m \angle A = 70.5;$

$m \angle B = 19.5$

10. $m \angle A = 63.4; m \angle B = 26.6$

11. 26.6

12. 63.4

13. 49.4

Reteaching 8-4

1. $DF = 2.7, EF = 7.5$

2. $XZ = 8.7, YZ = 10$

3. $RS = 8.4, ST = 13.1$

4. 5.8

5. 30, 60

6. 16.6;

73.4

8. 2.1; 3.4

9a. 7.1

9b. $\sqrt{51}$

Reteaching 8-5

1. 42.9 ft

2. 1113.8 ft

3. 2.2°

4. 42.0 ft

5. 76.1 ft

6. 19.1 ft

Reteaching 8-6

1. 12.8 mi/h at 51.3° off directly across

2a. 10 mi

2b. 36.9° west of due north

3. The bird’s path shifts 31.0° east of due south.

4. 4 mi/h

Enrichment 8-1

Friar Tuck is the winner. By using the Pythagorean Theorem, Friar Tuck’s string length is $4 \sqrt{2} + 2 \sqrt{5}$ or $\approx 10.1$ in. Little John’s string length is $4 \sqrt{5} + 2 \sqrt{2}$ or $\approx 11.8$ in.

Enrichment 8-2

1. $\sqrt{2}$

2. $\sqrt{3}$

3. $\sqrt{4}$ or 2

4. $\sqrt{5}$

5.

Enrichment 8-3

1. 3

2. $-2$

3. $45^\circ$

4. $135^\circ$; definition of supplementary angles

5. $45^\circ$; vertical angles are congruent.

6. $135^\circ$; vertical angles are congruent.

7.

Enrichment 8-4

1. 5.8 cm

2. 3.4 cm

3. 7.5 cm

4. 47

5. 25.5

6. 107.5

7. 0.731

8. 0.431

9. 0.951

10. 7.9

11. 7.9

12. 7.9

13. They are equal.

14. The sides of a triangle are proportional to the sides of their opposite angles.

15. 70

16. 15.55 km

17. 11.83 km

18. Lighthouse Q; it is closer.
Chapter 8 Answers (continued)

Enrichment 8-5
1. the top of the tower 2. PM
3. 
4. the angle of elevation from B to the top of the tower; 30°-60°-90°; scalene right triangle 5. the angle of elevation from A to the top of the tower; 45°-45°-90°; isosceles right triangle 6. tan 60° = \( \frac{x}{y} \); tan 45° = 1
7. tan 45° = \( \frac{x}{y} \); \( y + 100 \)
8. \( x = \tan 60^\circ \); \( x = \tan 45^\circ \) (y + 100)
9. \( 136.6 \); \( 100 \)
10. 236.6; the height of the tower
11. yes

Enrichment 8-6
1. (0.5, 0.5) 2. (−0.25, 1.80) 3. (−2.25, 1.80)
4. 2.9 km; 38.7° south of east

Chapter Project

Activity 1: Building
\( \triangle 1 \) and \( \triangle 2 \) are complementary, and \( \angle 2 \) and \( \angle 3 \) are complementary, so \( m\angle 1 = m\angle 3 \).

Activity 2: Measuring
Check students’ work.

Activity 3: Comparing
Check students’ work.

Activity 4: Researching
Check students’ work.

✓ Checkpoint Quiz 1
1. obtuse 2. right 3. acute 4. \( \tan A = \frac{4}{3} \), sin \( A = \frac{4}{5} \), cos \( A = \frac{3}{5} \); tan \( B = \frac{3}{4} \), sin \( B = \frac{3}{5} \), cos \( B = \frac{4}{5} \)
5. \( \tan A = 2 \), sin \( A = 0.8955 \), cos \( A = 0.4478 \); tan \( B = 0.5 \), sin \( B = 0.5 \), cos \( B = 0.8833 \)
6. \( \tan A = 0.8833 \), sin \( A = 0.6625 \), cos \( A = 0.75 \); tan \( B = 1.1321 \), sin \( B = 0.75 \), cos \( B = 0.6625 \)
7. 11.6 8. 41° 9. 16.5

✓ Checkpoint Quiz 2
1. (7, 79) 2. (16, −9) 3. (−60, 148) 4. 31.6, 71.6° north of east 5. 16.1, 29.7° south of west 6. 120.4, 48.4° north of west 7. 22.5 ft

Chapter Test, Form A
1. right 2. acute 3. obtuse 4. \( 2\sqrt{41} \) 5. 3
6. \( x = \frac{7\sqrt{2}}{2} \); \( y = \frac{7\sqrt{2}}{2} \) 7. \( x = 4 \); \( y = 4\sqrt{3} \)
8. \( \sin A = \frac{2\sqrt{42}}{22} \); \( \cos A = \frac{19}{22} \); \( \tan A = \frac{2\sqrt{42}}{19} \)
9. \( \sin A = \frac{2}{3} \); \( \cos A = \frac{\sqrt{3}}{3} \); \( \tan A = \frac{2\sqrt{3}}{3} \)
10. 67
11. 20.9 12. 76.0
13. 26.6 14a. angle of elevation from the person to the top of the tree 14b. angle of depression from the top of the tree to the birds 14c. angle of elevation from the top of the tree to the birds
15. 102 ft 16. 143 ft
17. Sample: The side opposite \( \angle X \) is the same as the side adjacent to \( \angle Y \). The side adjacent to \( \angle X \) is the same as the side opposite \( \angle Y \).

So, \( \tan x^\circ = \text{opposite} \angle X \); \( \tan \angle Y \) = \( \text{adjacent} \angle X \);
18. (92.9, 83.6) 19. (120.4, −258.3) 20. \( \approx 211.9 \) mi; \( \approx 19^\circ \) south of west
21. (11, 0) 22. (4, 3)

Chapter Test, Form B
1. right 2. obtuse 3. \( 2\sqrt{85} \) 4. \( x = 4\sqrt{3} \); \( y = 8\sqrt{3} \)
5. \( \sin T = \frac{6\sqrt{2}}{19} \); \( \sin T = \frac{17}{19} \); \( \tan T = \frac{6\sqrt{2}}{17} \)
6. \( \sin T = \frac{14}{17} \); \( \cos T = \frac{\sqrt{193}}{17} \); \( \tan T = \frac{14\sqrt{193}}{193} \)
7. 60
8. 15.3 9. 63 10. 36.9 11. 78.7 12a. angle of depression from the cloud to the person in the castle 12b. angle of elevation from the person in the cloud to the cloud 12c. angle of depression from the person in the castle to the person on the ground 12d. angle of elevation from the person on the ground to the castle 13. \( \approx 1710 \) ft
14. \( \sin x^\circ = \frac{\text{opposite}}{\text{hypotenuse}} \); \( \cos x^\circ = \frac{\text{adjacent}}{\text{hypotenuse}} \)
15. (−92.3, 59.9) 16. \( \approx 18.0 \) mi; \( \approx 33.7^\circ \) north of east
17. (1, 10)
18. Sample:

![Graph](image)

Alternative Assessment, Form C

**TASK 1 Scoring Guide:**

\[ x = 57.3 \]

3 Student gives correct answer and shows a valid method.
2 Student gives generally correct work that contains minor errors.
1 Student gives an incorrect answer, and the method is not correct.
0 Student makes little or no effort.

**TASK 2 Scoring Guide:**

111 ft

3 Student gives a correct answer and shows a valid method.
2 Student gives generally correct work that contains minor errors.
1 Student gives an incorrect answer, and the method is not correct.
0 Student makes little or no effort.

**TASK 3 Scoring Guide:**

\[ x = 1.53 \text{ mi}; y = 0.52 \text{ mi} \]

3 Student gives correct answer and shows a valid method.
2 Student gives generally correct work that contains minor errors.
1 Student gives an incorrect answer, and the method is not correct.
0 Student makes little or no effort.

**TASK 4 Scoring Guide:**

\begin{align*}
\text{a. } & \overrightarrow{a} (12, -3); \overrightarrow{b} (8, 8) \\
\text{b. } & \overrightarrow{a} (12, 4); \overrightarrow{b} (11, 3) \\
\text{c. } & \overrightarrow{a} \approx 14^\circ \text{ south of east}; \overrightarrow{b} 45^\circ \text{ north of east}
\end{align*}

3 Student gives correct answers.
2 Student gives answers that are mostly correct.
1 Student gives mostly incorrect answers.
0 Student makes little or no effort.

**Cumulative Review**


14. 10 15. Sample: You may have used the branches to climb into the tree. 16. Sample: Suppose that an equilateral triangle has an obtuse angle. Then one angle in the triangle has measure greater than 90. This contradicts the fact that, in an equilateral triangle, the measure of each angle is 60.

17. Sample: The side opposite the 90° angle is also the hypotenuse, and there are two adjacent sides. 18. Sample: The sum of the three angles of a triangle is 180°. On a sphere, the sum of the three angles will be greater than 180°.
Chapter 9 Answers

Practice 9-1
1. No; the triangles are not the same size.  2. Yes; the hexagons are the same shape and size.  3. Yes; the ovals are the same shape and size.  4a. \( \angle C' \) and \( \angle F' \)  4b. \( CD \) and \( C'D' \), \( DE \) and \( D'E' \), \( EF \) and \( E'F' \), \( CF \) and \( C'F' \)  5a. \( MN \) and \( M'N' \), \( NO \) and \( N'O' \), \( MO \) and \( M'O' \)  6. \((x, y) \rightarrow (x - 2, y - 4)\)  7. \((x, y) \rightarrow (x - 2, y - 2)\)  8. \((x, y) \rightarrow (x - 3, y - 1)\)  9. \((x, y) \rightarrow (x + 4, y - 2)\)  10. \((x, y) \rightarrow (x - 5, y + 1)\)  11. \((x, y) \rightarrow (x + 2, y + 2)\)  12. \(W'(-2, 2), X'(-1, 4), Y'(3, 3), Z'(2, 1)\)  13. \(J'(-5, 0), K'(-3, 4), L'(-3, -2)\)  14. \(M'(3, -2), N'(6, -2), P'(7, -7), Q'(4, -6)\)  15. \((x, y) \rightarrow (x + 4.2, y + 11.2)\)  16. \((x, y) \rightarrow (x + 13, y - 13)\)  17. \((x, y) \rightarrow (x, y)\)  18. \((x, y) \rightarrow (x + 3, y + 3)\)  19a. \(P'(-5, -1)\)  19b. \(P'(0, 8), N'(-5, 2), Q'(2, 3)\)

Practice 9-2
1. \((-3, -2)\)  2. \((-2, -3)\)  3. \((-1, -4)\)  4. \((4, -2)\)  5. \((4, -1)\)  6. \((3, -4)\)  7a.

Practice 9-3
1. \(I\)  2. \(I\)  3. \(I\)  4. \(GH\)  5. \(G\)  6. \(ST\)  7.

Geometry Chapter 9

Answers
Chapter 9 Answers (continued)

Practice 9-4
1. The helmet has reflectional symmetry.
2. The teapot has reflectional symmetry.
3. The hat has both rotational and reflectional symmetry.
4. The hairbrush has reflectional symmetry.
5. 

6. 

7. This figure has no lines of symmetry.
8. 

9. 

10. 

11. 

12. 

13.
Chapter 9 Answers (continued)

14. line symmetry and 45° rotational symmetry

15. 180° rotational symmetry

16. line symmetry

17. 18.

19. 20.

21. Practice 9-5

1. $L'(-2, -2)$, $M'(-1, 0)$, $N'(2, -1)$, $O'(0, -1)$

2. $L'(-30, -30)$, $M'(-15, 0)$, $N'(30, -15)$, $O'(0, -15)$

3. $L'(-12, -12)$, $M'(-6, 0)$, $N'(12, -6)$, $O'(0, -6)$

4. $\frac{2}{3}$

5. $\frac{5}{3}$

6. 2

7. yes

8. no

9. no

10. R

11. A

12. P

13. $P'(-12, -12)$, $Q'(-6, 0)$, $R'(0, -6)$

14. $P'(-\frac{1}{2}, \frac{1}{2})$,

15. $Q'(1\frac{1}{2}, -\frac{1}{2})$, $R'(1\frac{1}{2}, 2)$

16. $P'(-2, 1)$, $Q'(-1, 0)$, $R'(0, 1)$

Practice 9-6

1. I. D II. C III. B IV. A

2. I. B II. A III. C IV. D

3. B

4. $\ell \parallel m$

5. J

6. $\ell \parallel m$
Chapter 9 Answers (continued)

7. reflection
   
8. translation
   
9. glide reflection
   
10. reflection
   
11. rotation
   
12. glide reflection

Practice 9-7

1. translational symmetry
   
2. line symmetry, rotational symmetry, translational symmetry, glide reflectional symmetry
   
3. translational symmetry
   
4. line symmetry, rotational symmetry, translational symmetry, glide reflectional symmetry
   
5. line symmetry, rotational symmetry, translational symmetry, glide reflectional symmetry
   
6. rotational symmetry, translational symmetry, glide reflectional symmetry
   
7. translational symmetry
   
8. translational symmetry
Chapter 9 Answers (continued)

9.–11. Samples:

9. 

10. 

11. 

12. yes 13. yes 14. no 15. yes 16. no 17. no

Reteaching 9-1

1.–5. Check students’ work. 6. $A'(0, -3), B'(1, 1), C'(4, -1), D'(5, -4)$ 7. $A'(4, -2), B'(5, 2), C'(8, 0), D'(9, -3)$ 8. $A'(3, 5), B'(4, 9), C'(7, 7), D'(8, 4)$

Reteaching 9-2

6. reflection across $x$-axis: $F'(-1, -3), G'(-5, -1), H'(-3, -5)$; reflection across $y$-axis: $F'(1, 3), G'(5, 1), H'(3, 5)$ 7. reflection across $x$-axis: $C'(2, -4), D'(5, -2), E'(6, -3)$; reflection across $y$-axis: $C'(-2, 4), D'(-5, 2), E'(-6, 3)$ 8. reflection across $x$-axis: $J'(-1, 5), K'(-2, 3), L'(-4, 6)$; reflection across $y$-axis: $J'(1, -5), K'(2, -3), L'(4, -6)$

Reteaching 9-3

6. 

7. Sample:

Reteaching 9-4

1. two lines of symmetry (vertical and horizontal), $180^\circ$ rotational symmetry (point symmetry) 2. one line of symmetry (horizontal) 3. one line of symmetry (vertical) 4. two lines of symmetry (vertical and horizontal), $180^\circ$ rotational symmetry (point symmetry) 5. one line of symmetry (vertical) 6. one line of symmetry (vertical)

Reteaching 9-5

1. Check students’ work. 2.

3. 

4. 

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Chapter 9 Answers (continued)

Reteaching 9-6

Reteaching 9-7
1.

2. Sample:

3.

4.

5. line symmetry across the dashed lines, rotational symmetry around points, translational symmetry, glide reflectional symmetry

Enrichment 9-1
1. \((x, y) \rightarrow (x - 7, y - 2)\)  2. \((x, y) \rightarrow (x + 4, y - 7)\)  3. \((x, y) \rightarrow (x - 10, y + 5)\)  4. \((x, y) \rightarrow (x - 3, y - 4)\)  5. Foster  6. yes  7. Wilson  8. \((x, y) \rightarrow (x - 7, -3)\)  9. C

Enrichment 9-2
1. \((-3, -1)\)  2. \((-1, 0)\)  3. \((5, 3)\)  4. the midpoint formula: \(M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\)  5.

6. \(y = \frac{1}{2}x + \frac{1}{2}\)

7.

y = -4
Chapter 9 Answers (continued)

8. $y = -x$

9. $x = 1$

10. $y = -2x - 2$

11. $y = 3x + 2$

12. $y = -\frac{1}{2}x - 1$

Enrichment 9-3
1. $(0, -2, 3)$  2. $(-2, -2, 3)$  3. $(-2, -2, 0)$
4. $(0, -2, 0)$  5. $(0, -6, 0)$  6. $(0, -6, 3)$
7. $(-2, -6, 3)$  8. $(-2, -6, 0)$  9. $(2, 0, 3)$
10. $(2, -2, 3)$  11. $(2, -2, 0)$  12. $(2, 0, 0)$
13. $(6, 0, 0)$  14. $(6, 0, 3)$  15. $(6, -2, 3)$
16. $(6, -2, 0)$

Enrichment 9-4
1. yes; rotational and point symmetry  2. no  3. yes; rotational and point symmetry  4. yes; rotational and point symmetry  5. no  6. no  7. diamonds  8. 12
9. Seven of diamonds; this does not have symmetry because the diamond in the middle is toward either the bottom or top of the card, and when you rotate the card 180°, the position will be reversed.  10. All the face cards have symmetry.
11. yes; 2, 4, 10  12. No; Sample: When you look at the card one way, three of the points of the hearts are pointing down, and five are pointing up. When you rotate the card 180°, five of the points of the hearts are pointing down, and three are pointing up.
13. No; because the number and suit of each card are placed in opposite corners, none of the cards have line symmetry.
14. You can add a backward 3 with the small club below it to the two empty corners to create line symmetry, or you can remove the 3 with the small club below it from each of the two corners.

Enrichment 9-5
1a. $(2, 0, 3)$  1b. $(0, 0, 2)$  1c. $(0, 2, 3)$  1d. $(2, 2, 3)$
1e. $(2, 0, 0)$  1f. $(0, 0, 0)$  1g. $(0, 2, 0)$  1h. $(2, 2, 0)$
2a. $(4, 0, 4)$  2b. $(0, 0, 4)$  2c. $(0, 4, 4)$  2d. $(4, 4, 4)$
2e. $(4, 0, 0)$  2f. $(0, 0, 0)$  2g. $(0, 4, 0)$  2h. $(4, 4, 0)$
3. $(0, 0, 0)$  4. 2  5. Each edge in the image is double that in the preimage.
6. Samples: Three faces are coplanar for image and preimage; three faces are parallel; each face in the image has two times the perimeter and four times the area of the corresponding face in the preimage.
7. Surface area of image = 96 sq. units; surface area of preimage = 24 sq. units; the ratio of the surface areas is 4 : 1.
8. Volume of image = 64 cubic units; volume of preimage = 8 cubic units; the ratio of the volumes is 8 : 1. 9. Dilations in three dimensions are proportional to dilations in two dimensions.

Enrichment 9-6
1.–9. Check students’ work. 10. 12-3; rotation; twice
11. 130 12. 100 13. 160

Enrichment 9-7

Activity 3: Investigating
a, b

Activity 4: Classifying
a. mg
b. 12

Activity 5: Creating
Check students’ work.

✔ Checkpoint Quiz 1
1. No; the figures are not the same size. 2. yes
3. yes 4. \((x, y) \rightarrow (x - 3, y - 2)\) 5. 6 units right and 4 units down 6. a resultant translation of 4 units right and 11 units up
7.
8.
9.
10. \(T'(8, -5), Q'(2, -3)\) and \(R'(4, 0)\)

✔ Checkpoint Quiz 2
1. line symmetry; rotational; 180°
2. line symmetry; rotational; 180°
3. line symmetry
4. rotational; 180° 5. translational; no turns
6. rotational; preimage turns 7. translational
8. rotational, reflectional, translational 9. translational

Chapter Test, Form A
1. \(P'(13, 0), Q'(10, -2), R'(11, -4), S'(13, -6)\)
2. \(P'(-1, -2), Q'(2, -4), R'(1, -6), S'(-1, -8)\)
3. \(P'(0, -5), Q'(2, -2), R'(4, -3), S'(6, -5)\)
4. \(P'(-2.5, 0), Q'(-1, -1), R'(-1.5, -2), S'(-2.5, -3)\)
5. \(P'(5, 2), Q'(3, 2), R'(3, -2), S'(5, -4)\)
6. \(P'(-11, 3), Q'(-8, 1), R'(-9, -1), S'(-11, -3)\)
7. glide reflection
8. translation 9. translation 10. rotation

Chapter Project

Activity 1: Investigating
a. vertical
b. vertical
c. both

Activity 2: Modeling
a. Ancient Egyptian ornament
b. Oriental design
c. Greek vase design

Activity 3: Investigating
a, b

Activity 4: Classifying
a. mg
b. 12

Activity 5: Creating
Check students’ work.

✔ Checkpoint Quiz 1
1. No; the figures are not the same size. 2. yes
3. yes 4. \((x, y) \rightarrow (x - 3, y - 2)\) 5. 6 units right and 4 units down 6. a resultant translation of 4 units right and 11 units up
7.
8.
9.
10. \(T'(8, -5), Q'(2, -3)\) and \(R'(4, 0)\)

✔ Checkpoint Quiz 2
1. line symmetry; rotational; 180°
2. line symmetry; rotational; 180°
3. line symmetry
4. rotational; 180° 5. translational; no turns
6. rotational; preimage turns 7. translational
8. rotational, reflectional, translational 9. translational

Chapter Test, Form A
1. \(P'(13, 0), Q'(10, -2), R'(11, -4), S'(13, -6)\)
2. \(P'(-1, -2), Q'(2, -4), R'(1, -6), S'(-1, -8)\)
3. \(P'(0, -5), Q'(2, -2), R'(4, -3), S'(6, -5)\)
4. \(P'(-2.5, 0), Q'(-1, -1), R'(-1.5, -2), S'(-2.5, -3)\)
5. \(P'(5, 2), Q'(3, 2), R'(3, -2), S'(5, -4)\)
6. \(P'(-11, 3), Q'(-8, 1), R'(-9, -1), S'(-11, -3)\)
7. glide reflection
8. translation 9. translation 10. rotation
Chapter 9 Answers (continued)

11.

12. line symmetry  13. rotational symmetry
14a. Any two of the following: ABCDEHIKOMOTUVWXYZ
14b. Any two of the following: HINOSXZ
15. Translations can be performed on the pieces because they can slide. Rotations can be performed on the pieces because each piece can be turned. Reflections and glide reflections cannot be performed on the pieces because the back of a puzzle piece is useless in solving the puzzle. Dilations are impossible because the pieces cannot change size.
16. A  17. Sample: glide reflectional, rotational, line, and translational

18.  \(L'(-3, 6), M'(6, 6), N'(3, -9)\)  19.  \(L' \left(\frac{1}{3}, -\frac{3}{2}\right), M' \left(\frac{3}{4}, 1\right), N' \left(\frac{3}{2}, -\frac{1}{2}\right)\)
20.  \(L'(-18, -18), M'(-18, 18), N'(0, 0)\)
21.  \(L'(3, -2), M'(0, -\frac{5}{3}), N'(\frac{7}{3}, -2)\)
22.  \(Y'Z' = 15 \text{ cm}; X'Z' = 24 \text{ cm}; \text{ scale factor } = \frac{3}{7}\)
23.  \(X'Y' = 25 \text{ in.}; Y'Z' = 40 \text{ in.}; \text{ scale factor } = \frac{5}{3}\)
24.  \((-1, 3)\)  25.  \((5, 4)\)  26.  \((-2, -7)\)  27.  \((2, -7)\)
28.  \((1, -1)\)  29.  \((-3, 1)\)

Chapter Test, Form B

1. Yes; the figures are the same shape and size.  2. No; the figures are not the same size.  3. Yes; the figures are the same shape and size.  4. \((x, y) \rightarrow (x - 6, y + 1)\)
5. \((x, y) \rightarrow (x + 3, y - 7)\)  6. \(A'(0, -3), B'(-4, 0), C'(-6, 4), D'(-3, -5)\)  7. \(A'(6, 3), B'(10, 0), C'(12, 4), D'(9, 5)\)
8. \(A'(3, 8), B'(-1, 5), C'(-3, 9), D'(0, 10)\)
9. \(A'(5, 2), B'(-1, -1), C'(1, -3), D'(2, 4)\)  10. \(A'(-3, 0), B'(0, -4), C'(-4, 6), D'(-5, 3)\)  11. \(A'(0, 3), B'(4, 6), C'(6, 2), D'(3, 1)\)
12. \((x, y) \rightarrow (x + 4, y - 3)\)
13. \((x, y) \rightarrow (x - 3, y + 6)\)  14. rhombus and two triangles; translation
15. \(A'\)
16. \(B'\)
17. 180° rotational symmetry  18. line symmetry
19. line symmetry; 180° rotational symmetry

Alternative Assessment, Form C

TASK 1: Scoring Guide

20; 200 cm; all distances are magnified 20 times.

3 Student gives correct answers and a clear and accurate explanation.
2 Student gives answers and an explanation that are largely correct.
1 Student gives answers or an explanation that may contain serious errors.
0 Student makes little or no effort.

TASK 2: Scoring Guide

Sample:

3 Student draws a figure that accurately reflects the stated conditions.
2 Student draws a figure that is mostly correct but falls just short of satisfying all the stated conditions.
1 Student draws a figure that has significant flaws relative to the stated conditions.
0 Student makes little or no effort.
TASK 3: Scoring Guide
Sample:
The car can undergo translations and rotations as it moves and turns. But it can undergo neither a dilation, which would change its size, nor a reflection, which would, for example, put the steering wheel on the passenger side.

3 Student gives a correct and thorough explanation.
2 Student gives an explanation that demonstrates understanding of transformations but may contain minor errors or omissions.
1 Student gives an explanation containing major errors or omissions.
0 Student makes little or no effort.

TASK 4: Scoring Guide
Sample:

a. translation:
translation vector: \( (1, 0) \)

b. dilation:
scale factor: 2

c. reflection:
line of reflection: \( y = 1 \)

d. glide reflection:
The glide reflection is equivalent to a translation \((2, 0)\) followed by a reflection across the line \( y = -2 \).

e. rotation:
degrees of rotation: 90

Cumulative Review

19.
20. Equilateral triangle, square, and regular hexagon; the measure of each interior angle is a factor of 360.
21. If a triangle is not a right triangle, then the Pythagorean Theorem cannot be applied.
22.
23. 21.21
Chapter 10 Answers

Practice 10-1
1. 8 2. 8 3. 60 4. 240 5. 2.635 6. 1.92
7. 8.125 8. 9 9. 1500 10. 8.75 11. 3.44
12. 7 13. 110 14. 30 15. 16 16. 119
17. 12 18. 48 19. 40 20. 56

Practice 10-2
1. 48 cm² 2. 784 in² 3. 11.4 ft² 4. 90 m²
5. 2400 in² 6. 374 ft² 7. 160 cm² 8. 176.25 in²
9. 54 ft² 10. 42.5 square units 11. 96√3 square units
12. 36√3 square units 13. 45 cm² 14. 226.2 in²
15. 49,500 m²

Practice 10-3
1. \( x = 7; y = 3.5\sqrt{3} \)
2. \( a = 60; c = \frac{8\sqrt{3}}{3}; d = \frac{8\sqrt{3}}{3} \)
3. \( p = 4\sqrt{3}; q = 8; x = 30 \)
4. \( m\angle 1 = 45; m\angle 2 = 45; m\angle 3 = 90; m\angle 4 = 90 \)
5. \( m\angle 5 = 60; m\angle 6 = 90; m\angle 7 = 120 \)
6. \( m\angle 8 = 60; m\angle 9 = 30; m\angle 10 = 60 \)
7. \( 12\sqrt{3} \)
8. \( 36.75\sqrt{3} \)
9. \( 75\sqrt{3} \)
10. 120 in²
11. 137 in²
12. 97 in²

Practice 10-4
1. \( 4:5; 16:25 \)
2. \( 5:3; 25:9 \)
3. \( 3:4; 9:16 \)
4. \( 1:2 \)
5. \( 9:5 \)
6. \( 1:6 \)
7. \( 8:3 \)
8. \( \frac{576}{5} \text{ in}^2 \)
9. \( \frac{2700}{7} \text{ cm}^2 \)
10. \( \frac{1152}{25} \text{ in}^2 \)
11. 2:3
12. 1:2
13. 2:5
14. 108 ft²

Practice 10-5
1. 174.8 cm²
2. 578 ft²
3. 1250.5 mm²
4. 192.6 m²
5. 1131.4 in²
6. 419.2 cm²
7. 324.9 in²
8. 162 cm²
9. 37.6 m²
10. 30.1 m²
11. 55.4 km²
12. 357.6 in²
13. 9.7 mm²
14. 384.0 in²
15. 54.5 cm²
16. 9.1 ft²
17. 80.9 m²
18. 83.1 m²
19. 65.0 ft²
20. 119.3 ft²
21. 8 m³
22. 55.4 ft²

Practice 10-6
1. \( 32\pi \)
2. \( 16\pi \)
3. \( 7.8\pi \)
4. Samples: \( \triangle \overline{DAF}, \overline{ABF} \)
5. Samples: \( \overline{FE}, \overline{FD} \)
6. Samples: \( \overline{FEB}, \overline{EDA} \)
7. Samples: \( \overline{FE}, \overline{ED} \)
8. \( \triangle \overline{ACB} \)
9. \( \angle FC \)
10. 32
11. 43
12. 54
13. 39
14. 71
15. 90
16. 50
17. 220
18. 140
19. 220
20. 130
21. 6\pi \text{ in.}
22. 16\pi \text{ cm}
23. \( \frac{9}{4}\pi \text{ m} \)

Practice 10-7
1. \( 49\pi \)
2. 21.2
3. \( \frac{49}{3}\pi \)
4. \( \frac{49}{3}\pi - 21.2 \)
5. \( \frac{1}{3}\pi \)
6. \( \frac{7}{3}\pi \)
7. \( \frac{110}{16}\pi \)
8. \( \frac{17}{10}\pi - \frac{7}{8}\pi \)
9. \( \frac{4}{9}\pi \)
10. \( \frac{18}{5}\pi \)
11. \( \frac{9}{6}\pi \)
12. \( \frac{15}{2}\pi \)
13. \( \frac{1}{3}\pi \)
14. \( 6\pi \)

Practice 10-8
1. 8.7%
2. 10.9%
3. 15.3%
4. \( \frac{3}{16} \)
5. \( \frac{7}{9} \)
6. \( \frac{5}{7} \)
7. \( \frac{7}{9} \)
8. 1
9. \( \frac{5}{7} \)
10. 22.2%
11. 4.0%
12. 37.5%
13. 20%
14. 50%
15. 40%
16. 33\% 

Reteaching 10-1
1. Samples:

2. Samples:

3. 20 cm²
4. 55 in²
5. 6 m
6. 8 ft
7. 4 in.
8. Samples:

9. 5 in.
10. 23 cm
11. 9 ft

Reteaching 10-2
1. Sample:

2. Sample: bases = 6 cm and 10 cm, height = 6 cm
3. Sample: 48 cm²
4. Check students’ work.
5. Sample:

6. Sample: diagonals = 4 cm and 9 cm
7. Sample: 18 cm²
8. Check students’ work.
9. 144 ft²
10. 112 in²
11. 75 m²
12. 98.4 cm²
Chapter 10 Answers (continued)

Reteaching 10-3
1.–2. Sample:

3. Sample: apothem = 2.5, radius = 3.5, side = 5
4. yes 5. They are approximately equal.

Reteaching 10-4
1. 4 : 3; 16 : 9 2. 8 : 5; 64 : 25 3. 9 : 5; 81 : 25
4. 4 : 3 5. 4 : 5 6. 3 : 5 7. 16 : 9 8. 16 : 25
9. 9 : 25

Reteaching 10-5
1. 19.3 in.² 2. 123.1 cm² 3. 86.6 in.² 4. 172.0 in.²
5. 363.3 in.² 6. 126.3 in.² 7a. 81 in.² 7b. area of square = s²

Reteaching 10-6
1. Sample:

2. Sample: 135° 3. Sample: 2 cm 4. Sample: \( \frac{3\pi}{2} \) cm = 4.7 cm 5. Sample: 4.5 cm 6. They are approximately equal.
7. \( \frac{5\pi}{2} \) 8. \( \frac{10\pi}{3} \) 9. 5π 10. \( \frac{20\pi}{3} \) 11. \( \frac{5\pi}{2} \)
12. \( \frac{25\pi}{6} \) 13. \( \frac{15\pi}{2} \)

Reteaching 10-7
1.–2. Sample:

3. Sample: 50 cm² 4. Sample: 50.3 cm² 5. They are approximately equal. 6. Check students’ work.
7. 25\(\pi\) ft² 8. 4\(\pi\) in.² 9. 64\(\pi\) m² 10. 20.25\(\pi\) ft²
11. 324\(\pi\) cm² 12. 64\(\pi\) in.² 13. 14 cm 14. 615 cm²

Reteaching 10-8
1. Check students’ work. 2. \( \approx 21\% \) 3. Check students’ work. 4. Sample: They are approximately equal.
5. Sample: The ratio would tend to be close to 21%. 6. \( \approx 43\% \)

Enrichment 10-1
1. each segment is the hypotenuse of a right triangle with legs \( a \) and \( b \). 2. No; the angles must be shown to be right angles.
3. They are congruent; SSS 4. 180° 5. 90°; acute angles of a right triangle are complementary.
6. CPCTC 7. \( m\angle SPQ = 180 - (m\angle APS + m\angle PSA) \) = 180 – 90 = 90° 8. Repeat steps similar to those in Exercises 4–7 to show that \( \angle PQR, \angle QRS, \) and \( \angle RSP \) are right angles.
9. \( a + b; (a + b)^2 = a^2 + 2ab + b^2 \)
10. \( c; c^2 \)
11. \( a, b, \frac{1}{2}ab \) 12. The areas of congruent triangles are equal.
13. \( a^2 + 2ab + b^2 = 4\left(\frac{1}{2}ab\right) + c^2 \) or \( a^2 + 2ab + b^2 = 2ab + c^2 \) 14. By subtracting 2ab from both sides, you obtain \( a^2 + b^2 = c^2 \) (The Pythagorean Theorem).

Enrichment 10-2
1. Choose an interior point \( X; \) construct \( PX \parallel BC \), \( XQ \parallel AC \), and \( XR || AB \). 2. A triangle is a trapezoid with one side where \( b_1 = 0 \). Hence \( A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}hb \).
3. Area of \( \triangle BDE \) = Area \( \triangle ABC \) – Area \( \triangle BDC 
= \left[ \frac{1}{2}h(y + x + b_1) \right] - \left[ \frac{1}{2}h(y + x) \right] 
= \frac{1}{2}hy + \frac{1}{2}hx + \frac{1}{2}hb - (\frac{1}{2}hy + \frac{1}{2}hx) 
= \frac{1}{2}hb 

Enrichment 10-3
1. 19.5 square units 2. 47 square units 3. 73 square units

Enrichment 10-4
1. \( \frac{1}{4} \) 2. \( \frac{2}{3} \) 3. \( \frac{4}{5} \) 4. \( \frac{1}{9} \)

Enrichment 10-5
1. 36 cm² 2. \( 4\sqrt{3} \) cm or about 6.93 cm 3. 6 cm
4. \( A = 6\sin\theta \), \( 0^\circ < \theta < 180^\circ \)

Enrichment 10-6
1.–5. \[ \begin{array}{|c|c|c|c|}
\hline
Object & Radius (r) & Diameter (d) & Circumference (C) & \frac{C}{d} \\
\hline
1. & Quarter & 1.15 cm & 2.3 cm & 7.222 cm & 3.14 \\
2. & CD & 2\frac{1}{2} in. & 4\frac{1}{2} in. & 14.92 in. & -3.14 \\
3. & Circle with a radius of 4 in. & 4 in. & 8 in. & 8\pi & \pi \\
4. & Check student’s work. & & & -3.14 \\
5. & Check student’s work. & & & -3.14 \\
\hline
\end{array} \]
6. They are approximately equal. 7. \( \pi = 3.14 \)
8. \[ \begin{array}{|c|c|c|}
\hline
Object & Radius (r) & Diameter (d) & Circumference (C) & \frac{C}{d} \\
\hline
1. & Quarter & 1.15 cm & 2.3 cm & 7.222 cm & 3.14 \\
2. & CD & 2\frac{1}{2} in. & 4\frac{1}{2} in. & 14.92 in. & -3.14 \\
3. & Circle with a radius of 4 in. & 4 in. & 8 in. & 8\pi & \pi \\
4. & Check student’s work. & & & -3.14 \\
5. & Check student’s work. & & & -3.14 \\
\hline
\end{array} \]
Chapter 10 Answers (continued)

Enrichment 10-7
1. $\pi$ 2. $\sqrt{3}$ 3. $\frac{3\sqrt{3}}{2}$ 4. $\pi - \frac{3\sqrt{3}}{2}$ 5. $\frac{1}{2}$
6. $\frac{\sqrt{3}}{4}$ 7. 4 8. 1 9. $\pi$ 10. $\sqrt{2}$ 11. 2
12. $\pi - 2$

Enrichment 10-8
1. 0.06 or $\frac{1}{18}$ 2. 0.41 3. 0.63 or $\frac{\sqrt{3}}{1 + \sqrt{3}}$ 4. 0.21

Chapter Project
Activity 1: Modeling
36 in.$^2$; 30.25 in.$^2$; the seams in the finished quilt block account for the difference.

Activity 2: Creating
Check students’ work.

Activity 3: Researching
Check students’ work.

☑ Checkpoint Quiz 1
1. 37.2 m$^2$ 2. 20 ft$^2$ 3. 63 m$^2$ 4. 96 in.$^2$
5. 200 ft$^2$ 6. 648 m$^2$ 7. 3:7:9:49

☑ Checkpoint Quiz 2
1. 119.3 ft$^2$ 2. 293.9 m$^2$ 3. 523.1 in.$^2$ 4. 6.9 in.
5. 20.9 ft 6. 10 ft mi; 25 ft mi$^2$ 7. 18 ft cm; 81 ft cm$^2$
8. $\frac{3}{4}$ or about 21% 9. $\frac{1}{2}$ or 50%

Chapter 10 Test, Form A
1. 15.6 ft$^2$ 2. 41.6 cm$^2$ 3. 192 ft$^2$ 4. 80 square units
5. 120 square units 6. 50 square units
7. $\frac{64}{9}$ 8. $\frac{9}{1}$ 9. D 10. 18.8 cm$^2$ 11. 19.1 in.$^2$
12. 268.4 ft$^2$ 13. 166.3 in.$^2$ 14. 172.0 cm$^2$
15. Sample: Draw an altitude to form a $30^\circ$-$60^\circ$-$90^\circ$ triangle.
The altitude is $4\sqrt{3}$, and the area is $32\sqrt{3}$.
16. 13.5 cm
17. 20 square units 18. $12\pi$ 19. $36\pi$ 20. $\frac{4}{3}\pi$
21. $\frac{3}{2}\pi$ 22. $\frac{5}{3}\pi$ 23. 4.44 square units 24. 21.5%
25. 9.1%

Chapter 10 Test, Form B
1. 110.9 ft$^2$ 2. 649.5 cm$^2$; 649.6 cm$^2$; or 649.7 cm$^2$
3. 40.2 ft$^2$ 4. 60 ft$^2$ 5. 92 cm$^2$ 6. 72 in.$^2$
7. $\frac{16}{81}$ 8. $\frac{1}{4}$ 9. A 10. 72.7 in.$^2$ 11. 42.2 cm$^2$
12. 21.6 cm 13. 385 cm$^2$; 386 cm$^2$; or 387 cm$^2$
14. $36\pi$ 15. 324$\pi$ 16. $4\pi$ 17. $18\pi$
18. 75.4 cm$^2$ 19. $144\pi$ cm$^2$ 20. $4\pi$ in.$^2$

Alternative Assessment, Form C

TASK 1: Scoring Guide
Sample:

A parallelogram with base $b$ and height $h$ has the same area as two triangles with base $b$ and height $h$. Thus, the area of the parallelogram is: $A = 2\left(\frac{1}{2}bh\right) = bh$.

b.

A trapezoid with bases $b_1$ and $b_2$ and height $h$ has the same area as two triangles. The bases of the two triangles are $b_1$ and $b_2$, and the heights of both are $h$. Thus, the area of the trapezoid is: $A = \frac{1}{2}b_1h + \frac{1}{2}b_2h = \frac{1}{2}h(b_1 + b_2)$.

3 Student gives accurate explanations accompanied by clear and correct diagrams.
2 Student gives generally correct explanations, although there may be some errors in either the explanations or the diagrams.
1 Student gives inaccurate or incomplete explanations and diagrams.
0 Student makes little or no attempt.

TASK 2: Scoring Guide
Sample: $C(2, 8), D(-2, 5)$. The slopes of $\overline{AB}$ and $\overline{CD}$ are equal, and the slopes of $\overline{BC}$ and $\overline{AD}$ are equal, so $ABCD$ is a parallelogram. The slopes of $\overline{AB}$ and $\overline{BC}$ are opposite reciprocals, so $\overline{AB} \perp \overline{BC}$, and $ABCD$ is a rectangle. $AB = 5$ and $BC = 5$, so the area of $ABCD$ is 25 square units.

3 Student gives correct answers and provides complete and accurate reasoning.
2 Student gives generally correct answers and reasoning.
1 Student gives generally incorrect answers and reasoning.
0 Student makes little or no attempt.

TASK 3: Scoring Guide
Sample:

a. $\triangle QXY$ is a $45^\circ$-$45^\circ$-$90^\circ$ triangle,
so the apothem $QY = XY = \frac{1}{\sqrt{2}}$ or $\sqrt{2}$.
The perimeter is $4(2XY) = 4\left(\frac{\sqrt{2}}{2}\right) = 4\sqrt{2}$.
The area is $(2XY)^2 = 4 \cdot \left(\frac{\sqrt{2}}{2}\right)^2 = 4\left(\frac{1}{2}\right) = 2$. 

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Geometry Chapter 10
Answers
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Chapter 10 Answers (continued)

b. \(\triangle PAB\) forms a 30°-60°-90° triangle, so \(AB = \frac{1}{\sqrt{3}}\) or \(\frac{\sqrt{3}}{3}\).

The perimeter is \(6(2AB) = 12\sqrt{3} = 4\sqrt{3}\).

The area is \(\frac{1}{2}ap = \frac{1}{2}(4\sqrt{3}) = 2\sqrt{3}\).

3 Student gives clear and correct diagrams and explanations.
2 Student gives diagrams and explanations that may contain some minor errors.
1 Student gives diagrams or explanations that contain major errors or omissions.
0 Student makes little or no effort.

TASK 4: Scoring Guide
Sample:
a. a circle
b. \(a\) and \(r\) are nearly equal; \(p\) and \(C\) are nearly equal.
c. If the number of sides is great enough that the polygon becomes indistinguishable from a circle, the two formulas are equivalent, and one can be derived from the other.

For example:
\[
A = \frac{1}{2}ap \\
= \frac{1}{2}rC \\
= \frac{1}{2}r(2\pi r) \\
= \pi r^2
\]

3 Student gives correct answers to parts a and b and gives an adequate explanation relating the two formulas in part c.
2 Student gives correct answers to parts a and b but exhibits difficulty with the logic of part c.
1 Student gives answers and an explanation that may contain significant errors.
0 Student makes little or no effort.

Cumulative Review
14. J  15. A  16. J  17. Sample: You may have used the branches to climb into the tree. 18. Sample:
Suppose that an equilateral triangle has an obtuse angle. Then one angle in the triangle has measure greater than 90. This contradicts the fact that, in an equilateral triangle, the measure of each angle is 60.

19. Sample:

20. \(16\pi - 24\sqrt{3}\)  21. 61.5 m²  22. Euclid
Chapter 11 Answers

Practice 11-1
8. triangle  9. rectangle  10. rectangle
11. rectangle
12. rectangle

Practice 11-2
1. 62.8 cm²  2. 552.9 ft²  3. 226.2 cm²  4. 113.1 m²
5. 44.0 ft²  6. 84.8 cm²  7a. 320 m²  7b. 440 m²
8a. 576 in²  8b. 684 in²  9a. 216 mm²
9b. 264 mm²  10a. 48 cm²  10b. 60 cm²
11a. 1500 m²  11b. 1800 m²  12a. 2000 ft²
12b. 2120 ft²

Practice 11-3
1. 64 m²  2. 620 cm²  3. 188 ft²  4. 36π cm²
5. 800π cm²  6. 300π in²  7. 109.6 m²  8. 160.0 cm²
9. 216 mm²  10. 264 mm²

Practice 11-4
1. 1131.0 m³  2. 94,247.8 cm³  3. 7.1 in³  4. 1781.3 in³
5. 785.4 cm³  6. 603.2 cm³  7. 120 in³  8. 35 ft³
9. 1415.8 in³  10. 784 cm³

Practice 11-5
1. 34,992 cm³  2. 400 in³  3. 10,240 in³
4. 4800 yd³  5. 150 m³  6. 48 cm³
7. 628.3 cm³  8. 314.2 in³  9. 4955.3 m³
10. 1415.8 in³  11. 1005.3 m³  12. 18.8 ft³
13. 20  14. 2  15. 6

Practice 11-6
1. 2463.0 in²  2. 6,157,521.6 m²  3. 12.6 cm²
4. 1256.6 m²  5. 50.3 ft²  6. 153.9 m²  7. 1435.5 m³
8. 268,082.6 m²  9. 7238.2 m²  10. 141.3 cm²
11. 2,572,354.6 cm³  12. 904,810.7 yd³  13. 546 ft²
14. 399 m²  15. 1318 cm²  16. 233 cm³
17. 7124 cm³  18. 5547 cm³  19. 42 cm³

Practice 11-7
1. 7 : 9  2. 5 : 8  3. yes; 4 : 7  4. yes; 4 : 3
5. not similar  6. yes; 4 : 5  7. 125 cm³  8. 256 in³
9. 432 ft²  10. 25 ft²  11. 90 m²  12. 600 cm²
13. 54,000 lb  14. 16 lb

Reteaching 11-1
1. Sample:

```
[Diagram of a shape with dimensions and calculations]
```

2. Check students’ work.
3. vertices = 6; faces = 5; edges = 9
4. \(E = V + F - 2; 9 \div 6 + 5 - 2; 9 = 9\)

```
[Diagram of a shape with calculations]
```

Reteaching 11-2
1. –2. Check students’ work.
3. L.A. = 300 cm²; S.A. = 350 cm²
4. 722.6 cm²
5. 153.3 m²

Reteaching 11-3
1. Sample:

```
[Diagram of a shape with calculations]
```

2. Check students’ work.
3. Sample: base = 3 cm, slant height = 4 cm
4. Sample: 33 cm³
5. 896 cm²
6. 314.2 m²
7. 416.2 in²
8. 349.3 ft²

Reteaching 11-4
1. 490 cm³
2. 242.5 cm³
3. 735 m³
4. 2544.7 in³
5. 756 ft³
6. 2261.9 cm³

Reteaching 11-5
1. B  2. 301.6 cm³  3. 1296 ft³  4. 48 in³
5. 211.6 m³
Chapter 11 Answers (continued)

Reteaching 11-6

1.–4. Check students’ work. 5. four circles 6. They are the same. 7. \( V = 113.1 \text{ in.}^3; S.A. = 113.1 \text{ in.}^2 \)
8. \( V = 523.6 \text{ cm}^3; S.A. = 314.2 \text{ cm}^2 \) 9. \( V = 7238.2 \text{ m}^3; S.A. = 1809.6 \text{ m}^2 \)
10. \( V = 8181.2 \text{ ft}^3; S.A. = 1963.5 \text{ ft}^2 \)
11. \( V = 1047.4 \text{ in.}^3; S.A. = 498.8 \text{ in.}^2 \) 12. \( V = 310.3 \text{ mm}^3; S.A. = 221.7 \text{ mm}^2 \)

Reteaching 11-7

1. 2 : 1 2. 3 : 2 3. 64 : 125 4. 512 : 125 5. \( 250 \sqrt{2} : 27 \) 6. 16 : 9 7. 49 : 25

Enrichment 11-1

1. 8 2. 9 3. 6 4. 11 5. 4 6. 18 7. 7 8. 0 9. 10 10. 15 11. 3 12. \( \frac{24}{5} - 0.5x \) in. 13. \( \frac{96}{13} \) in. 14. 1.5 \text{ in.} 15. 96 in.\(^2\); no matter what the size of the base is, the surface area of the rectangular prism will always be 96 in.\(^2\) as long as the entire piece of paper is used.

Enrichment 11-2

1. 96 in.\(^2\) 2. 8 in.\(^2\) 3. 88 in.\(^2\) 4. 4 5. 22 in.\(^2\) 6. 11 in. 7. 96 in.\(^2\) 8. \( 25 \text{ in.}^2 \) 9. \( (96 - 2s)^2 \) in.\(^2\) 10. 4 11. \( (24 - 0.5s)^2 \) in.\(^2\) 12. \( s \) in. 13. \( \frac{24}{5} - 0.5s \) in. 14. 96 in.\(^2\) 15. 96 in.\(^2\); no matter what the size of the base is, the surface area of the rectangular prism will always be 96 in.\(^2\) as long as the entire piece of paper is used.

Enrichment 11-3

1. \( a^2 \) 2. \( b^2 \) 3. trapezoid 4. \( \frac{1}{2}(a + b) \) 5. \( 2(a + b) \) 6. \( a^2 + b^2 + 2(a + b) \) 7. \( \pi R^2 \) 8. \( \pi r^2 \) 9. \( \pi Rr - \pi rl \) 10. \( \pi(R^2 + r^2 + Rl + rl) \) 11. 152 cm\(^2\) 12. 722.6 in.\(^2\) 13. 506.6 ft\(^2\) 14. 1952 m\(^2\)

Enrichment 11-4

1. 48 cubic units 2. 30 cubic units 3. 64 cubic units 4. 15 cubic units 5. 90 cubic units 6. 25 cubic units

Enrichment 11-5

1. on a line perpendicular to the base at the center of the square 2., 4., 5., 6., 7., 8., 9., 10., 11., 12., 13., 14., 15.

Enrichment 11-6

1. \( \frac{4}{3}\pi r^3 \) 2. \( \frac{5}{3}\pi R^3 \) 3. \( \frac{4}{3}\pi(r^3 - R^3) \) 4. 266,000,000,000 \text{ mi}^3 5. \( \frac{4}{3}\pi(r - R)(r^2 + rR + R^2) \) 6. \( \frac{4}{3}\pi d(3R^2) = 4\pi dR^3 \) 7.

<table>
<thead>
<tr>
<th>( R )</th>
<th>( d )</th>
<th>Exact Volume of Shell</th>
<th>Approximate Volume of Shell</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.1</td>
<td>126.93</td>
<td>125.66</td>
</tr>
<tr>
<td>50</td>
<td>1</td>
<td>32,048.44</td>
<td>31,415.93</td>
</tr>
<tr>
<td>100</td>
<td>3</td>
<td>388,413.95</td>
<td>376,991.12</td>
</tr>
</tbody>
</table>

8. 0.421 in.\(^3\) 9. 1.34 in.\(^3\)

Enrichment 11-7

1. 180 ft; 120 ft 2. \$118.13 3. 6.53 in. 4. 5.52 in. 5. \$5.40 6. 8.14 in. 7. 13.58 in. 8. 16.98 in. 9. 3.43 in. 10. 19.48 in.
Chapter 11 Answers (continued)

Chapter Project

Activity 1: Measuring
Check students’ work.

Activity 2: Analyzing
Sample: $\begin{array}{|c|c|c|c|c|} \hline l & w & d & V & S.A \ : \ S.A. \\
\hline 6 & 6 & 6 & 216 & 1 : 1 \\
2 & 3 & 6 & 216 & 372 \ : \ 18 \ : \ 31 \\
3 & 3 & 24 & 216 & 306 \ : \ 36 \ : \ 51 \\
4 & 6 & 9 & 216 & 228 \ : \ 18 \ : \ 19 \\
\hline \end{array}$

1. 6 by 6 by 6; 2 by 3 by 36; the lower the ratio, the more material is being used to enclose the same volume.
2. to minimize the cost of packaging
3. Sample: A cereal box is used for promoting the product, so it needs a large surface area on the front.

Activity 3: Investigating
Check students’ work.

Activity 4: Creating
Check students’ work.

✔ Checkpoint Quiz 1

1.

[Diagram]

2.

[Diagram]

3. $88 \text{ in.}^2$
4. $80\pi \text{ m}^2$
5. $1814.4\pi \text{ in.}^2$

6.

[Diagram]

7. [Diagram]

8. rectangle

✔ Checkpoint Quiz 2

1. $201.1 \text{ m}^2; 268.1 \text{ m}^3$
2. $452.4 \text{ ft}^2; 402.1 \text{ ft}^3$
3. $158.7 \text{ in.}^2; 84 \text{ in.}^3$
4. $534.1 \text{ yd}^2; 942.5 \text{ yd}^3$
5. $384 \text{ m}^2; 384 \text{ m}^3$
6. $113.1 \text{ cm}^2; 50.3 \text{ cm}^3$
7. $21.2 \text{ in.}^3$

Chapter Test, Form A

1. Sample:

2. Sample:

3. The lateral area of a paint roller determines the amount of paint it can spread on a wall in one revolution. Because the lateral area of roller A is $24\pi \text{ in.}^2$ and the lateral area of roller B is $21\pi \text{ in.}^2$, they both spread the same amount of paint in one revolution.

4. $V = 400.0 \text{ cm}^3; S.A. = 360.0 \text{ cm}^2$
5. $V = 254.5 \text{ in.}^3; S.A. = 226.2 \text{ in.}^2$
6. $V = 1050.0 \text{ cm}^3; S.A. = 628.3 \text{ ft}^2$
7. $V = 113.1 \text{ m}^3; S.A. = 174 \text{ cm}^2$
8. $V = 1050.0 \text{ cm}^3; S.A. = 734.0 \text{ cm}^2$
9. $V = 113.1 \text{ m}^3; S.A. = 384 \text{ m}^2$
10. $V = 119 \text{ cm}^3; S.A. = 113.1 \text{ m}^2$
11. a cylinder with radius 2 units and height 3 units

12. $V = 251.3 \text{ cm}^2$

13. $729 : 64$
14. $9 : 4$
15. $137.2 \text{ cm}^3$
16. $S.A. = 603.2 \text{ m}^2; V = 1206.4 \text{ m}^3$
17. $S.A. = 3327.7 \text{ cm}^2; V = 12,250.6 \text{ cm}^3$

18. $18 \text{ cm}$

Chapter Test, Form B

1. Sample:

2. Sample:

3. The lateral area of a paint roller determines the amount of paint it can spread on a wall in one revolution. Because the lateral area of roller A is $22\pi \text{ in.}^2$ and the lateral area of roller B is $21\pi \text{ in.}^2$, roller A spreads more paint on a wall in one revolution.

4. $V = 523.6 \text{ m}^3; S.A. = 314.2 \text{ m}^2$
5. $V = 1583.4 \text{ cm}^3; S.A. = 754.0 \text{ cm}^2$
6. $V = 67.0 \text{ ft}^3; S.A. = 138.2 \text{ ft}^2$
7. $V = 880.0 \text{ in.}^3; S.A. = 696.0 \text{ in.}^2$
8. half of a sphere and a cone
9. Two regular cans of soup each have a volume of $461.8 \text{ cm}^3$ for a total of $923.6 \text{ cm}^3$. The single family-size can with its volume of $942.5 \text{ cm}^3$ is larger than the total volume of the two regular cans.

10. $D$
11. $S.A. = 1357.2 \text{ m}^2; V = 4071.5 \text{ m}^3$
12. $S.A. = 3296.8 \text{ cm}^2; V = 12,113.3 \text{ cm}^3$

13. $729 : 64$
14. $9 : 4$
15. $137.2 \text{ cm}^3$
16. $S.A. = 603.2 \text{ m}^2; V = 1206.4 \text{ m}^3$
17. $S.A. = 3327.7 \text{ cm}^2; V = 12,250.6 \text{ cm}^3$

18. $18 \text{ cm}$

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Alternative Assessment, Form C

**TASK 1: Scoring Guide**

Sample:

![Diagram of a prism and cylinder]

Prism: \( S.A. = 60 \text{ cm}^2; V = 24 \text{ cm}^3 \)

Cylinder: \( S.A. = 10\pi \text{ cm}^2 \) or about 31.4 cm\(^2\); \( V = 4\pi \text{ cm}^3 \) or about 12.6 cm\(^3\)

3 Student gives accurate drawings and calculations.
2 Student gives drawings and calculations that may contain minor errors.
1 Student gives drawings and calculations that contain significant errors or omissions.
0 Student makes little or no attempt.

**TASK 2: Scoring Guide**

Sample:

Pyramids have polygonal sides, whereas cones have rounded sides. Each of these figures has a vertex, and each has volume \( \frac{1}{3}Bh \). The base of a pyramid is a polygon, and the base of a cone is a circle.

![Diagram of a pyramid and a cone]

A pyramid with square base, height \( h \), and slant height \( s \)

\[
S.A. = x^2 + 4\left(\frac{1}{2}xs\right) = x^2 + 2xs \\
V = \frac{1}{3}x^2h
\]

A cone with radius \( r \), height \( h \), and slant height \( s \)

\[
S.A. = \pi r^2 + \pi rs \\
V = \frac{1}{3}\pi r^2h
\]

3 Student gives accurate drawings and formulas.
2 Student gives drawings and formulas that may contain minor errors.
1 Student gives drawings and formulas that contain significant errors.
0 Student makes little or no attempt.

**TASK 3: Scoring Guide**

Sample:

![Diagram of a construction]

a. 

![Diagram of a net]

3 Student draws an accurate net and gives correct answers.
2 Student draws a net and gives answers that may contain minor errors.
1 Student draws a net and does calculations incorrectly.
0 Student makes little or no attempt.

**TASK 4: Scoring Guide**

Foundation drawing:

![Diagram of a drawing]

3 Student draws completely accurate figures.
2 Student draws generally accurate figures, but these may contain a minor error.
1 Student draws figures containing significant errors.
0 Student makes little or no attempt.

**Cumulative Review**

Chapter 12 Answers

Practice 12-1
1. 32 2. 50 3. 72 4. 15 5. \( \sqrt{91} \) 6. 6 7. \( \sqrt{634} \) 8. \( \sqrt{901} \) 9. \( 4\sqrt{3} \) 10. circumscribed 11. inscribed 12. circumscribed 13. 24 cm 14. 28 in. 15. 52 ft

Practice 12-2
1. \( r = 13; m\overline{AB} \approx 134.8 \) 2. \( r = 3\sqrt{5}; m\overline{AB} \approx 53.1 \) 3. \( r = \frac{\sqrt{41}}{2}; m\overline{AB} \approx 102.7 \) 4. 3 5. 4.5 6. 3 7. 20.8 8. 13.7 9. 8.5 10. \( \angle Q \equiv \angle T \) 11. \( PR \equiv SU \) 12. Construct \( \overline{CD} \) and \( \overline{AB} \), then by HL, \( \triangle OEB \equiv \triangle OFD \), and by CPCTC, \( OE = OF \). 13. Because congruent arcs have congruent chords, \( AB = BC = CA \). Then, because an equilateral triangle is equiangular, \( m\angle ABC = m\angle BCA = m\angle CAB \).

Practice 12-3
1. \( \angle A \) and \( \angle D \); \( \angle B \) and \( \angle C \) 2. \( \angle ADB \) and \( \angle CDB \) 3. \( \angle ADB \) and \( \angle CAD \) 4. 55 5. \( x = 45; y = 50; z = 85 \) 6. \( x = 90; y = 70 \) 7. \( x = 120; y = 60; z = 60 \) 8. \( x = 120; y = 100; z = 140 \) 9. \( x = 50; y = 80; z = 80 \) 10. \( m\angle C = 85 \) 11. \( m\angle E = 170 \) 12. \( m\angle D = 85 \) 13. \( m\angle A = 90 \) 14. \( m\angle B = 80 \) 15. \( m\angle C = 90 \) 16. \( m\angle D = 100 \)

Practice 12-4
1. 87 2. 35 3. 45 4. 120 5. 72 6. 186 7. \( x = 58; y = 59; z = 63 \) 8. \( x = 30; y = 66 \) 9. \( x = 30; y = 30; z = 120 \) 10. \( x = 16; y = 52 \) 11. \( x = 138; y = 111; z = 111 \) 12. \( x = 30; y = 60 \) 13. 10 14. 14.8 15. 4.7 16. 4 17. 3.2 18. 6

Practice 12-5
1. \( C(0,0), r = 6 \) 2. \( C(2,7), r = 7 \) 3. \( C(-1,-6), r = 4 \) 4. \( C(-3,11), r = 2\sqrt{3} \) 5. \( x^2 + y^2 = 49 \) 6. \( (x - 4)^2 + (y - 3)^2 = 64 \) 7. \( (x - 5)^2 + (y - 3)^2 = 4 \) 8. \( (x + 5)^2 + (y - 4)^2 = \frac{1}{4} \) 9. \( (x + 2)^2 + (y + 5)^2 = 2 \) 10. \( (x + 1)^2 + (y - 6)^2 = 5 \) 11. \( x^2 + y^2 = 4 \) 12. \( (x + 3)^2 + (y - 3)^2 = 1 \) 13. \( x^2 + (y - 3)^2 = 16 \) 14. \( (x - 7)^2 + (y + 2)^2 = 4 \) 15. \( x^2 + (y + 20)^2 = 100 \) 16. \( (x + 4)^2 + (y + 6)^2 = 25 \) 17. \( x^2 + y^2 = 25 \) 18. \( (x - 5)^2 + (y - 9)^2 = 9 \) 19. \( (x + 4)^2 + (y + 3)^2 = 61 \) 20. \( (x - 7)^2 + (y + 2)^2 = 80 \) 21. \( (x + 4)^2 + (y + 3)^2 = 4 \) 22. \( (x - 4)^2 + (y - 1)^2 = 16 \)
11. a third line parallel to the two given lines and midway between them  
12. a sphere whose center is the given point and whose radius is the given distance  
13. the points within, but not on, a circle of radius 1 in. centered at the given point  

Reteaching 12-1  
1. $\sqrt{157}$  
2. $\sqrt{741}$  
3. $\sqrt{2}$  
4. 7.5  

Reteaching 12-2  
1. $5\sqrt{5}$  
2. $12\sqrt{2}$  
3. $2\sqrt{14}$  
4. 11.83 cm  
5. 2.54 cm  
6. 8.66 in.  
7. 2.78 in.  

Reteaching 12-3  
1. 87  
2. 40  
3. 60  
4. 55  
5. $x = 94, y = 80$  
6. 120  
7. 40  
8. 20  
9. 70  

Reteaching 12-4  
1. 93  
2. 156  
3. 42  
4. 35  
5. 60  
6. 55  
7. $x = 36; y = 60; z = 48$  
8. $x = 64; y = 64; z = 52$  
9. $x = 46; y = 90; z = 44$  

Reteaching 12-5  
1. $(x - 3)^2 + (y - 11)^2 = 4$  
2. $(x + 5)^2 + y^2 = 225$  
3. $(x - 6)^2 + (y + 6)^2 = 7$  
4. $x^2 + y^2 = 20$  
5. $(x + 2)^2 + (y + 2)^2 = 4$  
6. $(x - 3)^2 + (y - 1)^2 = 50$  
7. $(x - 5)^2 + (y - 2)^2 = 36$  
8. $(x - 4)^2 + (y - 2)^2 = 25$  
9. $(x + 2)^2 + (y - 3)^2 = 25$  
10. $C(-3, -5); r = 5$  
11. $C(0,0); r = 0.2$  
12. $C(4, 0); r = \sqrt{6}$  
13. $C(3, 5); r = 4$  

Reteaching 12-6  
1.–5. Check students’ work.  
6. the line perpendicular to $\overline{AB}$ at its midpoint  
7. one, the midpoint of $\overline{AB}$
Chapter 12 Answers (continued)

Enrichment 12-1
1. Given 2. Two points determine a line segment.
3. Two tangents drawn to a circle from an external point are congruent. 4. Radii of a circle are congruent.
5. A radius and a tangent drawn to the same point of contact form a right angle. 6. Definition of a square. 7. Sides of a square are congruent. 8. Addition Property
9. Segment Addition Postulate 10. Substitution Property
11. Addition Property 12. Substitution Property
13. Subtraction Property

Enrichment 12-2
1a. 1 1b. 3 1c. 6 1d. 10 1e. 15
2. 1, 3, 6, 10, 15 3. Triangular numbers
4. \[ \frac{N(N - 1)}{2} \]
5. From each point, \((N - 1)\) chords may be drawn. So, a total of \(N(N - 1)\) chords may be drawn for \(N\) points. Because each chord has been drawn twice, divide by 2.
6. \(N(N - 1)\)

Enrichment 12-3
1. Given 2. Two points determine a line segment.
3. The measures of exterior angles of a triangle equal the sum of the measures of the remote interior angles.
4. Radii of a circle are congruent. 5. Definition of isosceles triangle. 6. Base angles of an isosceles triangle are congruent. 7. Definition of congruent angles.
8. Base angles of an isosceles triangle are congruent. 9. Definition of congruent angles. 10. The measures of exterior angles of a triangle equal the sum of the measures of the remote interior angles.
11. Substitution Property
12. Division Property 13. Drawing diameter \(DE\) passing through \(A\) so that radius \(OB = AB\)

Enrichment 12-4
1. \(2x + 2x + 8 + x + x - 32 = 360; 6x - 24 = 360; 6x = 384\) 2. \(x = 64\) 3. 128 4. 136 5. 64
6. 32 7. Chords \(AB, BD\); 16 8. Secants \(EB, EC\); 52
9. Tangent \(FB\) and chord \(AB\); 64 10. Chords \(BD, AC\); 84
11. Chords \(AC, BC\); 64 12. Tangent \(FB\) and secant \(FD\); 36
13. Chords \(AC, BD\); 96 14. \(AF, AE\); 100
15. \(DE, DA\); 48 16. Tangent \(FB\) and chord \(BC\); 68

Enrichment 12-5
1a. The edge of the fountain pool 1b. Plaza
1c. Fountain pool 2. A circle with center \((-3, 4)\) and a radius of 3
3. All the interior points of a circle with center \((4, 0)\) and a radius of \(\sqrt{12}\)
4. All points outside a circle with center \((0, 0)\) and a radius of \(\sqrt{18}\)
5. A circle and all its interior points with center \((0, -2)\) and a radius of 5
6. \(x^2 + y^2 = 100\) 7. \(x^2 + y^2 < 6\)
8. The sergeant can draw a circle with center \((0, 0)\) and a radius of 2. The team will search in the circle.

Enrichment 12-6
1–9. Check students’ drawings. 1. \(\odot P\) with a radius of 4 in.
2. Two lines parallel to line \(l\), one 2 in. above \(l\) and the other 2 in. below \(l\)
3. 4 points 4. Two lines parallel to line \(l\), one 4 in. above \(l\) and the other 4 in. below \(l\)
5. 2 points 6. Two lines parallel to line \(l\), one 6 in. above \(l\) and the other 6 in. below \(l\)
7. Empty set 8. 6 points
9. Empty set

Chapter Project
Activity 1: Doing
Check students’ work.

Activity 2: Exploring
Both figures are made from 9 circles with the same center. In Figure A, the smallest circle and alternate rings are black. In Figure B, vertical and horizontal tangents to the smallest circle are drawn. Then the tangents at \(45^\circ\) angles to the first ones are drawn. Alternating sections of the 6 outer rings are black, as is the smallest ring.
Check students’ work.

Activity 3: Constructing
Check students’ work.

✓ Checkpoint Quiz 1
1. 52 in. 2. 44 cm 3. 40 m 4. 6 5. 12
6. 11.5 7. \(x = 37, y = 100\) 8. \(x = 116, y = 88\)
9. 7 10. \(x = 120\)

✓ Checkpoint Quiz 2
1. \(x = 140\) 2. \(x = 75, y = 105\)
3. \(x = 20, y = 70\) 4. \(x^2 + (y - 1)^2 = 6.76\)
5. \((x + 3)^2 + (y - 2)^2 = 100\)
6. 12 7. \(\frac{51}{7}\) 8. 20

Chapter Test, Form A
1. \(C(0, 0); r = 10\) 2. \(C(11, -6); r = 9\)
3. \(C(-1, -4); r = \sqrt{7}\) 4. \((x + 3)^2 + (y - 2)^2 = 1\)
5. \((x - 2)^2 + (y - 1)^2 = 16\) 6. \(x^2 + (y - 2)^2 = 9\)
7. \(C = 31.4; A = 78.5\) 8. \((x + 3)^2 + (y - 4)^2 = 41\)
9. 10. 90
Chapter 12 Answers (continued)

Chapter Test, Form B

1. inscribed  2. circumscribed  3. \( \angle R; \overline{QS} \)  4. \( \angle M; \overline{LN} \)
5. center \((0, 0); r = 5\)  6. center \((3, 5); r = 10\)
7. \( x^2 + y^2 = 9\)  8. \( (x - 4)^2 + (y + 1)^2 = 4\)
9. 37.7 units; 113.1 square units  10. \( x^2 + (y + 1)^2 = 25\)
11. center \((-3, 2); r = 3\)

Alternative Assessment, Form C

TASK 1: Scoring Guide

(a) center \((2, 5)\), radius = 4  (b) The distance from point \((x, y)\) on a circle to the center \((2, 5)\) is given by the formula \(D = \sqrt{(x - 2)^2 + (y - 5)^2}\). In any circle, the distance, or radius, is the same for all points on the circle. Therefore the distance formula, when applied to the circle in this problem, becomes \(4 = \sqrt{(x - 2)^2 + (y - 5)^2}\) for all points \((x, y)\) on the circle. Squaring both sides of this equation yields the equation of the circle, \(16 = (x - 2)^2 + (y - 5)^2\).

3 Student gives accurate answers and a correct explanation.
2 Student gives answers or an explanation that may contain minor errors.
1 Student gives wrong answers or an incomplete or inaccurate explanation.
0 Student makes little or no effort.

TASK 2: Scoring Guide

(a) Construct the perpendicular bisector of two chords. The point at which they meet is the center of the circle.  (b) Because the pentagon is regular, all chords are congruent. Therefore the corresponding arcs are congruent. Because there are 5 congruent arcs in the circle, each arc, and in particular \(AB\), has measure 72. To find \(AB\), call the center \(O\) and consider triangle \(AOB\). Because \(AB\) has measure 72, so does angle \(AOB\). Because \(OA = OB\), \(\triangle AOB\) is isosceles. Therefore, the bisector of angle \(AOB\) is perpendicular to (and bisects) chord \(AB\), forming a 36°-54°-90° triangle. Then \(\sin 36 = \frac{\overline{AB}}{\overline{OA}} = \frac{\overline{AB}}{12}\), yielding \(\overline{AB} = 7.05\).
Chapter 12 Answers (continued)

3 Student devises a correct method and gives a correct answer and a valid explanation.
2 Student devises a method, gives an answer, and gives an explanation that may contain some errors.
1 Student gives a method, an answer, and an explanation that may contain major errors or omissions.
0 Student makes little or no effort.

**TASK 3: Scoring Guide**
(a) Angles A and C are inscribed angles. Each is inscribed in a different arc, but together the arc in which they are inscribed comprise the entire circle. Therefore, \( m\angle A + m\angle C = \frac{1}{2}(360) = 180 \). And, because \( ABCD \) is a parallelogram, angles A and C are congruent. Finally, if angles are congruent and supplementary, then they are right. (b) The diagonals of \( ABCD \) bisect each other because \( ABCD \) is a parallelogram. Suppose that they intersect at point X. Then \( AX = CX \) and \( BX = DX \). Also, because \( ABCD \) is inscribed in a circle, the diagonals are chords, and therefore \( AX \cdot CX = BX \cdot DX \). Substituting yields \( AX^2 = BX^2 \), and therefore \( AX = BX = CX = DX \). Then \( AC = BD \). (c) Either (a) or (b) allows you to conclude that \( ABCD \) must be a rectangle.

3 Student gives valid and accurate arguments and explanations.
2 Student gives arguments that, although basically valid, may contain minor flaws.
1 Student gives arguments containing major flaws.
0 Student makes little or no attempt.

**TASK 4: Scoring Guide**
(a) Because the circumscribing lines are tangent to the circle, \( AB = AX \), \( CB = CY \), and \( DX = DY \). Then, because it is given that \( AB = BC \), we have \( AX = CY \). Adding segments together yields \( DA = DC \), showing that \( \triangle ACD \) is isosceles. Secondly, because \( XY \) has measure 100, \( XBY \) has measure 260. Then \( m\angle D = \frac{1}{2}(260 - 100) = 80 \), and because \( \triangle ACD \) is isosceles, \( m\angle A = m\angle C = 50 \). Finally, \( AC = 20 \) because \( AB = BC = 10 \), and, by trigonometry, \( \cos 50 = \frac{AB}{AD} \), \( AD = \frac{AB}{\cos 50} = 10 \times 0.6428 = 15.56 \). And again, \( AD = CD \). (b) Because angle A has measure 50 and line \( AB \) is tangent at \( B \), triangle \( AOB \) is a \( 25^\circ \)-65\(^\circ\)-90\(^\circ\) triangle. Therefore, by trigonometry, \( \tan 25 = \frac{OB}{AB} \), \( OB = 10 \cdot \tan 25 = 4.66 \).

3 Student gives accurate answers and explanations.
2 Student gives answers and explanations that may contain minor errors.
1 Student gives answers and explanations that contain significant errors.
0 Student makes little or no attempt.

**Cumulative Review**

Chapter 1

Practice 1-1
1. 47, 53  2. 1.00001, 1.00001  3. 42, 54  4. −64, 128  
5. 22, 29  6. 63.5, 63.75  7. Sample: 2 or 3  8. 51 or 49  
9. 6 or 8  10. A or AA  11. D or G  12. Y or A  
13. any hexagon  14. circle with 8 equally spaced diameters  
15. a 168.75° angle  16. 21 hand-shakes  17. \( h = \frac{n(n - 1)}{2} \)  
18. 34  19. Sample: The farther out you go, the closer the ratio gets to a number that is approximately 0.618.  
20. 0, 1, 1, 2, 3, 5, 8, 13

Guided Problem Solving 1-1
1. The pattern is easier to visualize.  2. The graph will go up.  
3. Use Years for the horizontal axis.  4. Use Number of Stations for the vertical axis.  5. increasing  6. greater  
7. Yes. Since the number of stations increases steadily from 1950 to 2000, we can be confident that the number of stations in 2010 will be greater than in 2000.  8. Patterns are necessary to reach a conclusion through inductive reasoning.  
9. (any list of numbers without a pattern would apply) 2,435; 16,439; 16,454; 3,765; 210,564

Practice 1-2
1. 2. 3. 4.  

Guided Problem Solving 1-2
1. They represent three-dimensional objects on a two-dimensional surface.  2. nine  3. See the figure in 4, below.  
4. 5. Yes. It is similar to the foundation drawing, except there are no numbers.  6. no  
7. 8. Yes.  9.  

Practice 1-3
1. \( \overline{AC} \)  2. any two of the following: \( \overline{ABD}, \overline{DBC}, \overline{CBE}, \overline{ABE}, \overline{ECD}, \overline{ADE}, \overline{ACE}, \overline{ACD} \)  
3. Points E, B, and D are collinear.  4. yes  5. yes  6. no  7. no  8. yes  9. no
Guided Problem Solving 1-3
1. Collinear points lie on the same line. 2. Answers may vary. (Some people might note that the $y$-coordinate of two of the points is the same so that the third point must have the same $y$-coordinate to be collinear. Since it does not, the points are not collinear.) 3. horizontal 4. No. 5. No. 6. All points must have the same $y$-coordinate, $-3$. 7. No. 8. $\left(1, -\frac{1}{2}\right)$

Practice 1-4
1. true 2. false 3. true 4. false 5. false 6. false
7. $\overline{JK}$, $\overline{HG}$ 8. $\overline{EH}$ 9. any three of the following pairs: $\overline{EF}$ and $\overline{JK}$, $\overline{JK}$ and $\overline{FG}$, $\overline{HG}$ and $\overline{JE}$, $\overline{HG}$ and $\overline{JK}$ and $\overline{EK}$, $\overline{JE}$ and $\overline{FG}$, $\overline{EH}$ and $\overline{FG}$, $\overline{JK}$ and $\overline{HG}$; $\overline{EH}$ and $\overline{FG}$, $\overline{JK}$ and $\overline{HG}$; $\overline{EH}$ and $\overline{FG}$, $\overline{JK}$ and $\overline{HG}$; $\overline{EH}$ and $\overline{FG}$, $\overline{JK}$ and $\overline{HG}$ 10. planes $A$ and $B$ 11. planes $A$ and $C$; planes $B$ and $C$ 12. planes $A$ and $C$ 13. planes $B$ and $C$ 14. Sample: $\overline{EG}$ 15. 6 16. $\overline{EF}$ and $\overline{ED}$ or $\overline{EG}$ and $\overline{ED}$ 17. $\overline{FE}$, $\overline{FD}$ 18. $\overline{GF}$, $\overline{GD}$ 19. yes 20. Sample:

21. Sample:

Guided Problem Solving 1-4
1. Opposite rays are two collinear rays with the same endpoint. 2. a line 3–4. See graph in Exercise 5 answer.

5. Answers may vary. Sample: $(0, 0)$ (Answers will be coordinates $(x, y)$, where $y = \frac{3}{2}x$, $x < 2$.)

6. yes 7. $L(4, 2)$

Practice 1-5
1. 4 2. 13 3. 20 4. 6 5. 22 6. $-10$ or $6$ 7. $-1$ or $1$ 8. 3; 4; no 9. 6; 6; yes 10. $C$ or $-2$ 11. 15 12. 31 13. 14 14. $x = 11\frac{2}{3}; AB = 31$; $BC = 31$ 15. $x = 35\frac{2}{3}; AB = 103; BC = 103$

Guided Problem Solving 1-5
1. $\overline{AD} \cong \overline{DC}$ 2. $\overline{AD} = \overline{DC}$ 3. Segment Addition Postulate. 4. Since $\overline{AD} = \overline{DC}$, $\overline{AC} = 2(\overline{AD})$. 5. $AC = 2(12) = 24$ 6. $y = 15$ 7. $DC = AD = 12$ 8. Answers may vary. 9. $ED = 11$, $DB = 11$, $EB = 22$

Practice 1-6
1. any three of the following: $\angle O$, $\angle MOP$, $\angle POM$, $\angle 1$ 2. $\angle OAB$ 3. $\angle EOC$ 4. $\angle DOC$ 5. 51 6. 90
7. 17 8. 107 9. 141 10. 68 11. $\angle ABD$, $\angle DBE$, $\angle EBF$, $\angle DBF$, $\angle FBC$ 12. $\angle ABE$, $\angle DBC$ 13. $\angle ABE$, $\angle EBC$ 14. $x = 5$; $8; 21; 13$ 15. $x = 9$; $85; 35; 120$

Guided Problem Solving 1-6
1. Angle Addition Postulate 2. supplementary angles 3. $m \angle RQS + m \angle TQS = 180$ 4. $(2x + 4) + (6x + 20) = 180$ 5. $x = 19.5$ 6. $m \angle RQS = 43; m \angle TQS = 137$ 7. The sum of the angle measures should be $180; m \angle RQS + m \angle TQS = 43 + 137 = 180$. 8a. $x = 11$ 8b. $m \angle AOB = 17$; $m \angle COB = 73$
Guided Problem Solving 1-7
1. $\angle DBC \equiv \angle ABC$
2. complementary angles
3. $\angle CBD$
4. $m\angle CBD = m\angle CBA = 41$
5. $m\angle ABD = m\angle CBA + m\angle CBD = 41 + 41 = 82$
6. $m\angle ABE + m\angle CBA = 90$
   $m\angle ABE + 41 = 90$
   $m\angle ABE = 49$
7. $m\angle DBF = m\angle ABE = 49$
8. Answers may vary. Sample:
The sum of the measures of the complementary angles should be 90 and the sum of the measures of the supplementary angles should be 180.
9. $m\angle CBD = 21$, $m\angle FBD = 69$, $m\angle CBA = 21$, and $m\angle EBA = 69$
Practice 1-8
1–5.

6. \(5\sqrt{2} = 7.1\) 7. \(2\sqrt{17} = 8.2\) 8. 12 9. 8 10. 12
11. \(\sqrt{26} = 5.1\) 12. (5, 5) 13. \((\frac{1}{2}, 1)\) 14. \((10\frac{1}{2}, -5)\)
15. \((-2\frac{1}{2}, 6)\) 16. \((-0.3, 3.4)\) 17. \((2\frac{7}{8}, -5)\) 18. \((5, -2)\)
19. (4, 10) 20. (3, 4) 21. yes; \(AB = BC = CD = DA = 6\) 22. \(\sqrt{401} \approx 20.025\)

23.

24. \(= 24.7\) 25. (3.5, 3)

Guided Problem Solving 1-8
1. Distance Formula 2. The distance \(d\) between two points \(A(x_1,y_1)\) and \(B(x_2,y_2)\) is \(d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\).
3. No; the differences are opposites but the squares of the differences are the same.
4. \(XY = \sqrt{(5 - (-6))^2 + (-2 - 9)^2}\) 5. To the nearest tenth, \(XY = 15.6\) units. 6. To the nearest tenth, \(XZ = 12.0\) units. 7. \(Z\) is closer to \(X\). 8. The results are the same; e.g., \(XY = \sqrt{(-6 - 5)^2 + (9 - (-2))^2} = \sqrt{242}\), or about 15.6 units, as before.
9. \(YZ = \sqrt{(17 - (-6))^2 + (-3 - 9)^2} = \sqrt{673}\); to the nearest tenth, \(YZ = 25.9\) units. To the nearest tenth, \(XY + YZ + XZ = 53.5\) units.

Practice 1-9
1. 792 in.\(^2\) 2. 3240 in.\(^2\) 3. 2.4 m\(^2\) 4. \(32\pi\) 5. 16\(\pi\) 6. 7.8\(\pi\) 7. 26 cm; 42 cm\(^2\) 8. 46 cm; 42 cm\(^2\) 9. 29 in.; 42 in.\(^2\) 10. 40 ft; 51 ft\(^2\) 11. 40 m; 99 m\(^2\) 12. 40 m; 91 m\(^2\) 13. 68; 285 14. 26; 22 15. 30; 44 16. 156.25\(\pi\) 17. 10,000\(\pi\) 18. \(\frac{\pi}{16}\) 19. 48; 28 20. 36 21. 26; 13

Guided Problem Solving 1-9
1. six 2. It is a two-dimensional pattern you can fold to form a three-dimensional object. 3. rectangles
4.

5. 208 in.\(^2\) 6. They are equal. 7. 208 in.\(^2\) 8. Answers will vary. Sample: \(2(4 \cdot 6) + 2(4 \cdot 8) + 2(6 \cdot 8)\); the results are the same, 208 in.\(^2\).

1A: Graphic Organizer
1. Tools of Geometry 2. Answers may vary. Sample: patterns and inductive reasoning; measuring segments and angles; basic constructions; and the coordinate plane 3. Check students’ work.

1B: Reading Comprehension
1. Answer may vary. Sample: \(AB \parallel CD, EF \parallel GH\), \(JK \equiv LM, JL \equiv KM, m\angle JLF + m\angle FJK = 180^\circ, \angle HKM \equiv \angle KMD, D \cap \angle CD \equiv \angle D\). 2. Points \(A, M,\) and \(S\) are collinear. 3. \(AB, HT,\) and \(LN\) intersect at point \(M\).

1C: Reading/Writing Math Symbols
1. Line \(BC\) is parallel to line \(MN\). 2. Line \(CD\) 3. Line segment \(GH\) 4. Ray \(AB\) 5. The length of segment \(XY\) is greater than the length of segment \(ST\). 6. \(MN = XY\) 7. \(GH = 2(KL)\) 8. \(ST \perp UV\) 9. plane \(ABC \parallel plane XYZ\). 10. \(AB \parallel DE\)

1D: Visual Vocabulary Practice

1E: Vocabulary Check
Net: A two-dimensional pattern that you can fold to form a three-dimensional figure. Conjecture: A conclusion reached using inductive reasoning. Collinear points: Points that lie on the same line. Midpoint: A point that divides a line segment into two congruent segments. Postulate: An accepted statement of fact.
Chapter 2

Practice 2-1
1. Sample: It is 12:00 noon on a rainy day.
2. Sample: The car will not start because of a dead battery.
3. Sample: 6
4. If you are strong, then you drink milk.
5. If a rectangle is a square, then it has four sides the same length.
6. If you are tired, then you did not sleep.
7. If $x = 26$, then $x - 4 = 22$; true.
8. If $x > 0$, then $|x| > 0$; true.
9. If $m$ is positive, then $m^2$ is positive; true.
10. If $2y - 1 = 5$, then $y = 3$; true.
11. If $x > 0$, then point $A$ is in the first quadrant; false.
12. If lines are parallel, then their slopes are equal; true.
13. If you have a sibling, then you are a twin; false; a brother or sister born on a different day.
14. siblings
twin

15. Hypothesis: If you like to shop; conclusion: Visit Pigeon Forge outlets in Tennessee.
16. Pigeon Forge outlets are good for shopping.
17. If you visit Pigeon Forge outlets, then you like to shop.
18. It is not necessarily true. People may go to Pigeon Forge outlets because the people they are with want to go there.
19. Drinking Sustain makes you train harder and run faster.
20. If you drink Sustain, then you will train harder and run faster.
21. If you train harder and run faster, then you drink Sustain.

Guided Problem Solving 2-1
1. Hypothesis: $x$ is an integer divisible by 3.
2. Conclusion: $x^2$ is an integer divisible by 3.
3. Yes, it is true. Since 3 is a factor of $x$, it must be a factor of $x \cdot x = x^2$.
4. If $x^2$ is an integer divisible by 3 then $x$ is an integer divisible by 3.
5. The converse is false. Counterexamples may vary. Let $x^2 = 3$. Then $x = \sqrt{3}$, which is not an integer and is not divisible by 3.
6. No. The conditional is true, so there is no such counterexample.
7. No. By definition, a general statement is false if a counterexample can be provided.
8. If $5x + 3 = 23$, then $x = 4$. The original statement and the converse are both true.

Practice 2-2
1. Two angles have the same measure if and only if they are congruent.
2. $2x - 5 = 11$ if and only if $x = 8$.
3. The converse, “If $|n| = 17$, then $n = 17$,” is not true.
4. A figure has eight sides if and only if it is an octagon.
5. If a whole number is a multiple of 5, then its last digit is either 0 or 5. If a whole number has a last digit of 0 or 5, then it is a multiple of 5.
6. If two lines are perpendicular, then the lines form four right angles. If two lines form four right angles, then the lines are perpendicular.
7. If you live in Texas, then you live in the largest state in the contiguous United States.
8. Sample: Other vehicles, such as trucks, fit this description.
9. Sample: Other objects, such as spheres, are round.
10. Sample: “Pleasing, smooth, and rigid” all are too vague.
11. Sample: Baseball also fits this definition.
12. Sample: Pleasing, smooth, and rigid all are too vague.
13. yes 14. no 15. yes

Guided Problem Solving 2-2
1. A good definition is clearly understood, precise, and reversible.
2. $\angle 3$ and $\angle 4$, $\angle 5$ and $\angle 6$.
3. No.
4. They are not supplementary. 5. A linear pair has a common vertex, shares a common side, and is supplementary. 6. yes
7. yes; yes 8. linear pairs: \( \angle 1 \) and \( \angle 2 \), \( \angle 3 \) and \( \angle 4 \); not linear pairs: \( \angle 1 \) and \( \angle 3 \), \( \angle 1 \) and \( \angle 4 \), \( \angle 2 \) and \( \angle 3 \), \( \angle 2 \) and \( \angle 4 \)

**Practice 2-3**

1. \( \angle A \) and \( \angle B \) are supplementary. 2. Football practice is canceled for Monday. 3. \( \triangle DEF \) is a right triangle. 4. If you liked the movie, then you enjoyed yourself. 5. If two lines are not parallel, then they intersect at a point. 6. If you vacation at the beach, then you like Florida. 7. not possible 8. Tamika lives in Nebraska. 9. not possible 10. It is not freezing outside. 11. Shannon lives in the smallest state in the United States. 12. On Thursday, the track team warms up by jogging 2 miles.

**Guided Problem Solving 2-3**

1. conditional; hypothesis 2. Yes 3. Beth will go. 4. Anita, Beth, Aisha, Ramon 5. No; only two students went. 6. Beth, Aisha, Ramon; no—only two went. 7. Aisha, Ramon 8. The answer is reasonable. It is not possible for another pair to go to the concert. 9. Ramon

**Practice 2-4**


**Guided Problem Solving 2-4**

1. Angle Addition Postulate 2. Substitution Property of Equality; Simplify; Addition Property of Equality; Division Property of Equality 3. 40 4. yes; yes 5. 13; 13

**Practice 2-5**

1. 30 2. 15 3. 20 4. 6 5. 16 6. 9 7. \( m \angle A = 135 \); \( m \angle B = 45 \) 8. \( m \angle A = 36 \); \( m \angle B = 144 \) 9. \( m \angle A = 75 \); \( m \angle B = 15 \) 10. \( m \angle A = 10 \); \( m \angle B = 80 \) 11. \( m \angle PMO = 55 \); \( m \angle PMQ = 125 \); \( m \angle QMN = 55 \) 12. \( m \angle BOC = m \angle COE + m \angle BOC = m \angle COD = 45 \); \( m \angle AOB = m \angle DOE = 45 \) 13. \( m \angle BWC = m \angle CWD \); \( m \angle AOB + m \angle BWC = 180 \); \( m \angle CWD + m \angle DWA = 180 \); \( m \angle AWB = m \angle AWD \)

**Guided Problem Solving 2-5**

1. 90 2. See graph in Exercise 5 answer. 3. on the positive y-axis 4. Answers may vary. \( B \) can be any point on the positive y-axis. Sample: \( B(0, 3) \).
Chapter 3

Practice 3-1
1. corresponding angles  2. alternate interior angles  
3. same-side interior angles  4. alternate interior angles  
5. same-side interior angles  6. corresponding angles  
7. \( \angle 1 \) and \( \angle 5 \), \( \angle 2 \) and \( \angle 6 \), \( \angle 3 \) and \( \angle 7 \)  
8. \( \angle 4 \) and \( \angle 6 \), \( \angle 3 \) and \( \angle 5 \)  
9. \( \angle 4 \) and \( \angle 5 \)  
10. \( m \angle 1 = 100 \), alternate interior angles; \( m \angle 2 = 100 \), corresponding angles or vertical angles  
11. \( m \angle 1 = 75 \), alternate interior angles; \( m \angle 2 = 75 \), vertical angles or corresponding angles  
12. \( m \angle 1 = 135 \), corresponding angles; \( m \angle 2 = 135 \), vertical angles  
13. \( x = 103 \), \( 103^\circ \)  
14. \( x = 24 \), \( 12^\circ \), \( 168^\circ \)  
15. \( x = 30 \), \( 85 \), \( 85^\circ \)  
16a. Alternate Interior Angles Theorem  16b. Vertical angles are congruent.  
16c. Transitive Property of Congruence

Guided Problem Solving 3-1
1. The top and bottom sides are parallel, and the left and right sides are parallel.  
2. The two diagonals are transversals, and also each side of the parallelogram is a transversal for the two sides adjacent to it.  
3. Corresponding angles, interior and exterior angles are formed.  
4. \( v \), \( w \) and \( x \); By the Alternate Interior Angles Theorem, \( v = 42 \), \( w = 25 \) and \( x = 76 \).  
5. Answers may vary. Possible answer: By the Same-Side Interior Angles Theorem, \( (w + 42) + (y + 76) = 180 \). Since \( w = 25 \), \( y = 37 \). (The two \( y \)'s are equal by Theorem 3-1.)  
6. \( w = 25 \), \( y = 37 \), \( v = 42 \); yes  
7. \( v = 42 \), \( w = 35 \), \( x = 57 \), \( y = 46 \)

Practice 3-2
1a. same-side interior  1b. \( \overrightarrow{QR} \)  1c. \( \overrightarrow{TS} \)  1d. same-side interior  
1e. Same-Side Interior Angles  1f. \( \overrightarrow{TS} \)  1g. 3-5  
2. \( l \) and \( m \), Converse of Same-Side Interior Angles Theorem  
3. none  
4. \( BC \) and \( AD \), Converse of Same-Side Interior Angles Theorem  
5. \( \overrightarrow{RT} \) and \( \overrightarrow{HU} \), Converse of Corresponding Angles Postulate  
6. \( \overrightarrow{BH} \) and \( \overrightarrow{CI} \), Converse of Corresponding Angles Postulate  
7. \( a \) and \( b \), Converse of Same-Side Interior Angles Theorem  
8. 43  
9. 90  
10. 38  
11. 100  
12. 70  
13. 48

Guided Problem Solving 3-2
1. \( \ell \) and \( m \), transversals  
2. \( x \); the angles measuring \( 19x^\circ \) and \( 17x^\circ \)  
3. \( 5 \), \( 180^\circ \)  
4. \( 17x \), \( 180^\circ \)  
5. \( 180^\circ \)  
6. \( 17x \)  
7. \( 180 - 19x = 17x \) or \( 19x + 17x = 180 \)  
8. \( x = 5 \)  
9. With \( x = 5 \), \( 19x = 95 \) and \( 17x = 85 \).  
10. \( x = 6 \)

Practice 3-3
1. True. Every avenue will be parallel to Founders Avenue, and therefore every avenue will be perpendicular to Center City Boulevard, and therefore every avenue will be perpendicular to any street that is parallel to Center City Boulevard.  
2. Not necessarily true. No information has been given about the spacing of the streets.  
3. True. The fact that one intersection is perpendicular, plus the fact that every street belongs to one of two groups of parallel streets, is enough to guarantee that all intersections are perpendicular.  
4. True. Opposite sides of each block must be of the same type (avenue...
or boulevard), and adjacent sides must be of opposite type.
5. Not necessarily true. If there are more than three avenues and more than three boulevards, there will be some blocks bordered by neither Center City Boulevard nor Founders Avenue. 6. \(a \perp e\)  7. \(a \parallel e\)  8. \(a \perp e\)  9. \(a \parallel e\)  10. \(a \perp e\)  11. \(a \parallel e\)  12. If the number of \(\perp\) statements is even, then \(\ell_1 \parallel \ell_2\). If it is odd, then \(\ell_1 \perp \ell_2\). 13. The proof can instead use alternate interior angles or alternate exterior angles (if the angles are congruent, the lines are parallel) or same-side interior or same-side exterior angles (if the angles are supplementary, the lines are parallel).

14. It is possible.

Guided Problem Solving 3-3
1. supplementary angles  2. right angle  3. Any one of the following: Postulate 3-1, or Theorem 3-1, 3-2, 3-3 or 3-4  4. 90 5. It is congruent; Postulate 3-1  6. 90  7. \(a \perp c\)  8. It is true for any line parallel to \(b\).  9. Yes. The point is that a transversal cannot be perpendicular to just one of two parallel lines. It has to be perpendicular to both, or else to neither.

Practice 3-4
1. 125  2. 69  3. 143  4. 129  5. 140  6. 136  7. \(x = 35\); \(y = 145\); \(z = 25\)  8. \(a = 55\); \(b = 97\); \(c = 83\)  9. \(v = 118\); \(w = 37\); \(t = 62\)  10. 50  11. 88  12. \(m \angle 3 = 22\); \(m \angle 4 = 22\); \(m \angle 5 = 88\)  13. 57.1  14. 136  15. \(m \angle 1 = 33\); \(m \angle 2 = 52\)  16. isosceles  17. obtuse scalene  18. right scalene  19. obtuse isosceles  20. equiangular equilateral

Guided Problem Solving 3-4
1. three  2. 180  3. right triangle  4. \(z = 90\); Because it is given in the figure that \(\overline{BD} \perp \overline{AC}\).  5. Theorem 3-12, the Triangle Angle-Sum Theorem  6. \(x = 38\)  7. \(y = 36\)  8. \(\triangle ABD\) is a 36-54-90 right triangle. \(\triangle BCD\) is a 38-52-90 right triangle.  9. 74  10. \(\triangle ABC\) is a 52-54-74 acute triangle.  11. Yes, all three are acute angles, with \(\angle ABC\) visibly larger than \(\angle A\) and \(\angle C\).  12. \(\angle BCD\)

Practice 3-5
1. \(x = 120\); \(y = 60\)  2. \(n = 51\frac{3}{2}\)  3. \(a = 108\); \(b = 72\)  4. 109  5. 133  6. 129  7. 129  8. 47  9. 127  10. 30  11. 150  12. 6  13. 5  14. 8  15. \(BEDC\)  16. \(\angle FAE\)  17. \(\angle FAE\) and \(\angle BAE\)  18. \(ABCDE\)  19. 20

Guided Problem Solving 3-5
1. A theater stage, consisting of a large platform surrounding a smaller platform. The shapes in the bottom part of the figure may represent a ramp for actors to enter and exit.  2. The measures of angles 1 and 2  3. 8; octagon  4. \((8 - 2)180 = 1080\) degrees  5. 135  6. 45  7. Yes, angle 1 is an obtuse angle and angle 2 is an acute angle.  8. trapezoids  9. 360

Practice 3-6
1. \(y = \frac{1}{3}x - 7\)  2. \(y = -2x + 12\)  3. \(y = 7x - 18\)  4. \(y = -\frac{1}{2}x - 3\)  5. \(y = \frac{1}{6}x - \frac{3}{2}\)  6. \(y = \frac{1}{3}x - 2\)  7. \(y = 4x - 13\)  8. \(y = -x + 6\)  9. \(y = \frac{1}{3}x + 2\)  10. \(y = x - 1\)  11. \(y = -\frac{3}{4}x + 5\)  12. \(y = 2x - 3\)  13. \(y = 2x - 7\)  14. \(y = -\frac{3}{4}x + 2\)  15. \(y = 5x + 4\)  16. \(y = -\frac{3}{2}x - 3\)  17. \(y = 5x + 4\)  18. \(y = -\frac{3}{2}x - 3\)
1. Neither; $3 \neq \frac{1}{3} \cdot \frac{1}{3} = -1$
2. Perpendicular; $\frac{1}{2} \cdot -2 = -1$
3. Parallel; $-\frac{2}{3} \neq -\frac{2}{3}$
4. Parallel; $-1 \neq -1$
5. Perpendicular; $y = 2$ is a horizontal line, $x = 0$ is a vertical line
6. Parallel; $-\frac{1}{2} = -\frac{1}{2} = -\frac{1}{2}$
7. Neither; $1 \neq \frac{1}{2} \cdot \frac{1}{2} = -1$
8. Parallel; $-\frac{2}{3} = -\frac{2}{3}$
9. Perpendicular; $-1 \cdot 1 = -1$
10. Neither; $\frac{1}{2} \neq \frac{5}{2} \cdot \frac{5}{2} = -1$
11. Neither; $-\frac{2}{3} = -\frac{2}{3} = -\frac{2}{3}$

45a. $m = 0.10$  
45b. the amount of money the worker is paid for each box loaded onto the truck  
45c. $b = 3.90$  
45d. the base amount the worker is paid per hour  
46. $y = -\frac{2}{3} x + 8$

Guided Problem Solving 3-6

1. [Graph showing a triangle with vertices labeled A, B, and C]

2. $m = \frac{y_2 - y_1}{x_2 - x_1}$  
3. $y - y_1 = m(x-x_1)$
4. Slope of line $AB = \frac{5}{2}$, slope of line $BC = -\frac{5}{2}$. The absolute values of the slopes are the same, but one slope is positive and the other is negative.  
5. Point-slope form: $y - 0 = \frac{5}{2}(x - 0)$; slope-intercept form: $y = \frac{5}{2}x$
6. Point-slope form: $y - 5 = -\frac{5}{2}(x - 2)$ or $y - 0 = -\frac{5}{2}(x - 4)$; slope-intercept form: $y = -\frac{5}{2}x + 10$
7. Of line $\overrightarrow{AB}$; $(0, 0)$
8. $\triangle ABC$ appears to be an isosceles triangle, which is consistent with a horizontal base and two remaining sides having slopes of equal magnitude and opposite sign.
9. Slope of $\overrightarrow{BC}$; $(0, 10)$
10. $y = 0$; $y$-intercept $= (0, 0)$ just as for line $\overrightarrow{AB}$ (they intersect on the $y$-axis).
12. neither; $6 \neq -\frac{1}{2}$, $6 \cdot -\frac{1}{2} \neq -1$
13. neither; $\frac{2}{3} \neq 4$, $\frac{2}{3} \cdot 4 \neq -1$
14. parallel; $\frac{1}{2} = \frac{1}{2}$
15. $y = \frac{2}{3}x$
16. $y = -\frac{3}{4}x + 24$
17. $y = -x - 3$
18. $y = \frac{2}{3}x + 6$
19. $y = 0$
20. $y = 2x - 4$
21. $y = 2x$

Guided Problem Solving 3-7

1. 

2. a right angle  3. $m_1 \cdot m_2 = -1$
4. sides $GH$ and $GK$
5. Slope of $GH = \frac{3}{5}$, slope of $GK = -\frac{8}{3}$
6. Product $= -\frac{8}{5} \neq -1$. Sides $GH$ and $GK$ are not perpendicular.
7. $\triangle GKH$ has no pair of perpendicular sides. It is not a right triangle.
8. $\angle HGK$, approximately 80°
9. $\angle HGK$; approximately 80°
10. Slope of $LM = -\frac{3}{2}$ and slope of $LN = -\frac{2}{7}$. The product of the slopes is $-1$, so $LM$ and $LN$ are perpendicular.

Practice 3-8

1. – 3. 4. – 6.

7. – 9.
15. \[ \text{Guided Problem Solving 3-8} \]

1. a line segment of length \( c \)  
2. Construct a quadrilateral with one pair of parallel sides of length \( c \), and then examine the other pair.  
3. The procedure is given on p. 181 of the text.  
4. Adjust the compass to exactly span line segment \( c \), end to end. Then tighten down the compass adjustment as necessary.  
5.  
6.  
7. They appear to be both congruent and parallel.  
8. The answers to Step 7 are confirmed.  
9. Yes; a parallelogram

3A: Graphic Organizer

1. Parallel and Perpendicular Lines  
2. Answers may vary. Sample: properties of parallel lines; finding the measures of angles in triangles; classifying polygons; and graphing lines  
3. Check students’ work.

3D: Visual Vocabulary Practice/High-Use

1. property  
2. conclusion  
3. describe  
4. formula  
5. measure  
6. approximate  
7. compare  
8. contradiction  
9. pattern

3E: Vocabulary Check

Transversal: A line that intersects two coplanar lines in two points.  
Alternate interior angles: Nonadjacent interior angles that lie on opposite sides of the transversal.  
Same-side interior angles: Interior angles that lie on the same side of a transversal between two lines.  
Corresponding angles: Angles that lie on the same side of a transversal between two lines, in corresponding positions.  
Flow proof: A convincing argument that uses deductive reasoning, in which arrows show the logical connections between the statements.

Chapter 4

Practice 4-1

1. \( m \angle 1 = 110; m \angle 2 = 120 \)  
2. \( m \angle 3 = 90; m \angle 4 = 135 \)  
3. \( m \angle 5 = 140; m \angle 6 = 90; m \angle 7 = 40; m \angle 8 = 90 \)  
4. \( \angle A \equiv \angle S, \angle T \equiv \angle D \)  
5. \( \angle C \equiv \angle J, \angle A \equiv \angle S, \angle T \equiv \angle D \)  
6. \( \angle W \equiv \angle M, \angle X \equiv \angle K, \angle Y \equiv \angle L, \angle Z \equiv \angle M \)  
7. Yes; \( \angle GHJ \equiv \angle IHJ \)  
8. Given  
9. Theorem 4-1  
10. Given  
11. Definition of \( \equiv \)  
12. Definition of \( \equiv \) triangles

3C: Reading/Writing Math Symbols

1. \( m \perp n \)  
2. \( m \angle 1 + m \angle 2 = 180 \)  
3. \( AB \parallel CD \)  
4. \( m \angle MNP + m \angle MNQ = 90 \)  
5. \( \angle 3 \equiv \angle EFD \)  
6. Line 1 is parallel to line 2.  
7. The measure of angle \( ABC \) is equal to the measure of angle \( XYZ \).

8. Line \( AB \) is perpendicular to line \( DF \).  
9. Angle \( ABC \) and angle \( ABD \) are complementary.  
10. Angle 2 is a right angle, or the measure of angle 2 is 90°.  
11. Sample answer: \( \overline{CB} \parallel \overline{DG}, m \angle BAF = m \angle GFA \)
Practice 4-2
1. \( \triangle ADB \cong \triangle CDB \) by SAS  2. not possible  3. not possible  4. \( \triangle TUS \cong \triangle XWV \) by SSS  5. not possible  6. \( \triangle DEC \cong \triangle GHF \) by SAS  7. \( \angle MKL \cong \angle KML \) by SAS  8. \( \angle PRN \cong \angle PRQ \) by SSS  9. not possible  10. \( \angle C \)  11. \( \overline{AB} \) and \( \overline{BC} \)  12. \( \angle A \) and \( \angle B \)  13. \( \overline{AC} \)  14a. Given  14b. Reflexive Property of Congruence  14c. SAS Postulate

Guided Problem Solving 4-2
1. \( \overline{TS} \parallel \overline{OP} \) and \( \overline{SP} \) bisects \( \angle ISO \).  2. Prove whatever additional facts can be proven about \( \triangle ISP \) and \( \triangle OSP \), based on the given information.  3. \( \overline{TS} \parallel \overline{OS} \)  4. \( \angle ISP \cong \angle OSP \)  5. \( \overline{SP} \)  6. \( \triangle ISP \cong \triangle OSP \) by Postulate 4-2, the Side-Angle-Side (SAS) Postulate  7. It does not matter. The Side-Angle-Side Postulate applies whether or not they are collinear.  8. It does follow, because \( \triangle ISP \cong \triangle OSP \) and because \( \overline{TP} \) and \( \overline{PQ} \) are corresponding parts.

Practice 4-3
1. not possible  2. ASA Postulate  3. AAS Theorem  4. AAS Theorem  5. not possible  6. not possible  7. ASA Postulate  8. not possible  9. AAS Theorem

10. Statements
   1. \( \angle K \cong \angle M \), \( \overline{KL} \parallel \overline{ML} \)
   2. \( \angle JKL \cong \angle PLM \)
   3. \( \angle JKL \cong \angle PLM \) by ASA Postulate

11. \( \angle Q \cong \angle S \)

Guided Problem Solving 4-3
1. corresponding angles and alternate interior angles  2. \( \angle EAB \) and \( \angle DBC \)  3. \( \angle EBA \) and \( \angle DCB \)  4. \( \angle EAB \) and \( \angle DBC \)  5. \( \angle EAB \cong \angle DBC \), \( \overline{AE} \parallel \overline{BD} \), and \( \angle E \cong \angle D \)  6. \( \angle EAB \cong \angle DBC \) by Postulate 4-3, the Angle-Side-Angle (ASA) Postulate  7. Yes; Theorem 4-2, the Angle-Side-Angle (ASA) Theorem; \( \angle EBA \cong \angle DCB \)  8. No, because now there is no way to demonstrate a second pair of congruent sides, nor a second pair of congruent angles.

Practice 4-4
1. \( \overline{BD} \) is a common side, so \( \triangle ADB \cong \triangle CDB \) by SAS, and \( \angle A \cong \angle C \) by CPCTC.  2. \( \overline{FH} \) is a common side, so \( \triangle FHE \cong \angle HFG \) by ASA, and \( \overline{HE} \parallel \overline{FG} \) by CPCTC.  3. \( \angle KLI \cong \angle PMN \) by ASA, so \( \angle K \cong \angle P \) by CPCTC.  4. \( \overline{Q} \) is a common side, so \( \overline{QTS} \cong \angle SRQ \) by AAS.  5. \( \overline{XY} \) is a common side, so \( \triangle UVX \cong \triangle WVX \) by SSS, and \( \angle U \cong \angle W \) by CPCTC.  6. \( \angle AYZ \) and \( \angle CAB \) are vertical angles, so \( \angle ABC \cong \angle AYZ \) by ASA, and \( \angle ZA \cong \angle AC \) by CPCTC.  7. \( \overline{EG} \) is a common side, so \( \triangle DEG \cong \triangle FEG \) by SAS, and \( \overline{FG} \parallel \overline{DG} \) by CPCTC.  8. \( \angle JKH \) and \( \angle LKM \) are vertical angles, so \( \angle LJM \equiv \angle MLK \) by SAS, and \( \overline{LJ} \cong \overline{KL} \) by CPCTC.  9. \( \overline{PR} \) is a common side, so \( \triangle PNR \cong \triangle QRP \) by SSS, and \( \angle N \cong \angle Q \) by CPCTC.  10. First, show that \( \angle ABC \) and \( \angle ECD \) are vertical angles. Then, show \( \triangle ABC \cong \triangle EDC \) by ASA. Last, show \( \angle A \cong \angle E \) by CPCTC.  11. First, show \( \overline{FH} \) as a common side. Then, show \( \triangle JFH \cong \triangle GHF \) by ASA. Last, show \( \overline{FG} \parallel \overline{FH} \) by CPCTC.

Guided Problem Solving 4-4
1. A compass with a fixed setting was used to draw two circular arcs, both centered at point \( P \) but crossing \( \ell \) in different locations, which were labeled \( A \) and \( B \). The compass was used again, with a wider setting, to draw two intersecting circular arcs, one centered at \( A \) and one at \( B \). The point at which the new arc intersected was labeled \( C \). Finally, line \( \overline{CP} \) was drawn.  2. Find equal lengths or distances and explain why \( \overline{CP} \) is perpendicular to \( \ell \).  3. \( \angle AC \parallel \angle DBC \)  4. \( \overline{AP} \parallel \overline{PB} \) and \( \overline{AC} \parallel \overline{BC} \), so \( \triangle APC \cong \triangle BPC \) by Postulate 4-1, the Side-Side-Side (SSS) Postulate  5. \( \triangle APC \cong \triangle BPC \) by CPCTC.  6. Since \( \triangle APC \cong \triangle BPC \), \( m\angle APC \equiv m\angle BPC \). \( m\angle APC \equiv m\angle BPC \equiv 180 \), it follows that \( m\angle APC = m\angle BPC = 90 \).  8. From the definition of perpendicular and the fact that \( m\angle APC = m\angle BPC = 90 \), the distances do not matter, so long as \( \overline{AP} \parallel \overline{BP} \) and \( \overline{AC} \parallel \overline{BC} \). That is what is required in order that \( \triangle APC \cong \triangle BPC \).  10. Draw a line, and use the construction technique of the problem to construct a second line perpendicular to the first. Then do the same thing again to construct a third line perpendicular to the second line. The first and third lines will be parallel, by Theorem 3-10.

Practice 4-5
1. \( x = 35; y = 35 \)  2. \( x = 80; y = 90 \)  3. \( t = 150 \)  4. \( r = 45; s = 45 \)  5. \( x = 55; y = 70; z = 125 \)  6. \( a = 132; b = 36; c = 60 \)  7. \( x = 6 \)  8. \( a = 30; b = 30 \)  9. \( c = 75 \)  10. \( AD = \angle D \parallel \angle E \)  11. \( \overline{CA}; \angle CAG \parallel \angle GBC \)  12. \( KT; \angle KII \parallel \angle KJI \)  13. \( DC; \angle CDE \parallel \angle CED \)  14. \( \overline{BA}; \angle ABJ \parallel \angle AJB \)  15. \( \overline{CB}; \angle BCH \parallel \angle BHC \)  16. \( 130 \)  17. \( 65 \)  18. \( 130 \)  19. \( 90 \)  20. \( x = 70; y = 55 \)  21. \( x = 70; y = 20 \)  22. \( x = 45; y = 45 \)
Guided Problem Solving 4-5
1. One angle is obtuse. The other two angles are acute and congruent. 2. Highlight an obtuse isosceles angle and find its angle measures, then find all the other angle measures represented in the figure. 3. Possible answer:

4. 60° 5. 30°, because the measure of each base angle is half the measure of an angle of the equilateral triangle. 6. 120° because the sum of the angles of the highlighted triangle must equal 180°. 7. The other measures are 90° and 150°.

Examples:

Practice 4-6
1. Statements
   1. $\overline{AB} \perp \overline{BC}$, $\overline{ED} \perp \overline{FE}$
   2. $\angle B$, $\angle E$ are right $\angle$s.
   3. $\overline{AC} \cong \overline{DF}$, $\overline{AB} \cong \overline{ED}$
   4. $\triangle ABC \cong \triangle DEF$

2. Statements
   1. $\overline{PS} \cong \overline{QR}$
   2. $\overline{SQ} \cong \overline{QS}$
   3. $\triangle POS \cong \triangle RSQ$

3. $\angle MJN$ and $\angle MJK$
   Perpendicular lines form right $\angle$s.
   $\overline{MN} \cong \overline{MK}$
   Given
   $\triangle MJN \cong \triangle MJK$
   HL Theorem
   $\overline{MJ} \cong \overline{MJ}$
   Reflexive Property of $\cong$

4. $\overline{GI} \cong \overline{JI}$
   Given
   $\overline{HH} \cong \overline{HH}$
   Reflexive Property of $\cong$
   $\triangle IHG \cong \triangle JHG$
   HL Theorem
   $\angle GHI \cong \angle JHI$
   Given
   $m\angle GHI + m\angle JHI = 180$
   Angle Addition Postulate

5. $\overline{RS} \cong \overline{VW}$
6. none
7. $m\angle C$ and $m\angle F = 90$
8. $\overline{GH} \cong \overline{JI}$
9. $\overline{LN} \cong \overline{PR}$
10. $\overline{ST} \cong \overline{UV}$ or $\overline{SV} \cong \overline{UT}$
11. $m\angle A$ and $m\angle X = 90$
12. $m\angle F$ and $m\angle D = 90$
13. $\overline{GI} \perp \overline{HH}$

Guided Problem Solving 4-6
1. Two congruent right triangles. Each one has a leg and a hypotenuse labeled with a variable expression. 2. The values of $x$ and $y$ for which the triangles are congruent by HL.
3. The two shorter legs are congruent. 4. $x = y + 1$
5. The hypotenuses are congruent. 6. $x + 3 = 3y$
7. $x = 3; y = 2$
8. $m\angle GHI$ and $m\angle JHI$ are right $\angle$s.

Practice 4-7
1. $\triangle ZWX \cong \triangle YXW$; SAS
2. $\triangle ABC \cong \triangle DCF$; ASA
3. $\triangle EJG \cong \triangle FKH$; ASA
4. $\triangle LNP \cong \triangle LMO$; SAS
5. $\triangle ADF \cong \triangle BGE$; SAS
6. $\triangle UVY \cong \triangle VUX$; ASA

7. $\overline{BC}$
8. $\overline{FG}$
9. $\angle L$
10. Sample:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $AX \cong AY$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $CX \perp AB, BY \perp AC$</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. $m \angle CAX$ and $m \angle BYA = 90$</td>
<td>3. Perpendicular lines form right $\angle$s.</td>
</tr>
<tr>
<td>4. $\angle A \cong \angle A$</td>
<td>4. Reflexive Property of $\cong$</td>
</tr>
<tr>
<td>5. $\triangle BYA \cong \triangle CAX$</td>
<td>5. ASA Postulate</td>
</tr>
</tbody>
</table>

11. Sample: Because $\overline{FD} \cong \overline{GE}$, $\angle HFG \cong \angle EGF$, and $\overline{FG} \cong \overline{GF}$, then $\triangle FGE \cong \triangle GFH$ by SAS. Thus, $\overline{FE} \cong \overline{GH}$ by CPCTC and $\overline{EH} \cong \overline{HE}$, then $\triangle GHE \cong \triangle FHE$ by SSS.

**Guided Problem Solving 4-7**

1. The figure, a list of parallel and perpendicular pairs of sides, and one known angle measure, namely $m \angle A = 56$
2. nine
3. They are congruent and have equal measures
4. $m \angle A = m \angle 1 = m \angle 2 = 56$
5. $m \angle 4 = 90$
6. $m \angle 3 = 34$
7. $m \angle DCE = 56$
8. $m \angle 5 = 22$
9. $m \angle FCG = 90$
10. $m \angle 6 = 34$
11. $m \angle 7 = 34$
$m \angle 8 = 68$, and $m \angle 9 = 112$
12. $m \angle 9 = 56 + 56 = 112$
13. $m \angle FIC = 180 - (m \angle 2 + m \angle 3) = 90$
$m \angle DHC = m \angle 4 = 90$; $m \angle FJC = 180 - m \angle 9 = 68$.
$m \angle BIG = m \angle FIC = 90$

**4A: Graphic Organizer**

1. Congruent Triangles
2. Answers may vary. Sample: congruent figures; triangle congruence by SSS, SAS, ASA, and AAS; proving parts of triangles congruent; the Isosceles Triangle Theorem
3. Check students’ work.

**4B: Reading Comprehension**

1. Yes. Using the Isosceles Triangle Theorem, $\angle W \cong \angle Y$. It is given that $\overline{WX} \parallel \overline{XY}$ and $\overline{WU} \parallel \overline{YV}$. Therefore $\triangle WUX \cong \triangle YVX$ by SAS. 2. There is not enough information. You need to know if $\overline{AC} \parallel \overline{EC}$, if $\angle A \cong \angle E$, or if $\angle B \cong \angle D$. 3. a

**4C: Reading/Writing Math Symbols**

1. Angle-Angle-Side
2. triangle $XYZ$
3. angle $PQR$
4. line segment $BD$
5. line $ST$
6. ray $WX$
7. hypotenuse-
8. line $3$
9. angle $6$
10. Angle-Side-Angle

**4D: Visual Vocabulary Practice**

1. theorem
2. congruent polygons
3. base angle of an isosceles triangle
4. CPCTC
5. postulate
6. vertex angle of an isosceles triangle
7. corollary
8. base of an isosceles triangle
9. legs of an isosceles triangle

**4E: Vocabulary Check**

Angle: Formed by two rays with the same endpoint.
Congruent angles: Angles that have the same measure.
Congruent segments: Segments that have the same length.
Corresponding polygons: Polygons that have corresponding sides congruent and corresponding angles congruent.
CPCTC: An abbreviation for “corresponding parts of congruent triangles are congruent.”

**4F: Vocabulary Review Puzzle**

1. postulate
2. hypotenuse
3. angle
4. vertex
5. side
6. leg
7. perpendicular
8. polygon
9. supplementary
10. parallel
11. corresponding

**Chapter 5**

**Practice 5-1**

1a. 8 cm
1b. 16 cm
1c. 14 cm
2a. 22.5 in.
2b. 15.5 in.
2c. 15.5 in.
3a. 9.5 cm
3b. 17.5 cm
3c. 14.5 cm
4. 17
5. 20.5
6. 7
7. 12$rac{1}{2}$
8. 42
9. 16.5
10a. 18
10b. 61
11. $\overline{GH} \parallel \overline{AC}$, $\overline{HI} \parallel \overline{BA}$, $\overline{GT} \parallel \overline{BC}$
12. $PR \parallel YZ$, $PQ \parallel XZ$, $XY \parallel RQ$

**Guided Problem Solving 5-1**

1. 30 units
2. The three sides of the large triangle are each bisected by intersections with the two line segments lying in the interior of the large triangle.
3. the value of $x$
4. They are called midsegments.
5. They are parallel, and the side labeled 30 is half the length of the side labeled 30.
6. $x = 60$
7. Yes; the side labeled $x$ appears to be about twice as long as the side labeled 30.
8. No. Those lengths are not fixed by the given information. (The triangle could be vertically stretched or shrunk without changing the lengths of the labeled sides.) All one can say is that the midsegment is half as long as the side it is parallel to.

**Practice 5-2**

1. $\overline{WV}$ is the perpendicular bisector of $\overline{XZ}$
2. 4
3. 7.5
4. 9
5. right triangle
6. 5
7. 17
8. 17
9. equidistant
10. isosceles triangle
11. 3.5
12. 21
13. 21
14. right triangle
15. $\overline{JP}$ is the bisector of $\angle LIN$
16. 9
17. 45
18. 45
19. 14
20. Sample: Point $M$ lies on $\overline{JP}$. 21. right isosceles triangle
Guided Problem Solving 5-2

1. 

![Diagram](image)

2. See answer to Step 1, above. 3. See answer to Step 1, above. 4. Plot a point and explain why it lies on the bisector of the angle at the origin. 5. line \( \ell \): \( y = -\frac{3}{4}x + \frac{25}{2} \); line \( m \): \( x = 10 \). 6. \((10, 5)\) 7. \( CA = CB = 5 \); yes 8. Theorem 5-5, the Converse of the Angle Bisector Theorem 9. \( m \angle AOC = m \angle BOC \approx 27 \). 10. Draw \( \ell \), \( m \), and \( c \), then draw \( \overline{OC} \). Since \( OA = OB = 10 \), it follows that \( \triangle OAC \cong \triangle OBC \), by HL. Then \( CA = CB \) and \( \angle AOC \cong \angle BOC \) by CPCTC.

Practice 5-3

1. \((-2, 2)\) 2. \((4, 0)\) 3. \((2, 1)\) 4. Check students’ work. The final result of the construction is shown.

5. altitude 6. median 7. none of these 8. perpendicular bisector 9. angle bisector 10. altitude 11a. \((2, 0)\) 11b. \((-2, -2)\) 12a. \((0, 0)\) 12b. \((3, -4)\) 13a. \((0, 0)\) 13b. \((0, 3)\)

Guided Problem Solving 5-3

1. the figure and a proof with some parts left blank 2. Fill in the blanks. 3. \( AB \) 4. Theorem 5-2, the Perpendicular Bisector Theorem 5. \( BC \); \( XC \) 6. the Transitive Property of Equality 7. Perpendicular Bisector. (This converse is Theorem 5-3.) 8. The point of the proof is to demonstrate that \( n \) runs through point \( X \). It would not be appropriate to show that fact as already given in the figure. 9. Nothing essential would change. Point \( X \) would lie outside \( \triangle ABC \) (below \( BC \)), but the proof would run just the same.

Practice 5-4

1. I and III 2. I and II 3. The angle measure is not 65. 4. Tina does not have her driver’s license. 5. The figure does not have eight sides. 6. The restaurant is open on Sunday. 7. \( \triangle ABC \) is congruent to \( \triangle XYZ \). 8. \( m \angle Y \leq 50 \) 9a. If two triangles are not congruent, then their corresponding angles are not congruent; false. 9b. If corresponding angles are not congruent, then the triangles are not congruent; true. 10a. If you do not live in Toronto, then you do not live in Canada; false. 10b. If you do not live in Canada, then you do not live in Toronto; true. 11. Assume that \( m \angle A \neq m \angle B \). 12. Assume that \( TUVW \) is not a trapezoid. 13. Assume that \( LM \) does not intersect \( NO \). 14. Assume that \( \triangle FGH \) is not equilateral. 15. Assume that it is not sunny outside. 16. Assume that \( \angle D \) is obtuse. 17. Assume that \( m \angle A \approx 90 \). This means that \( m \angle A + m \angle C \approx 180 \). This, in turn, means that the sum of the angles of \( \triangle ABC \) exceeds 180, which contradicts the Triangle Angle-Sum Theorem. So the assumption that \( m \angle A \approx 90 \) must be incorrect. Therefore, \( m \angle A < 90 \).

Guided Problem Solving 5-4

1. Ice is forming on the sidewalk in front of Toni’s house. 2. Use indirect reasoning to show that the temperature of the sidewalk surface must be \( 32°F \) or lower. 3. The temperature of the sidewalk in front of Toni’s house is greater than \( 32°F \). 4. Water is liquid (ice does not form) above \( 32°F \). 5. There is no ice forming on the sidewalk in front of Toni’s house. 6. The result from Step 5 contradicts the information identified as given in Step 1. 7. The temperature of the sidewalk in front of Toni’s house is less than or equal to \( 32°F \). 8. If the temperature is above \( 32°F \), water remains liquid. This is reliably true. Converse: If water remains liquid, the temperature is above \( 32°F \). This is not reliably true. Adding salt will cause water to remain liquid even below \( 32°F \). 9. Suppose two people are each the world’s tallest person. Call them person A and person B. Then person A would be taller than everyone else, including B, but by the same token B would be taller than A. It is a contradiction for two people each to be taller than the other. So it is impossible for two people each to be the World’s Tallest Person.

Practice 5-5

1. \( \angle M, \angle N \) 2. \( \angle C, \angle D \) 3. \( \angle S, \angle Q \) 4. \( \angle R, \angle P \) 5. \( \angle A, \angle T \) 6. \( \angle S, \angle A \) 7. yes; \( 4 + 7 > 8, 7 + 8 > 4, 8 + 4 > 7 \) 8. no; \( 6 + 10 > 17 \) 9. yes; \( 4 + 4 > 4 \) 10. yes; \( 1 + 9 > 9, 9 + 9 > 1, 9 + 1 > 9 \) 11. yes; \( 11 + 12 > 13, 12 + 13 > 11, 13 + 11 > 12 \) 12. no; \( 18 + 20 > 40 \) 13. no; \( 1.2 + 2.6 > 4.9 \) 14. no; \( 8.4 + 9.3 > 18 \) 15. no; \( 2.5 + 3.5 > 6 \) 16. \( BC, AB, AC \) 17. \( BO, BL, LO \) 18. \( RS, ST, RT \) 19. \( \triangle D, \triangle S, \triangle A \) 20. \( \triangle N, \triangle S, \triangle J \) 21. \( \angle R, \angle O, \angle P \) 22. \( 3 < x < 11 \) 23. \( 8 < x < 26 \) 24. \( 0 < x < 10 \) 25. \( 9 < x < 31 \) 26. \( 2 < x < 14 \) 27. \( 13 < x < 61 \)

Guided Problem Solving 5-5

1.
2. The side opposite the larger included angle is greater than the side opposite the smaller included angle. 3. The angle opposite the larger side is greater than the angle opposite the smaller side. 4. The opposite sides each have a length of nearly the sum of the other two side lengths. 5. The opposite sides are the same length. They are corresponding parts of triangles that are congruent by SAS.

5A: Graphic Organizer
1. Relationships Within Triangles 2. Answers may vary. Sample: midsegments of triangles; bisectors in triangles; concurrent lines, medians, and altitudes; and inverses, contrapositives, and indirect reasoning. 3. Check students’ work.

5B: Reading Comprehension
1. The width of the tar pit is 10 meters. 2. b

5C: Reading/Writing Math Symbols

5D: Visual Vocabulary Practice
1. median 2. negation 3. circumcenter 4. contrapositive 5. centroid 6. equivalent statements 7. incenter 8. inverse 9. altitude

5E: Vocabulary Check
Midpoint: A point that divides a line segment into two congruent segments.
Midsegment of a triangle: The segment that joins the midpoints of two sides of a triangle.
Proof: A convincing argument that uses deductive reasoning.
Coordinate proof: A proof in which a figure is drawn on a coordinate plane and the formulas for slope, midpoint, and distance are used to prove properties of the figure.
Distance from a point to a line: The length of the perpendicular segment from the point to the line.

5F: Vocabulary Review

Chapter 6
Practice 6-1
1. parallelogram 2. rectangle 3. quadrilateral 4. parallelogram, quadrilateral 5. kite, quadrilateral 6. rectangle, parallelogram, quadrilateral 7. trapezoid, isosceles trapezoid, quadrilateral 8. square, rectangle, parallelogram, rhombus, quadrilateral 9. \( x = 7 \), \( AB = BD = DC = CA = 11 \) 10. \( m = 9; s = 42 \); \( ON = LM = 26; OL = MN = 43 \) 11. \( f = 5; g = 11 \); \( FG = GH = HI = IF = 17 \) 12. parallelogram 13. rectangle 14. kite 15. parallelogram

Guided Problem Solving 6-1
1. A labeled figure, which shows an isosceles trapezoid 2. The nonparallel sides are congruent. 3. The measures of the angles and the lengths of the sides 4. \( m \angle G = c \) 5. \( c + (4c - 20) = 180 \) 6. 40 7. \( m \angle D = m \angle G = 40 \), \( m \angle E = m \angle F = 140 \) 8. \( a - 4 = 11 \) 9. 15 10. \( DE = FG = 11 \); \( EF = 15 \); \( DG = 32 \) 11. \( 40 + 40 + 140 + 140 = 560 \) \( = (4 - 2)180 \) 12. \( m \angle D = m \angle G = 39 \), \( m \angle E = m \angle F = 141 \)

Practice 6-2
1. 15 2. 32 3. 7 4. 8 5. 12 6. 9 7. 8 8. 35 9. 54 10. 34 11. 54 12. 34 13. 100 14. 40; 40 15. 70; 110; 70 16. 113; 45; 22 17. 115; 15; 50 18. 55; 105; 55 19. 61; 72; 108; 32 20. 32; 98; 50 21. 16 22. 35 23. 28 24. 4

Guided Problem Solving 6-2
1. The ratio of two different angle measures in a parallelogram. 2. The consecutive angles are supplementary. 3. \( \frac{9\pi}{x} \)
4. The measures of the angles. 5. The angles are supplementary angles, because they are consecutive. 6. \( x + 9\pi = 180 \) 7. 18 and 162 8. No; the lengths of the sides are irrelevant in this problem. 9. 30 and 150

Practice 6-3
1. no 2. yes 3. yes 4. no 5. yes 6. yes 7. \( x = 2 \); \( y = 3 \). 8. \( x = 6; y = 3 \). 9. \( x = 64; y = 10 \) 10. \( x = 8 \); the figure is a \( \Box \) because both pairs of opposite sides are congruent. 11. \( x = 40 \); the figure is not a \( \Box \) because one pair of opposite angles is not congruent. 12. \( x = 25 \); the figure is a \( \Box \) because the congruent opposite sides are \( \parallel \) by the converse of the Alternate Interior Angles Theorem. 13. Yes; the diagonals bisect each other. 14. No; the congruent opposite sides do not have to be \( \parallel \). 15. No; the figure could be a trapezoid. 16. Yes; both pairs of opposite sides are congruent. 17. Yes; both pairs of opposite sides are \( \parallel \) by the converse of the Alternate Interior Angles Theorem. 18. Yes; only one pair of opposite angles is congruent. 19. Yes; one pair of opposite sides is both congruent and \( \parallel \). 20. No; only one pair of opposite sides is congruent.
Guided Problem Solving 6-3
1. A labeled figure, which shows a quadrilateral that appears to be a parallelogram 2. The consecutive angles are supplementary. 3. Find values for x and y which make the quadrilateral a parallelogram 4. \( m\angle A + m\angle D = 180 \), so that \( \angle A \) and \( \angle D \) meet the requirements for same-side interior angles on the transversal of two parallel lines (Theorems 3-2 and 3-6). 5. \( \angle B \equiv \angle D \). 6. \( 3x + 10 + 5y = 180; 8x + 5 = 5y \). 7. \( x = 15, y = 25 \). 8. \( m\angle A = m\angle C = 55 \) and \( m\angle B = m\angle D = 125 \), which matches the appearance of the figure. 9. \( (3x + 10) + (8x + 5) = 180; \text{yes} \)

Practice 6-4
1a. rhombus 1b. 72; 54; 72 2a. rectangle 2b. 72; 40; 18; 144 3a. rectangle 3b. 37; 53; 106; 74 4a. rhombus 4b. 59; 90; 59 5a. rectangle 5b. 60; 60; 20 6a. rhombus 6b. 22; 68; 68; 90

7. Yes; the parallelogram is a rhombus. 8. Possible; opposite angles are congruent in a parallelogram. 9. Impossible; if the diagonals are perpendicular, then the parallelogram should be a rhombus, but the sides are not of equal length.

10. \( x = 7; HI = 7; IK = 7 \). \( x = 7; HI = 26 \). \( IK = 26 \). 11. \( x = 7; HI = 26; IK = 25 \). 12. \( x = -3; HI = 13; IK = 13 \). 13a. 90; 90; 29; 29 13b. 288 cm²

15a. 70; 90; 70 15b. 88 in² 16a. 38; 90; 90 16b. 260 m²

17. Possible; because opposite angles are congruent and supplementary, for the figure to be a parallelogram they must measure 90, the figure therefore must be a rectangle.

Guided Problem Solving 6-4
1. A labeled figure, which shows a parallelogram. One angle is a right angle, and two adjacent sides are congruent. Algebraic expressions are given for the lengths of three line segments. 2. diagonals 3. Find the values of x and y. 4. It is a square. Theorem 6-1 and the fact that \( AB \equiv AD \) imply that all four sides are congruent. Theorems 3-11 and 6-2 plus the fact that \( m\angle B = 90 \) imply that all four angles are right angles. 5. congruent; bisect 6. \( 4x - y + 1 = (2x - 1) + (3y + 5) \); \( 2x - 1 = 3y + 5 \) 7. \( x = 7; y = 2 \). 8. It was not necessary to know \( AB \equiv AD \), but it was necessary to know \( m\angle B = 90 \). The key fact, which enables the use of Theorem 6-11 in addition to Theorem 6-3, is that \( ABCD \) is a rectangle. It does not matter whether all four sides are congruent. 9. 40

Practice 6-5
1. 118; 62 2. 99; 81 3. 59; 121 4. 96; 84 5. 101; 79 6. 67; 113 7. \( x = 4 \). 8. \( x = 16; y = 116 \). 9. \( x = 1 \). 10. 105.5; 105.5 11. 90; 25 12. 118; 118 13. 90; 63; 63 14. 107; 107 15. 90; 51; 39 16. \( x = 8 \). 17. \( x = 7 \). 18. \( x = 28; y = 32 \)

Guided Problem Solving 6-5
1. Isosceles trapezoid \( ABCD \) with \( \overline{AB} \parallel \overline{DC} \). 2. \( \angle B \equiv \angle C \) and \( \angle BAD \equiv \angle D \). 3. \( \overline{AB} \equiv \overline{DC} \) is given. \( \overline{DC} \equiv \overline{AE} \) because opposite sides of a parallelogram are congruent (Theorem 6-1). \( \overline{AB} \equiv \overline{AE} \) is from the Transitive Property of Congruence. 4. Isosceles; \( \equiv \) because base angles of an isosceles triangle are congruent. 5. \( \angle 1 \equiv \angle 2 \) because corresponding angles on a transversal of two parallel lines are congruent. 6. \( \angle B \equiv \angle C \) by the Transitive Property of Congruence. 7. \( \angle BAD \) is a same-side interior angle with \( \angle B \), and \( \angle D \) is a same-side interior angle with \( \angle C \). 8. This is not a proof, because for \( AD > BC \) there is a similar proof with a line segment drawn from \( B \) to a point \( E \) lying on \( AD \).

9. The two drawn segments can be shown to be congruent, and then one has two congruent right triangles by the HL Theorem. \( \angle B \equiv \angle C \) follows by CPCTC and \( \angle BAD \equiv \angle D \) because they are supplements of congruent angles.

Practice 6-6
1. (1.5a, 2b); a 2. (1.5a, b); \( \sqrt{a^2 + 4b^2} \) 3. 5a, 0; 0 4. 0, 5a, b; \( \sqrt{a^2 + 4b^2} \) 5. 0 6. 1 7. \( -\frac{1}{2} \) 8. 2 9. \( \frac{7b}{5a} \) 10. \( \frac{-2b}{3a} \) 11. \( \frac{2b}{3a} \) 12. \( \frac{-2b}{3a} \) 13. \( E(a, 3b); I(a, 4a, 0) \) 14. \( O(3a, 2b); M(3a, -2b); E(3a, -2b) \) 15. \( D(4a, b); I(3a, 0) \) 16. \( T(0, 2b); A(4a, b); L(2a, 2b) \) 17. \( (-4a, b) \) 18. \( (-b, 0) \)

Guided Problem Solving 6-6
1. A rhombus with coordinates given for two vertices 2. A rhombus is a parallelogram with four congruent sides. 3. the coordinates of the other two vertices 4. They are the diagonals. 5. They bisect each other. 6. \( W(-2r, 0), Z(0, -2t) \); 7. No; neither Theorem 6-3 nor any other theorem or result would apply. 8. Slope of \( WX = 0 \) and slope of \( YZ \) is undefined. This confirms Theorem 6-10, which says that the diagonals of a rhombus are perpendicular.

Practice 6-7
1a. \( \frac{b}{q} \) 1b. \( y = mx + b; q = \frac{p}{q}(p) + b; b = q - \frac{p^2}{q} \). \( y = \frac{p}{q}x + q - \frac{p^2}{q} \). 1c. \( x = r + p; 1d. y = \frac{p}{q}(r + p) + q - \frac{p^2}{q}; y = \frac{p}{q} + \frac{p^2}{q} + q - \frac{p^2}{q} \). \( y = \frac{p}{q} + q; \) intersection at \( (r + p, \frac{p^2}{q} + q) \) 1e. \( \frac{p}{q} \). 1f. \( r, q \). 1g. \( y = mx + b; \) \( y = \frac{p}{q}r + b; b = q - \frac{p^2}{q} \). \( y = \frac{p}{q} + \frac{p}{q}; \) intersection at \( (r + p, \frac{p}{q} + q) \) 1i. \( (r + p, \frac{p}{q} + q) \) 2a. \( -2a, 0 \) 2b. \( -2a, 0 \) 2c. \( \frac{(-2a, 0)}{2} \) 2d. \( \frac{b}{q} \) 3a. \( -4a, 0 \) 3b. \( -2a, 3a \) 3c. \( \frac{3}{2} \) 3d. \( 2a, -a \) 3e. \( \frac{1}{2} \) 4. The coordinates for \( D \) are \( (0, 2b) \). The coordinates for \( C \) are \( (2a, 0) \). Given these coordinates, the lengths of \( DC \) and \( HT \) can be determined.
Guided Problem Solving 6-7
1. kite $DEFG$ with $DE = EF$ with the midpoint of each side identified. 2. A kite is a quadrilateral with two pairs of adjacent sides congruent and no opposite sides congruent. 3. The midpoints are the vertices of a rectangle. 4. The midpoints are the vertices of a rectangle. 5. $D(-2b, 2c), G(0, 0)$. 6. Slope of $KL = \text{slope of } NM = 0$, slopes of $KN$ and $LM$ are undefined. 7. Opposite sides are parallel; it is a rectangle. 8. Adjacent sides are perpendicular. 9. Right angles. 10. Answers will vary. Example: $a = 3, b = 2, c = 2$ yields the points $D(-4, 4), E(0, 6), F(4, 4), G(0, 0)$ with midpoints at $(-2, 2), (-2, 5), (2, 5), (2, 2)$. Connecting these midpoints forms a rectangle. 11. Construct $DF$ and $EG$. Slope of $DF = 0$, so $DF$ is horizontal. Slope of $EG$ is undefined, so $EG$ is vertical.

6A: Graphic Organizer
1. Quadrilaterals. 2. Answers may vary. Sample: classifying quadrilaterals; properties of parallelograms; proving that a quadrilateral is a parallelogram; and special parallelograms. 3. Check students' work.

6B: Reading Comprehension
1. $QT \equiv SR, QR \equiv ST, QT \parallel RS, QR \parallel TV$. 2. No, it cannot be proven that $\triangle QTV \equiv \triangle SRU$ because with the given information, only one side and one angle of the two triangles can be proven to be congruent. Another side or angle is needed. If it were given that $QUSV$ is a parallelogram, then the proof could be made. 3. All four sides are congruent. 4. Yes. Since $EG \cong EG$ by the Reflexive Property, $\triangle EFG \cong \triangle EHG$ by SSS. 5. $b$.

6C: Reading/Writing Math Symbols
1. $\triangle AEF, \angle HHI$, or $\overline{BII}$. 2. $\overline{DF}, \overline{FG}$, or $\overline{EG}$. 3. $G$. 4. $\overline{DE}$ 5. rhombus 6. rectangle 7. square 8. isosceles trapezoid

6D: Visual Vocabulary Practice/High-Use Academic Words
1. solve 2. deduce 3. equivalent 4. indirect 5. equal 6. analysis 7. identify 8. convert 9. common

6E: Visual Vocabulary Check

6F: Vocabulary Review Puzzle

![Image of a vocabulary puzzle with terms like parallelogram, trapezoid, centroid, square, hypotenuse, etc.]}
Chapter 7

Practice 7-1

Guided Problem Solving 7-1
1. ratios 3. denominator 4. 2 1 = 82,000,000 or 1,000,000 5. Cross-Product Property 6. 0.029 7. 0.348 8. yes 9. 21.912

Practice 7-2
1. △ABC ~ △XYZ with similarity ratio 2 : 1 2. △QMN ~ △RST with similarity ratio 5 : 3 3. Not similar; corresponding sides are not proportional. 4. Not similar; corresponding angles are not congruent. 5. △ABC ~ △KMN with similarity ratio 4 : 7 6. Not similar; corresponding sides are not proportional. 7. 1 8 9. 10. 11. 12. 13. 3.96 ft 14. 4.8 in. 15. 3.75 cm 16. 10 m 17. 5 3 18. 53 19. 7 20. 4 21. 53 22. 37 23. 5

Guided Problem Solving 7-2
1. equal 2. 6.14 3. 2.61; 6.14 4. 19.3662; 19.2182 5. no 6. no 7. 2.3706; 2.3525 8. Since the quotients are not equal, the ratios are not equal, and the bills are not similar rectangles. 9. 4.045

Practice 7-3
1. △AXB ≅ △RXQ because vertical angles are ≅, △A ≅ △R (Given). Therefore △AXB ~ △RXQ by the AA ~ Postulate. 2. Because \( \frac{MP}{TW} = \frac{PX}{WX} = \frac{XM}{AX} = \frac{3}{4} \), △MPX ~ △LWA by the SSS ~ Theorem. 3. △QMP ≅ △AMB because vertical sides are ≅. Then, because \( \frac{QM}{AM} = \frac{PM}{BM} = \frac{2}{1} \), △QMP ~ △AMB by the SAS ~ Theorem. 4. △M ≅ △A (Given). Because there are 180° in a triangle, \( \measuredangle J = 130° \), and △J ≅ △C. So △MJN ~ △ACB by the AA ~ Postulate. 5. Because \( AX = BX \) and \( CX = RX, \frac{AX}{RX} = \frac{BX}{CR} \). △AXB ≅ △CXR because vertical angles are ≅. Therefore △AXB ~ △CXR by the SAS ~ Theorem. 6. Because \( AB = BC = CA \) and \( XY = YZ = ZX, \frac{AB}{BC} = \frac{XY}{YZ} = \frac{ZC}{CA} \). Then △ABC ~ △XYZ by the SSS ~ Theorem. 7. 15 2 8. 5 3 9. 48 7 10. 55 3 11. 5 3 12. 36 13. 33 ft

Guided Problem Solving 7-3
1. no; N/A 2. yes; \( \overline{WT}, \overline{RS} \) 3. It is a trapezoid. 4. They are congruent. 5. They are parallel. 6. They are congruent. 7. \( \triangle RSZ \) and \( \triangle TWZ \). 8. AA ~ or Angle-Angle Similarity Postulate. 9. No; there is only one pair of congruent angles. 10. yes; parallelogram, rhombus, rectangle, and square

Practice 7-4
1. 16 2. 8 3. 3 4. 2 5. 10\( \sqrt{2} \) 6. 6\( \sqrt{5} \) 7. h 8. y 9. x 10. a 11. b 12. c 13. x = 6; y = 6\( \sqrt{3} \) 14. x = 8\( \sqrt{3} \); y = 4\( \sqrt{3} \) 15. \( \frac{a}{b} \) 16. x = 4\( \sqrt{2} \); y = \( \sqrt{55} \) 17. x = 3; y = \( \sqrt{6} \); z = \( \sqrt{2} \); 18. x = 8; y = 2\( \sqrt{2} \); z = 6\( \sqrt{2} \) 19. 25/1 in.

Guided Problem Solving 7-4
1. \( \triangle ABC, \triangle ACD, \triangle BCD \). 2. 1 3. 1; 1 4. 1 5. 1 6. 2; 1 7. 2 8. \( \sqrt{2} \) 9. \( \sqrt{2} \) 10. yes 11. no (This would require the Pythagorean Theorem.)

Practice 7-5
1. \( BE \) 2. \( EH \) 3. \( BC \) 4. \( JD \) 5. \( \frac{IG}{IJ} \) 6. \( BE \) 7. \( \frac{16}{5} \) 8. 4 9. 4 10. \( \frac{2a}{c} \) 11. \( x = \frac{2a}{c} \); y = 4 12. \( \frac{15}{4} \) 13. x = 6; y = 6 14. x = \( \frac{189}{5} \); y = \( \frac{198}{5} \) 15. 2 16. 4 17. 10

Guided Problem Solving 7-5
1. parallel 2. \( CE, BD \) 3. 6; 15 4. 90 5. yes 6. yes 7. The sides would not be parallel. 8. They are similar triangles.

7A: Graphic Organizer
1. Similarity 2. Answers may vary. Sample: ratios and proportions; similar polygons; proving triangles similar; and similarity in right triangles 3. Check students’ work.

7B: Reading Comprehension
1. \( \triangle DHB \sim \triangle ACB \) 2. AA similarity postulate 3. The triangles have two similar angles. 3. 1 : 2 4. 1 : 2 5. \( \frac{DB}{AB} = \frac{HB}{CB} \) 6. 250 ft 7. \( a \)

7C: Reading/Writing Math Symbols
1. no 2. no 3. yes 4. no 5. yes 6. no 7. yes, AAS 8. yes, Hypotenuse-Leg Theorem 9. yes, SAS or ASA or AAS 10. not possible

7D: Visual Vocabulary Practice

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7E: Vocabulary Check

Similarity ratio: The ratio of lengths of corresponding sides of similar polygons.

Cross-Product Property: The product of the extremes of a proportion is equal to the product of the means.

Ratio: A comparison of two quantities by division.

Golden rectangle: A rectangle that can be divided into a square and a rectangle that is similar to the original rectangle.

Scale: The ratio of any length in a scale drawing to the corresponding actual length.

7F: Vocabulary Review


Chapter 8

Practice 8-1

1. \(\sqrt{51}\) 2. \(4\sqrt{7}\) 3. \(2\sqrt{65}\) 4. \(\sqrt{7}\) 5. \(2\sqrt{21}\) 6. 18\(\sqrt{2}\) 7. 46 in. 8. 78 ft 9. 279 cm 10. 19 m 11. acute 12. right 13. obtuse 14. right 15. obtuse 16. acute

Guided Problem Solving 8-1

1. the sum of the lengths of the sides 2. Pythagorean Theorem 3. 7 cm 4. 4 cm \(\times\) 3 cm 5. \(c^2 = a^2 + b^2\) 6. 5 7. 12 cm 8. perimeter of rectangle = 14 cm; yes 9. Answers will vary; example: Draw a 4 cm \(\times\) 3 cm grid, copy the given figure, measure the lengths with a ruler, add them together. 10. 20 cm

Practice 8-2

1. \(x = 2; y = \sqrt{3}\) 2. \(a = 4.5; b = 4.5\sqrt{3}\) 3. \(c = \frac{10\sqrt{3}}{3}\), \(d = \frac{28\sqrt{3}}{3}\) 4. \(8\sqrt{2}\) 5. 14\(\sqrt{2}\) 6. \(\frac{28\sqrt{3}}{3}\) 7. 2 8. \(x = \sqrt{15}; y = \sqrt{15}\) 9. \(3\sqrt{2}\) 10. 42 cm 11. 5.9 in. 12. 10.4 ft, 12 ft 13. 170 in. 14. \(w = \frac{10\sqrt{3}}{3}; x = 5\) \(y = 5\sqrt{2}; z = \frac{5\sqrt{3}}{3}\) 15. \(a = 4; b = 3\) 16. \(p = 4\sqrt{3}\) \(q = 4\sqrt{3}; r = 8; s = 4\sqrt{6}\)

Guided Problem Solving 8-2

1. \(30^\circ-60^\circ-90^\circ\) triangle 2. \(f\) 3. \(h\) 4. \(\sqrt{3}\) 5. \(\frac{24}{\sqrt{3}}\) or \(8\sqrt{3}\) 6. 2 7. \(\frac{48}{\sqrt{3}}\) or \(16\sqrt{3}\) 8. 28 ft 9. 0.28 min 10. yes 11. 34 ft

Practice 8-3

1. \(\tan E = \frac{1}{2}; \tan F = \frac{1}{2}\) 2. \(\tan E = 1; \tan F = 1\) 3. \(\tan E = \frac{3}{2}; \tan F = \frac{5}{2}\) 4. 12. 45. 6. 31. 6. 14. 1. 7. 71 8. 2.3 9. 6.4 10. 78.7 11. 26.6 12. 71.6 13. 39 14. 72

Guided Problem Solving 8-3

1. \(\triangle ABC\)

Practice 8-4

1. \(\sin P = \frac{2\sqrt{10}}{5}, \cos P = \frac{3}{5}\) 2. \(\sin P = \frac{5}{7}, \cos P = \frac{12}{13}\) 3. \(\sin P = \frac{5}{13}, \cos P = \frac{12}{13}\) 4. \(\sin P = \frac{15}{17}, \cos P = \frac{8}{17}\) 5. \(\sin P = \frac{1}{2}, \cos P = \frac{1}{2}\) 6. \(\sin P = \frac{15}{17}, \cos P = \frac{8}{17}\) 7. 64. 8. 11.0 9. 7.0 10. 42 11. 7.8 12. 53 13. 6.6 14. 37 15. 11.0 16. 56 17. 11.5 18. 9.8

Guided Problem Solving 8-4

1. The sides are parallel. 2. sine 3. \(\sin 30^\circ = \frac{1}{2}\) 4. 3.0 5. yes 6. cosine 7. \(\cos x^\circ = \frac{3}{4}\) 8. 41 9. Answers may vary. Sample: \(\cos 60^\circ = \frac{1}{2} \sin 49^\circ = \frac{3}{4}\) 10. 5.2, 2.6

Practice 8-5

1a. angle of depression from the birds to the ship 1b. angle of elevation from the ship to the birds 1c. angle of depression from the ship to the submarine 1d. angle of elevation from the submarine to the ship 2a. angle of depression from the plane to the person 2b. angle of elevation from the person to the plane 2c. angle of depression from the person to the sailboat 2d. angle of elevation from the sailboat to the person 3. 116.6 ft 4. 84.8 ft 5. 46.7 ft 6. 31.2 yd 7. 127.8 m 8. 323.6 m

9a.

9b. 26 ft
Guided Problem Solving 8-5
1. $e = 1$, $d = 4$
2. congruent
3. $m\angle e = m\angle d$
4. $7x - 5 = 4(x + 7)$
5. $11$
6. $72$
7. $72$
8. yes
9. $44$, $44$

Practice 8-6
1. $(46.0, 46.0)$
2. $(-37.1, -19.7)$
3. $(89.2, -80.3)$
4. $38.6 \text{ mi/h}; 31.2^\circ \text{ north of east}$
5. $134.5 \text{ m}; 42.0^\circ \text{ south of west}$
6. $1.3 \text{ m/sec}; 38.7^\circ \text{ south of east}$
7. $55^\circ \text{ north of east}$
8. $20^\circ \text{ west of south}$
9. $33^\circ \text{ west of north}$

Guided Problem Solving 8-6
1. $\frac{x}{100} = \frac{y}{100}$
2. $100 \cos 30^\circ; 100 \sin 30^\circ$
3. $86.6$, $50$
4. $86.6$, $-50$
5. $(173.2, 0)$
6. $173$; due east
7. yes
8. $100$; due east

8A: Graphic Organizer
1. Right Triangles and Trigonometry
2. Answers may vary.
Sample: the Pythagorean Theorem; special right triangles; the tangent ratio; sine and cosine ratios; angles of elevation and depression; vectors
3. Check students’ work.

8B: Reading Comprehension
1. A
2. J
3. B
4. J
5. B
6. B
7. b

8C: Reading/Writing Math Symbols
1. $F$
2. $G$
3. $D$
4. A
5. C
6. B
7. H
8. E
9. $\sin^{-1} A = \frac{5}{12}$
10. $\triangle ABC \sim \triangle XYZ$
11. $m\angle A \approx 52^\circ$
12. $\tan Z = \frac{7}{24}$

8D: Visual Vocabulary Practice
1. $30^\circ$-$60^\circ$-$90^\circ$ triangle
2. inverse of tangent
3. congruent sides
4. tangent
5. Pythagorean Theorem
6. hypotenuse
7. $45^\circ$-$45^\circ$-$90^\circ$ triangle
8. Pythagorean triple
9. obtuse triangle

8E: Vocabulary Check
Obtuse triangle: A triangle with one angle whose measure is between $90$ and $180$.
Isosceles triangle: A triangle that has at least two congruent sides.
Hypotenuse: The side opposite the right angle in a right triangle.
Right triangle: A triangle that contains one right angle.
Pythagorean triple: A set of three nonzero whole numbers $a$, $b$, and $c$ that satisfy the equation $a^2 + b^2 = c^2$. 
Chapter 9

Practice 9-1
1. No; the triangles are not the same size. 2. Yes; the hexagons are the same shape and size. 3. Yes; the ovals are the same shape and size. 4a. $\angle C'$ and $\angle F'$ 4b. $CD$ and $C'D'$, $DE$ and $D'E'$, $EF$ and $E'F'$, $CF$ and $C'F'$
5a. $\angle M'$ and $\angle N'$ 5b. $MN$ and $M'N'$, $NO$ and $N'O'$, $MO$ and $M'O'$
6. (x, y) → (x + 4, y - 2) 7. (x, y) → (x - 2, y - 4) 8. (x, y) → (x - 3, y - 1) 9. (x, y) → (x + 4, y - 2) 10. (x, y) → (x - 5, y + 1) 11. (x, y) → (x + 2, y + 2) 12. $W'(-2, 2), X'(-1, 4), Y'(3, 3), Z'(2, 1)$
13. $J'(-5, 0), K'(-3, 4), L'(-3, -2)$ 14. $M'(3, -2)$, $N'(6, -2), P'(7, -7), Q'(4, -6)$ 15. (x, y) → (x + 4.2, y + 11.2) 16. (x, y) → (x + 13, y - 13) 17. (x, y) → (x, y + 18. (x, y) → (x + 3, y + 3) 19a. $P'(-3, -1)$ 19b. $P'(0, 8), N'(-5, 2), Q'(2, 3)$

Guided Problem Solving 9-1
1. the four vertices of a preimage and one of the vertices of the image 2. Graph the image and preimage 3. $C(4, 2)$ and $C'(0, 0)$ 4. $x = 4, y = 2, x + a = 0, y + b = 0$ 5. $a = -4; b = -2; (x, y) → (x - 4, y - 2)$ 6. $A'(-1, 4), B'(1, 3), D'(-2, 1)$

Practice 9-2
1. $(-3, -2)$ 2. $(-2, -3)$ 3. $(-1, -4)$ 4. $(4, -2)$ 5. $(4, -1)$ 6. $(3, -4)$ 7a.
Guided Problem Solving 9-2

1. a point at the origin, and two reflection lines
2. A reflection is an isometry in which a figure and its image have opposite orientations.
3. the image after two successive reflections

7. \((0, -6)\)
8. Yes. The \(x\)-coordinate remains 0 throughout.
9. \(O'(0, 0)\) and \(O'(0, 6)\)
10. 
11. 
12. 
13. \((-6, 4)\)
14. \((-8, 0)\)
15. \((0, -12)\)
Practice 9-3
1. \( I \) 2. \( I \) 3. \( I \) 4. \( \overline{GH} \) 5. \( \overline{ST} \)

3. \[ \begin{array}{c}
\text{y} \\
\text{x} \\
\text{O} \\
\text{A} \\
\text{U} \\
\text{S} \\
\text{P} \\
\text{T} \\
\text{T'} \\
\text{U'} \\
\text{S'} \\
\text{P'} \\
\end{array} \]

slope of \( \overline{OA} = \frac{2}{5} \)

4. \[ \begin{array}{c}
\text{y} \\
\text{x} \\
\text{O} \\
\text{B} \\
\text{C} \\
\text{D} \\
\end{array} \]

5. slope of \( \overline{OB} = -\frac{5}{12} \); slope of \( \overline{OC} = \frac{2}{5} \); slope of \( \overline{OD} = -\frac{5}{12} \)

the slopes of perpendicular line segments are negative reciprocals.

6. square 7. yes 8. \( B(2, 7), C(7, -2), D(-2, -7) \)

Guided Problem Solving 9-3
1. The coordinates of point \( A \), and three rotation transformations. It is assumed that the rotations are counterclockwise.
2. parallelogram, rhombus, square

Practice 9-4
1. The helmet has reflectional symmetry.
2. The teapot has reflectional symmetry.
3. The hat has both rotational and reflectional symmetry.
4. The hairbrush has reflectional symmetry.
5. \[ \begin{array}{c}
\text{L}\text{'} \\
\end{array} \]

7. This figure has no lines of symmetry.
8. line symmetry and 72° rotational symmetry

9. line symmetry
10. line symmetry and 90° rotational symmetry
11. line symmetry and 45° rotational symmetry
12. \(180°\) rotational symmetry
13. line symmetry
14. line symmetry and 45° rotational symmetry
15. \(180°\) rotational symmetry
16. line symmetry
17. line symmetry
18. \(180°\) rotational symmetry
19. line symmetry
20. \(180°\) rotational symmetry

Guided Problem Solving 9-4
1. the coordinates of one vertex of a figure that is symmetric about the \(y\)-axis
2. Line symmetry is the type of symmetry for which there is a reflection that maps a figure onto itself.
3. the coordinates of another vertex of the figure
4. images (and preimages)
5. reflection across the \(y\)-axis
6. \((-3, 4)\)
7. Yes
8. \((-6, 7)\)

Practice 9-5
1. \(L'(-2, -2), M'(-1, 0), N'(2, -1), O'(0, -1)\)
2. \(L'(-30, -30), M'(-15, 0), N'(30, -15), O'(0, -15)\)
3. \(L'(-12, -12), M'(-6, 0), N'(12, -6), O'(0, -6)\)
4. \(\frac{\frac{3}{2}}{\frac{1}{2}}\)
5. \(\frac{1}{2}\)
6. 2
7. yes
8. no
9. no
10. \(P(-12, -12), Q(-6, 0), R'(0, -6)\)
11. \(P'(-\frac{1}{2}, \frac{1}{4}), Q'(\frac{1}{2}, -\frac{1}{2}), R'(1, 2)\)
12. \(P'(-21, 6), Q'(3, 24), R'(-6, 6)\)
13. \(P'(-12, -12), Q'(-6, 0), R'(0, -6)\)
14. \(P'(-\frac{1}{2}, \frac{1}{4}), Q'(\frac{1}{2}, -\frac{1}{2}), R'(1, 2)\)
15. \(P'(-21, 6), Q'(3, 24), R'(-6, 6)\)
16. \(P'(-2, 1), Q'(-1, 0), R'(0, 1)\)

Guided Problem Solving 9-5
1. A description of a square projected onto a screen by an overhead projector, including the square’s area and the scale factor in relation to the square on the transparency.
2. The scale factor of a dilation is the number that describes the size change from an original figure to its image.
3. The area of the square on the transparency 4. smaller; The scale factor \(16 > 1\), so the dilation is an enlargement.
5. \(\frac{1}{16}, \frac{1}{16}\)
6. \(\frac{1}{256}\)\(^{\text{th}}\) of the square on the transparency has \(\frac{1}{16} \times \frac{1}{16} = \frac{1}{256}\) times the area. 7. \(\frac{3}{256}\) ft\(^2\)
8. The shape does not matter. Regardless of the shape, the figure is being enlarged by a factor of 16 in two directions, so that the screen image has \(16 \times 16 = 256\) as large an area as the figure on the transparency. 9. 2520 ft\(^2\)

**Practice 9-6**

1. I. D II. C III. B IV. A  
2. I. B II. A III. C IV. D

3. 

\[ \ell \parallel m \]

4. 

\[ \ell \parallel m \]

5. 

\[ \ell \parallel m \]

6. 

\[ \ell \parallel m \]

7. 

\[ \ell \parallel m \]

8. 

\[ \ell \parallel m \]

9. 

\[ \ell \parallel m \]

10. 

\[ \ell \parallel m \]

11. 

\[ \ell \parallel m \]

12. 

\[ \ell \parallel m \]

13. 

\[ \ell \parallel m \]

14. reflection  
15. rotation  
16. glide reflection  
17. translation
Guided Problem Solving 9-6
1. assorted triangles and a set of coordinate axes
2. a transformation
3. the transformation that maps one triangle onto another
4. They are congruent, which confirms that they could be related by one of the isometries listed.
   Corresponding vertices: E and P, D and Q, C and M.
5. counterclockwise; clockwise. The triangles have opposite orientations, so the isometry must be a reflection or glide reflection.
6. No; there is no reflection line that will work.
7. a glide reflection consisting of a translation \((x, y) \rightarrow (x + 11, y)\) followed by a reflection across line \(y = 0\) (the \(x\)-axis)
8. Yes, both are horizontal.
9. 180° rotation with center \((-2\frac{1}{2}, 0)\)

Practice 9-7
1. 
   translational symmetry
2. 
   line symmetry, rotational symmetry, translational symmetry, glide reflectional symmetry
3. 
   translational symmetry
4. 
   line symmetry, rotational symmetry, translational symmetry, glide reflectional symmetry
5. Possible answer:
6. Check students’ work.
7. Answers may vary. Sample:

Guided Problem Solving 9-7
1. two different-sized equilateral triangles and a trapezoid
2. A tessellation is a repeating pattern of figures that completely covers a plane, without gaps or overlaps.
3. Theorem 9-7 says that every quadrilateral tessellates. If the three polygons can be joined to form a quadrilateral (using each polygon at least once), that quadrilateral can be the basis of a tessellation.
4. 
5. Possible answer:
6. Check students’ work.
7. Answers may vary. Sample:

9A: Graphic Organizer
1. Transformations
2. Answers may vary. Sample: reflections; translations; compositions of reflections; and symmetry
3. Check students’ work.
9B: Reading Comprehension
1. D  2. E, R, 5, and 20  3. The center of the county is the center of the square mile bounded by roads 12, 13, K and L.  4. C  5. 10 miles  6. a

9C: Reading/Writing Math Symbols
1. reflection; opposite  2. rotation; same  3. glide reflection; opposite  4. translation; same  5. rotation; same  6. translation; same

9D: Visual Vocabulary Practice/High-Use Academic Words
1. theorem  2. symbols  3. reduce  4. application  5. simplify  6. explain  7. characteristic  8. table  9. investigate

9E: Vocabulary Check
Transformation: A change in the position, size, or shape of a geometric figure.
Preimage: The given figure before a transformation is applied.
Image: The resulting figure after a transformation is applied.
Isometry: A transformation in which an original figure and its image are congruent.
Composition: A transformation in which a second transformation is performed on the image of the first transformation.

9F: Vocabulary Review Puzzle

Chapter 10

Practice 10-1
1. 8  2. 8  3. 60  4. 240  5. 2.635  6. 1.92  7. 0.8125  8. 9  9. 1500  10. 8.75  11. 3.44  12. 7  13. 110  14. 30  15. 16  16. 119  17. 12  18. 48  19. 40  20. 56

Guided Problem Solving 10-1
1. \( \frac{1}{2}bh \)  2. Check students' work; \( B(7, 7) \)  3. a right angle  4. 7  5. \( \overrightarrow{OA} \)  6. 7  7. 24.5  8. a square  9. 49  10. \( \frac{1}{2} \)  11. 24.5  12. \( \frac{1}{2}b^2 \)
Practice 10-2
1. 48 cm² 2. 784 in.² 3. 11.4 ft² 4. 90 m² 5. 2400 in.²
6. 374 ft² 7. 160 cm² 8. 176.25 in.² 9. 54 ft² 10. 42.5 square units 11. 96√3 square units 12. 36√3 square units 13. 45 cm² 14. 226.2 in.² 15. 49,500 m²

Guided Problem Solving 10-2
1. \( \frac{1}{2}h(b_1 + b_2) \) 2. 3:4:2 3. 9 4. no 5. yes 6. 3

Practice 10-3
1. \( x = 7; y = 3.5\sqrt{3} \) 2. \( a = 60; c = \frac{8\sqrt{3}}{3}; d = \frac{8\sqrt{3}}{3} \)
3. \( p = 4\sqrt{3}; q = 8; x = 30 \) 4. \( m \angle 1 = 45; m \angle 2 = 45; m \angle 3 = 90; m \angle 4 = 90 \) 5. \( m \angle 5 = 60; m \angle 6 = 30; m \angle 7 = 120 \) 6. \( m \angle 8 = 60; m \angle 9 = 30; m \angle 10 = 60 \) 7. 12√3 8. 36.75√3 9. 75√3 10. 120 in.²
11. 137 in.² 12. 97 in.²

Guided Problem Solving 10-3
1. 10 2. The angles are all congruent and the sides are all congruent. 3. 360 4. 36 5. bisects 6. 2 7. 18 8. 180 9. 72 10. 180 11. 30; 15; 75

Practice 10-4
5. 9 : 5 6. 1 : 6 7. 8 : 3 8. \( \frac{576}{27} \) in.² 9. \( \frac{2700}{27} \) ft²
10. \( \frac{1152}{27} \) in.² 11. 2 : 3 12. 1 : 2 13. 2 : 5 14. 108 ft²

Guided Problem Solving 10-4
1. scale 2. Check students’ work. 3. Answers will vary. Let \( a \) be the side opposite the 30° angle, \( b \) be the side opposite the 60° angle, and \( c \) be the side corresponding to 200 yd. Then
\[ a \approx 20 \text{ mm}, \ b \approx 30 \text{ mm} \text{ and } c \approx 40 \text{ mm.} \]
4. \( \frac{200}{40} \text{ yd} = 5 \text{ yd} \)
5. \( a \approx 90 \text{ mm}, \ b \approx 16 \text{ mm} \text{ and } c \approx 320 \text{ mm}² \)
6. 450 7. 8000 8. yes 9. 11. 6000

Practice 10-5
1. 174.8 cm² 2. 578 ft² 3. 1250.5 mm² 4. 192.6 m²
5. 1131.4 in.² 6. 419.2 cm² 7. 324.9 in.² 8. 162 cm²
9. 37.6 m² 10. 30.1 m² 11. 55.4 km² 12. 357.6 in.²
13. 9.7 mm² 14. 384.0 in.² 15. 54.5 cm² 16. 9.1 ft²
17. 80.9 m² 18. 83.1 m² 19. 65.0 ft² 20. 119.3 ft²
21. 8 m² 22. 55.4 ft²

Guided Problem Solving 10-5
1. subtract 2. 50 3. \( \frac{1}{2}ap \) 4. 30 5. 4 6. \( \frac{4}{\tan 30°} \) 7. 166
8. 116 9. yes 10. 15⅓

Practice 10-6
1. 32π 2. 16π 3. 7.8π 4. Samples: \( \overline{DF}A, \overline{ABF} \)
5. Samples: \( \overline{FE}, \overline{FD} \) 6. Samples: \( \overline{EB}, \overline{EDA} \) 7. Samples: \( \overline{FE}, \overline{ED}, \overline{ACB}, \overline{FCD} \)
8. 103 9. 31 10. 51 11. 54 12. 54
20. 130 21. 6π in. 22. 16π cm 23. \( \frac{9}{2}π \) m

Guided Problem Solving 10-6
1. 11; 2 2. 11; 3 3. circumference 4. 31 5. 26 6. 41
7. 105 8. 102 9. 3027.15

Practice 10-7
1. \( 49\pi \) 2. 21.2 3. \( \frac{49}{6} \pi \) 4. \( \frac{49}{6} \pi - 21.2 \) 5. \( \frac{1}{10} \pi \) or \( \frac{\pi}{10} \) 6. \( \frac{1}{8} \)
7. \( \frac{1}{10} \pi \) or \( \frac{\pi}{10} \) 8. \( \frac{3}{10} \pi - \frac{1}{8} \)
9. 4π 10. \( 18 \pi \) 11. \( \frac{9}{5} \pi \) 12. \( \frac{3}{2} \pi \)
13. \( \frac{15}{2} \pi \) 14. 6π 15. \( \frac{93}{8} \pi \) 16. \( \frac{22}{3} \pi \) 17. \( \frac{π}{2} \) 18. 3
19. 2 20. 10

Guided Problem Solving 10-7
1. \( \frac{m \angle A}{360} \cdot \pi r^2 \) 2. 45 3. 25 4. 30 5. 27 6. a piece from the outer ring 7. yes 8. a piece from the top tier

Practice 10-8
1. 8.7% 2. 10.9% 3. 15.3% 4. 3 \( \frac{3}{10} \) 5. 7 \( \frac{7}{10} \) 6. \( \frac{2}{5} \) 7. \( \frac{1}{10} \)
8. 1 9. \( \frac{1}{5} \) 10. 22.2% 11. 4.0% 12. 37.5% 13. 20% 14. 50% 15. 40% 16. 33\% 17. 50% 18. 35% 19. 20% 20. 30% 21. 40% 22. 50%

Guided Problem Solving 10-8
1. outcomes 2. event 3. \( \overline{AF} \) 4. \( \overline{FG} \) 5. \( \overline{EG} \) 6. \( \overline{AE} \)
7. Check students’ work. 7. yes 8. \( \frac{3}{5} \) or about 67% 8. \( \frac{2}{3} \) or about 67%; yes 9. \( \frac{3}{5} \) or 60%

10A: Graphic Organizer
1. Area 2. Answers may vary. Sample: areas of parallelograms and triangles; areas of trapezoids, rhombuses, and kites; areas of regular polygons; perimeters and areas of similar figures; trigonometry and area; circles and arcs; areas of circles and sectors; geometric probability 3. Check students’ work.

10B: Reading Comprehension
11. \( a^2 + b^2 = c^2 \) 12. \( m \overline{AC} = 105° \) 13. \( \overline{MN} \parallel \overline{RS} \)
14. \( \sqrt{225} = 15 \) 15. \( \triangle JLK \) 16. \( \overline{XY} \parallel \overline{PQ} \)
17. \( \square \overline{ABCD} \) 18. \( 4^2 = 16 \) 19. b

10C: Reading/Writing Math Symbols
1. 2860 ft² 2. 33\% ft² 3. 86 bundles 4. 28\% squares 5. $9.00
6. $27.00 7. 45 minutes 8. 21.5 hours 9. $645 10. $1419
10D: Visual Vocabulary Practice
1. area of a triangle  2. radius  3. altitude of a parallelogram  4. area of a trapezoid  5. area of a kite or rhombus  6. similarity ratio  7. apothem  8. perimeter (of an octagon)  9. height of a triangle

10E: Vocabulary Check
Semicircle: Half a circle.  
Adjacent arcs: Arrows on the same circle with exactly one point in common.  
Apothem: The distance from the center to the side of a regular polygon.  
Circumference: The distance around a circle.  
Concentric circles: Circles that lie on the same plane and have the same center.

10F: Vocabulary Review Puzzle

Chapter 11

Practice 11-1

Guided Problem Solving 11-1
1. a two-dimensional network  2. Verify Euler’s Formula for the network shown.  3.  4.  5. 4 + 6 = 9 + 1; This is true.  6. F becomes 3, V stays at 6, E becomes 8. Euler’s Formula becomes 3 + 6 = 8 + 1, which is still true.  7. 4 + 4 = 6 + 2; This is true.

Practice 11-2
1. 62.8 cm²  2. 552.9 ft²  3. 226.2 cm²  4. 113.1 m²  5. 44.0 ft²  6. 84.8 cm²  7a. 320 m²  7b. 440 m²  8a. 576 in.²  8b. 684 in.²  9a. 216 mm²  9b. 264 mm²  10a. 48 cm²  10b. 60 cm²  11a. 1500 m²  11b. 1800 m²  12a. 2000 ft²  12b. 2120 ft²  13. 8π m²  14. 94.5π cm²  15. 290π ft²

Guided Problem Solving 11-2
1. a picture of a box, with dimensions  2. the amount of cardboard the box is made of, in square inches  3. A front and a back, each 4 in. × 7 ½ in.; a top and a bottom, each 4 in. × 1 in.; and one narrow side, 1 in. × 7 ½ in.  4. Front and back; each 30 in.²; top and bottom: each 4 in.²; one narrow side: 7 ½ in.²  5. 75½ in.²  6. 75½ in.² is about half a square foot (1 ft² = 144 in.²), which is reasonable. The answer is an approximation because the solution did not consider the overlap of cardboard at the glued joints, nor does it factor in the trapezoid-shaped cutout in the front and back sides.  7. 3½ in.²

Practice 11-3
1. 64 m²  2. 620 cm²  3. 188 ft²  4. 36π cm²  5. 800π m²  6. 300π in.²  7. 109.6 m²  8. 160.0 cm²  9. 387.7 ft²  10. 147.6 m²  11. 118.4 cm²  12. 668.2 ft²

Guided Problem Solving 11-3
1. a solid consisting of a cylinder and a cone  2. the surface area of the solid  3. The top of the solid is a circular base of the cylinder, A₁ = πr². The side of the solid is the lateral area of the cylinder, A₂ = 2πrh. The bottom of the solid is the lateral area of the cone, A₃ = πrl.  4. r = 5 ft; h = 6 ft; ℓ = √(5² + 12²) = 13 ft  5. Top: A₁ = 78.5 ft²; side: A₂ = 188.5 ft²; bottom: 204.2 ft²  6. 471 ft²  7. About 0.43, or a little less than half. This seems reasonable.  8. 628 ft²

Practice 11-4
1. 1131.0 m³  2. 94,247.8 cm³  3. 7.1 in.³  4. 1781.3 in.³  5. 785.4 cm³  6. 603.2 cm³  7. 120 in.³  8. 35 ft³  9. 42 m³  10. 1728 ft³  11. 784 cm³  12. 265 in.³  13. 76 ft³  14. 943 in.³  15. 1152 m³

Guided Problem Solving 11-4
1. dimensions for a layer of topsoil, and two options for buying the soil  2. which purchasing option is cheaper  3. $\frac{1}{3}$ ft  4. 1400 ft³  5. 467 bags; $1167.50  6. 1400 ft³ ≈ 52 yd³  7. $1164 (or slightly less if the topsoil volume is not rounded up to the next whole number)  8. buying in bulk  9. The two costs are quite close. It makes sense that buying in bulk would be slightly cheaper for a good-sized backyard.  10. Buying topsoil by the bag would be less expensive because it would cost $335 and in bulk would cost at least $345.93.

Practice 11-5
1. 34.992 cm³  2. 400 in.³  3. 10,240 in.³  4. 4800 yd³  5. 150 m³  6. 48 cm³  7. 628.3 cm³  8. 314.2 in.³  9. 4955.3 m³  10. 1415.8 in.³  11. 1005.3 m³  12. 18.8 ft³  13. 20  14. 2  15. 6
Guided Problem Solving 11-5
1. a description of a plumb bob, with dimensions 2. the plumb bob’s volume 3. equilateral; 2 cm 4. \( a = \sqrt{3} \text{ cm}; B = 6\sqrt{3} \text{ cm}^2 \) 5. Prism: \( V_1 = 36\sqrt{3} \text{ cm}^3 \); pyramid: \( V_2 = 6\sqrt{3} \text{ cm}^3 \); plum bob: \( V \approx 73 \text{ cm}^3 \) 6. \( \frac{1}{7} \); this seems reasonable 7. 42 cm³ 9. distance equals the square root of the quantity of the sum of the square of the difference of the \( x \)-coordinates, squared, plus the difference of the \( y \)-coordinates, squared; the distance between two points \((x_1, y_1)\) and \((x_2, y_2)\) 10. area = one-half the height times the sum of base one plus base two; area of a trapezoid 11. a

Practice 11-6
1. 2463.0 in² 2. 6,157,521.6 m² 3. 12.6 cm² 4. 1256.6 cm² 5. 50.3 ft² 6. 153.9 m² 7. 1436.8 m³ 8. 268,082.6 cm³ 9. 7238.2 m² 10. 14.1 cm³ 11. 2,572,354.6 cm³ 12. 904,810.7 yd³

Guided Problem Solving 11-6
1. the diameter of a hailstone, along with its density compared to normal ice 2. the hailstone’s weight 3. \( V = \frac{4}{3}\pi r^3 \) 4. 2.8 in. 5. 91.95 in.³ 6. 1.7 lb 7. The density of normal ice. This was given for comparison purposes only. 8. 0.01 oz

Practice 11-7
1. 7 : 9 2. 5 : 8 3. yes; 4 : 7 4. yes; 4 : 3 5. not similar 6. yes; 4 : 5 7. 125 cm³ 8. 256 in.³ 9. 432 ft³ 10. 25 ft² 11. 90 m² 12. 600 cm² 13. 54,000 lb 14. 16 lb

Guided Problem Solving 11-7
1. two different sizes for an image on a balloon, and the volume of air in the balloon for one of those sizes 2. the volume of air for the other size 3. The two sizes of the clown face have a ratio of 4 : 8, or 1 : 2 4. \( 1 : 8 \) 5. \( 108 \text{ in.}^3 \times 8 = 864 \text{ in.}^3 \) 6. Yes, the two clown face heights are lengths, measured in inches (not in.² or in.³). 7. 351 cm³

11A: Graphic Organizer
1. Surface Area and Volume 2. Answers may vary. Sample: space figures and nets; space figures and drawing; surface areas; and volumes. 3. Check students’ work.

11B: Reading Comprehension
1. Area = \( \frac{1}{2} \) base times height; area of a triangle 2. slope = the difference of the \( y \)-coordinates divided by the difference of the \( x \)-coordinates; slope of the line between 2 points 3. \( a \) squared plus \( b \) squared equals \( c \) squared, the Pythagorean Theorem; the lengths of the sides of a right triangle 4. area = \( \pi \) times radius squared; area of a circle 5. volume = \( \pi \) times radius squared times height; volume of a cylinder 6. the sum of the \( x \)-coordinates of two points divided by 2, and the sum of the \( y \)-coordinates of two points divided by 2; midpoint of a line segment with endpoints \((x_1, y_1)\) and \((x_2, y_2)\) 7. circumference = \( 2 \times \pi \) times radius; circumference of a circle 8. volume = one-third \( \pi \) times radius squared times height; volume of a cone

11C: Reading/Writing Math Symbols
1. \( 527.8 \text{ in}^2 \) 2. \( 136 \text{ ft}^2 \)

11D: Visual Vocabulary Practice

11E: Vocabulary Check
Face: A surface of a polyhedron. Edge: An intersection of two faces of a polyhedron. Altitude: For a prism or cylinder, a perpendicular segment that joins the planes of the bases. Surface area: For a prism, cylinder, pyramid, or cone, the sum of the lateral area and the areas of the bases. Volume: A measure of the space a figure occupies.

11F: Vocabulary Review

Chapter 12

Practice 12-1
1. 32 2. 50 3. 72 4. 15 5. \( \sqrt{91} \) 6. 6 7. \( \sqrt{634} \) 8. \( \sqrt{901} \) 9. \( 4\sqrt{3} \) 10. circumscribed 11. inscribed 12. circumscribed 13. 24 cm 14. 28 in. 15. 52 ft

Guided Problem Solving 12-1
1. The area of one is twice the area of the other. 2. inscribed; circumscribed

3. \( 5 \cdot 2r = 10r \) 6. \( 4r^2 \) 7. \( \sqrt{2}r \) 8. \( 2r^2 \) 9. \( 4r^2 = 2(2r^2) \): One area is double the other area. 10. yes 11. The same: The area of one circle is twice the area of the other circle.
Practice 12-2
1. \( r = 13; m\overline{AB} \approx 134.8 \) 2. \( r = 3\sqrt{5}; m\overline{AB} \approx 53.1 \)
3. \( r = \frac{\sqrt{41}}{2}; m\overline{AB} \approx 102.7 \) 4. 3 5. 4.5
6. 3 7. 20.8 8. 13.7 9. 8.5 10. \( \angle O \equiv \angle T; \overline{PR} \equiv \overline{SU} \) 11. \( \angle A \equiv \angle B; \overline{FC} \equiv \overline{KL} \) 12. Construct radii \( \overline{OD} \) and \( \overline{OB}. \overline{FD} = \overline{EB} \) because a diameter perpendicular to a chord bisects it, and chords \( \overline{CD} \) and \( \overline{AB} \) are congruent (given). Then, by HL, \( \triangle OEB \equiv \triangle ODF \), and by CPCTC, \( \overline{OE} = \overline{OF} \). 13. Because congruent arcs have congruent chords, \( \overline{AB} = \overline{BC} = \overline{CA} \). Then, because an equilateral triangle is equiangular, \( m\angle ABC = m\angle BCA = m\angle CAB \).

Guided Problem Solving 12-2
1. perpendicular 2. bisects 3. 4; 3 4. right 5. radius 6. Pythagorean Theorem 7. 5 8. yes 9. 8; 4

Practice 12-3
1. \( \angle A \) and \( \angle D; \angle B \) and \( \angle C \) 2. \( \angle ADB \) and \( \angle CAD \) 3. \( \angle ADB \) and \( \angle CAD \) 4. 55 5. 5 6. \( x = 90; y = 70 \) 7. 180 8. 70 9. \( x = 120; y = 60; z = 60 \) 10. \( x = 120; y = 100; z = 140 \) 11. \( x = 63; y = 63; z = 54 \) 12. \( x = 50; y = 80; z = 80 \)
13a. \( m\overline{AE} = 170 \) 13b. \( m\angle C = 85 \) 13c. \( m\angle B = 10 \)
13d. \( m\angle D = 85 \) 14a. \( m\angle A = 90 \) 14b. \( m\angle B = 80 \) 14c. \( m\angle C = 90 \) 14d. \( m\angle D = 100 \)

Guided Problem Solving 12-3
5. Check students’ work. 6. Check students’ work.
7. Check students’ work. 8. 90 9. Check students’ work.

Practice 12-4
1. 87 2. 35 3. 45 4. 120 5. \( x = 45; y = 50; \) 6. 90; \( y = 70 \) 7. \( x = 58; y = 58; z = 63 \) 8. \( x = 30; y = 66 \)
9. \( x = 30; y = 30; z = 120 \) 10. \( x = 16; y = 52 \)
11. \( x = 138; y = 111; z = 111 \) 12. \( x = 30; y = 60 \)
13. 10 14. 14.8 15. 4.7 16. 4 17. 3.2 18. 6

Guided Problem Solving 12-4
1. tangent 2. 360 3. \( \frac{1}{2} \) 4. 85 5. 2 6. 180 7. 95 8. 104; 86 75 9. 360 10. yes; The sum of the measures of the arcs should be 360. 11. Opposite angles are not supplementary.

Practice 12-5
1. \( C(0, 0) ; r = 6 \) 2. \( C(2, 7); r = 7 \) 3. \( C(-1, -6); r = 4 \)
4. \( C(-3, 11) ; r = 2\sqrt{3} \) 5. \( x^2 + y^2 = 49 \)
6. \( x^2 + y^2 = 64 \) 7. \( x^2 + y^2 = 4 \)
8. \( x^2 + y^2 = \frac{1}{4} \) 9. \( x^2 + y^2 = 2 \)
10. \( x^2 + y^2 = 5 \) 11. \( x^2 + y^2 = 4 \)
12. \( x^2 + y^2 = 25 \) 13. \( x^2 + y^2 = 9 \)
14. \( x^2 + y^2 = 61 \) 15. \( x^2 + y^2 = 80 \)
16. \( x^2 + y^2 = 4 \) 17. \( x^2 + y^2 = 16 \)
18. \( x^2 + y^2 = 25 \) 19. \( x^2 + y^2 = 9 \)
20. \( x^2 + y^2 = 61 \) 21. \( x^2 + y^2 = 80 \)
22. \( x^2 + y^2 = 4 \) 23. \( x^2 + y^2 = 16 \)
Guided Problem Solving 12-5
1. (2, 2); 5  2. 4 3  3. They are perpendicular.  4. \(-\frac{3}{4}\)  5. \(\frac{39}{4}\)
6. \(y = -\frac{3}{4}x + \frac{39}{4}\)  7. 8. \(y = -\frac{3}{4}x + \frac{39}{4}\); yes
9. \(x\)-intercept: \(a = \frac{25}{3}\); \(y\)-intercept: \(b = \frac{25}{4}\)

Practice 12-6
1. 2. 3. They are perpendicular.
4. 5.
6. 7. 13 8. ; yes
9. \(x\)-intercept: \(a = \frac{25}{3}\); \(y\)-intercept: \(b = \frac{25}{4}\)

Guided Problem Solving 12-6
1. a line  2. Check students’ work.  3. They are perpendicular.  4. \(-\frac{1}{2}\)  5. 2  6. the midpoint of \(PQ\)  7. (3, 2)
8. \(-4\)  9. \(y = 2x - 4\)  10. (2, 0); yes  11. \(y = \frac{1}{2}x + 2\)

12A: Graphic Organizer
1. Circles  2. Answers may vary. Sample: tangent lines; chords
   and arcs; inscribed angles; and angle measures and segment
   lengths  3. Check students’ work.

12B: Reading Comprehension
Answers may vary. Sample answers:
1. \(AD, CE\)  2. \(BF, CD, AD\)  3. \(HD\)  4. \(AH\)
5. \(AF, AB, BC\)  6. \(CDF, DAB, DAC\)  7. \(AC, ED\)
8. \(\angle AOC\)  9. \(\angle DCE\)  10. \(D\)  11. \(\angle DAH\)
12. \(\angle AGB\)  13. \(\angle AHC\)  14. \(\angle DCE\)  15. \(\angle A\)  16. a

12C: Reading/Writing Math Symbols
1. \(\overleftrightarrow{BC} \perp \overleftrightarrow{DE}\)  2. \(\overleftrightarrow{EF}\) is tangent to \(\bigcirc G\).
3. \(WX \cong ZY\)  4. \(\overline{AB} \perp \overline{XY}\)  5. \(HM = 4\)

12D: Visual Vocabulary Practice
1. tangent to a circle  2. standard form of an equation of a
   circle  3. secant  4. chord  5. circumscribed about
6. intercepted arc  7. inscribed in  8. inscribed angle
9. locus

12E: Vocabulary Check
**Inscribed in:** A circle inside a polygon with the sides of the
polygon tangent to the circle.
**Chord:** A segment whose endpoints are on a circle.
**Point of tangency:** The single point of intersection of a
tangent line and a circle.
**Tangent to a circle:** A line, segment, or ray in the plane of a
circle that intersects the circle in exactly one point.
**Circumscribed about:** Circle outside a polygon with the
vertices of the polygon on the circle.

12F: Vocabulary Review
Lesson 1-1 Patterns and Inductive Reasoning

Vocabulary

Inductive reasoning is reasoning based on patterns you observe.

A conjecture is a conclusion you reach using inductive reasoning.

A counterexample is an example for which the conjecture is incorrect.

Examples

4. Finding a Counterexample

Find a counterexample for the conjecture:

A conjecture is that the product of 5 and any positive integer ends in 5.

Using Inductive Reasoning

The sum of the first two cubes equals the square of the sum of the first two counting numbers. The sum of the first three cubes equals the square of the sum of the first three counting numbers. This pattern continues for the fourth and fifth rows. So a conjecture might be that the sum of the cubes of the first 25 counting numbers equals the square of the sum of the first 25 counting numbers. Is it true? Is $1^3 + 2^3 + 3^3 + \ldots + 25^3 = (1 + 2 + 3 + \ldots + 25)^2$?

Therefore, this statement is false. A possible counterexample is $3^3 + 4^3$. The sum of the squares of two consecutive numbers is $(x^2 + (x + 1)^2)$. Since $4^2 + 5^2 = 16 + 25 = 41$, the conjecture is false. Since $4^2 + 5^2$ are not the same, this conjecture is incorrect.

The conjecture is that 41 is the square of the next consecutive number.

Begin with 4. The next consecutive number is 4 + 1 = 5.

The sum of the squares of these two consecutive numbers is $4^2 + 5^2 = 16 + 25 = 41$.

So, since $4^2 + 5^2$ are not the same, this conjecture is incorrect.

Quick Check

a. Find the next two terms in each sequence:
   1. Monday, Tuesday, Wednesday, , ,...
   2. 2, 4, 8, 16, 32, ..., , , ,...

b. Write the data in a table. Find a pattern.

<table>
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<th>Year</th>
<th>Price</th>
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<tr>
<td>2000</td>
<td>$8.00</td>
</tr>
<tr>
<td>2001</td>
<td>$9.00</td>
</tr>
<tr>
<td>2002</td>
<td>$11.00</td>
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</tbody>
</table>

Each year the price increased by $1.00. A possible conjecture is that the price in 2003 will increase by $1.00. If so, the price in 2003 would be $11.00 + $1.00 = $12.00.

A net is a two-dimensional pattern you can fold to form a three-dimensional figure.

A foundation drawing shows the shape of a three-dimensional figure. A top view, front view, and right-side view are the top, front, and right-side views of a three-dimensional figure.

A translation drawing shows the base of a structure and the height of each part.

A counterexample is any example for which the conjecture is incorrect.

A conjecture is an example for which the conjecture is correct.
Orthographic Drawing
Make an orthographic drawing of the isometric drawing at right.
Orthographic drawings flatten the depth of a figure. An orthographic drawing shows three views. Because no edge of the isometric drawing is hidden in the top, front, and right views, all lines are solid.

Foundation Drawing
Make a foundation drawing for the isometric drawing.
To make a foundation drawing, use the top view of the orthographic drawing.
Because the top view is formed from squares, show squares in the foundation drawing.
Identify the square that represents the tallest part. Write the number 2 in the back square to indicate that the back section is cubes high.
Write the number 1 in each of the two front squares to indicate that each front section is cube high.

Identifying Solids from Nets
Is the pattern a net for a cube? If so, name two letters that will be opposite faces.

Critical Thinking Which drawing did you use to answer part (a), the foundation drawing or the isometric drawing? Explain.

Drawing a Net
Draw a net for the figure with a square base and four isosceles triangle faces.
Label the net with its dimensions.

Quick Check
1. Make an isometric drawing of the cube structure below.

Answers may vary. Sample: the foundation drawing; you can just add the three numbers.

Identifying Solids from Nets
Is the pattern a net for a cube? If so, name two letters that will be opposite faces.

Critical Thinking Which drawing did you use to answer part (a), the foundation drawing or the isometric drawing? Explain.

Drawing a Net
Draw a net for the figure with a square base and four isosceles triangle faces.
Label the net with its dimensions.

Quick Check
1. Make an isometric drawing of the cube structure below.

Answers may vary. Sample: the foundation drawing; you can just add the three numbers.
Lesson 1-3

Points, Lines, and Planes

Vocabulary and Key Concepts

Postulate 5-1
Through any two points there is exactly one line.

Postulate 5-2
If two lines intersect, then they intersect in exactly one point.

Postulate 5-3
If two planes intersect, then they intersect in exactly one line.

Postulate 5-4
Through any three noncollinear points there is exactly one plane.

A point is a location. Space is the set of all points. A line is a set of points that extend in two opposite directions without end.

Collinear points are points that lie in the same line.

A line is named using any two points on the line such as \( \overrightarrow{AB} \) or \( \overleftrightarrow{AB} \).

A plane is a flat surface that has no thickness.

Examples

1. Identifying Collinear Points. In the figure at right, name three points that are collinear and three points that are noncollinear.

2. Naming a Plane. Name the plane shown in two different ways.

3. Identifying Parallel Lines. Parallel planes are planes that do not intersect. Opposite rays are two collinear rays with the same endpoint.

4. Identifying Perpendicular Lines. Perpendicular lines are lines that intersect to form a right angle.

Name: ____________________ Class: ____________________ Date: ________________
Lesson 1-5 Measuring Segments

Vocabulary and Key Concepts

Postulate 1-5: Ruler Postulate
The points on a line can be put into one-to-one correspondence with the real numbers so that the distance between any two points is the absolute value of the difference of the corresponding number.

Postulate 1-6: Segment Addition Postulate
If three points, A, B, and C, are collinear and B is between A and C, then AB + BC = AC.

Examples

Naming Segments and Rays
Name the segments and rays in the figure.

- \( AB \) and \( BC \)
- \( \overrightarrow{AB} \) and \( \overrightarrow{BC} \)
- \( \overrightarrow{BA} \) and \( \overrightarrow{CB} \)

Quick Check

1. Critical Thinking

Name all segments that are parallel to \( \overrightarrow{AB} \). Name all segments that are skew to \( \overrightarrow{AB} \).

- \( \overrightarrow{AD} \) and \( \overrightarrow{AE} \)
- \( \overrightarrow{BC} \) and \( \overrightarrow{CD} \)

2. Comparing Segment Lengths

Find \( AB \) and \( BC \). Are \( AB \) and \( BC \) congruent?

- \( AB = 8 \) inches
- \( BC = 3 \) inches
- Yes, they are congruent.

Using the Segment Addition Postulate

If \( AB = 25 \) and \( AN = 12 \), find the values of \( x \). Then find \( AV \) and \( NB \).

- \( AN = 12 \)
- \( AB = 25 \)
- \( AV = 12 \)
- \( NB = 13 \)

Check Check

Using the figure in Example 1, \( \overrightarrow{EF} \) and \( \overrightarrow{FG} \) form a line. Are they opposite rays? Explain.

- No, they do not have the same endpoint.

Lesson Objectives

1. Critical Thinking
2. Comparing Segment Lengths
3. Using the Segment Addition Postulate

Local Standards: ____________________________________

Topic: Measuring Physical Attributes

Measurement

NAEP 2005 Strand: ____________________________________

<table>
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<tr>
<td>1-6</td>
<td>Segment Addition Postulate</td>
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<tr>
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Geometry: All-In-One Answers Version A (continued)

Lesson 1-6 Measuring Angles

**Definitions:**
- **Midpoint:** The point on a line segment that divides it into two equal parts.
- **Segment Addition Postulate:** If point is in the interior of , then .

**Quick Check:**
1. Find .

**Examples:**
- **Naming Angles:** Name the angle at right in four ways.
- **Measuring Angles:** Find the measure of each angle.
- **Classifying Angles:** Classify each angle as acute, right, obtuse, or straight.

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**Quick Check**

1. a. Name two other ways to state the same angle: \( \angle A = \angle D \).

2. Find the measures of the angle. Classify it as acute, right, obtuse, or straight.

3. If \( m \angle DEG = 145 \), find \( m \angle GEF \).

4. Name an angle or angles in the diagram supplementary to each of the following:
   a. \( \angle DVA \)
   b. \( \angle AOB \)
   c. \( \angle DGO \) or \( \angle DOE \)

5. Can you make each conclusion from the information in the diagram? Explain.
   a. Yes; the markings show they are congruent.
   b. No, 3 angles have the same measure in the name to determine they are congruent.

**Making Conclusions From a Diagram**

Can you make each conclusion from the diagram?

- a. \( \angle A = \angle C \)
  - Yes, the markings show they are congruent.

- b. \( \angle B \) and \( \angle ACD \) are supplementary
  - Yes, there are no markings.

- c. \( \angle ACD = \angle B + \angle C = 180 \)
  - Yes, you can conclude that the angles are supplementary from the diagram.

- d. \( \angle A = \angle C \)
  - Yes, the markings show they are congruent.

**Examples**

- **Constructing Congruent Segments**
  - **Step 1** Draw a ray with endpoint \( Y \).
  - **Step 2** Open the compass to the length \( XY \).
  - **Step 3** With the same compass setting, put the compass point on point \( Y \). Draw an arc that intersects the ray. Label the point of intersection \( W \).
  - \( XY = YW \)

- **Constructing Congruent Angles**
  - **Step 1** Draw a ray with endpoint \( Z \).
  - **Step 2** With the compass point on point \( Z \), draw an arc that intersects the ray.
  - **Step 3** With the same compass setting, put the compass point on point \( Z \). Draw an arc that intersects the arc drawn in Step 2. Label the point of intersection \( P \).
  - **Step 4** Open the compass to the length \( ZP \). Keeping the same compass setting, put the compass point on point \( Z \). Draw an arc that intersects the arc drawn in Step 3. Label the point of intersection \( X \).
  - **Step 5** Draw \( ZX \) to complete \( \angle Y \).
  - \( \angle Y = \angle Z \)
Lesson 1-8 The Coordinate Plane

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Examples.

Key Concepts.

Lesson 1-8 The Coordinate Plane

Lesson Objectives:
- Find the distance between two points in the coordinate plane.
- Find the coordinates of the midpoint of a segment in the coordinate plane.

NASP 2005 Strand: Measurement
Topic: Measuring Physical Attributes
Local Standards:

Formula: The Distance Formula
The distance between two points \((x_1, y_1)\) and \((x_2, y_2)\) is

\[d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\]

Formula: The Midpoint Formula
The coordinates of the midpoint \(M\) of \(\overline{AB}\) with endpoints \((x_1, y_1)\) and \((x_2, y_2)\) are the following:

\[M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\]

Example 1
Applying the Distance Formula: How far is the subway ride from Oak to Symphony? Round to the nearest tenth. Each unit represents 1 mile.

Oak has coordinates \((0, 0)\), so \(x_1 = 0\) and \(y_1 = 0\) represent Oak.
Symphony has coordinates \((6, 3)\), so \(x_2 = 6\) and \(y_2 = 3\) represent Symphony.

\[d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\]

\[d = \sqrt{6^2 + 3^2}\]

\[d = \sqrt{36 + 9}\]

\[d = \sqrt{45}\]

\[d \approx 6.7\]

The distance from Oak to Symphony is approximately 6.7 miles.

Finding the Midpoint: \(\overline{AB}\) has endpoints \((0, 0)\) and \((6, 3)\). Find the coordinates of its midpoint \(M\) using the Midpoint Formula.

\[M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\]

\[M = \left(\frac{0 + 6}{2}, \frac{0 + 3}{2}\right)\]

\[M = (3, 1.5)\]

Finding the Endpoint: The midpoint of \(\overline{BC}\) is \((3, 1.5)\). One endpoint is \((0, 0)\). Find the coordinates of the other endpoint \(C\).

Use the Distance Formula:

\[d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\]

\[6.7 = \sqrt{(x_2 - 0)^2 + (1.5 - 0)^2}\]

\[6.7 = \sqrt{x_2^2 + 1.5^2}\]

\[45 = x_2^2 + 2.25\]

\[x_2^2 = 42.75\]

\[x_2 = \pm 6.5\]

Use the Midpoint Formula:

\[M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\]

\[3 = \frac{0 + x_2}{2}\]

\[6 = x_2\]

The coordinates of \(C\) are \((6.5, 1.5)\) or \((-6.5, 1.5)\).
Quick Check.

Find the coordinates of the midpoint of \( \overline{XY} \) with endpoints \( X(1, 5) \) and \( Y(6, 13) \).

b. Find the distance between Cedar and Maple.

3. The midpoint of \( \overline{XY} \) has coordinates \((4, -5)\). The coordinates of \( Y \) are \((2, -5)\).

Finding Circumference

A circle has a radius of 12 m. Find the circumference of the circle in terms of \( \pi \). Then find the circumference of the circle to the nearest tenth.

Postulate 5-9

If two figures are congruent, then their areas are equal.

Postulate 5-10

The area of a region is the sum of the areas of its non-overlapping parts.

Examples

i. Finding Circumference

- A circle has a radius of \( 2\text{ cm} \). Find the circumference of the circle in terms of \( \pi \). Then find the circumference of the circle to the nearest tenth.

- Circle with radius \( r \) and diameter of \( 2r \).

Formula for circumference of a circle

- \( C = \pi d \) or \( C = 2\pi r \).

ii. Finding Area

- The area of \( \triangle ABC \) is \( 22.5 \text{ in}^2 \). Use a calculator.

- The area of \( \triangle DEF \) is \( 15 \text{ in}^2 \).

Postulate 5-6

The area of a region is the sum of the areas of its non-overlapping parts.
Lesson 2-1 Conditional Statements

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4. Write the converse of the following conditional:

**Conditional:** If the measure of an angle is 180°, then it is a straight angle.  
**Converse:** If it is a straight angle, then its measure is 180°.  

- **Conclusion:** The measure of an angle is 180°.  
- **Hypothesis:** It is a straight angle.

A conditional is a *if-then* statement.  

The hypothesis is the part that follows *if* in a *if-then* statement.  

The conclusion is the part of an *if-then* statement (conditional) that follows then.  

The truth value of a statement is *true* or *false* according to whether the statement is true or false, respectively.

The converse exchanges the hypothesis and the conclusion.  

<table>
<thead>
<tr>
<th>Conditional</th>
<th>Example</th>
<th>Symbolic Form</th>
<th>You read it</th>
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<tbody>
<tr>
<td>If ( p ), then ( q ).</td>
<td>( p \rightarrow q )</td>
<td><em>If ( p ), then ( q ).</em></td>
<td>( q \rightarrow p )</td>
</tr>
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</table>

**Examples**

1. **Identifying the Hypothesis and the Conclusion.** Identify the hypothesis and conclusion of this conditional statement:

   **Conditional:** Two lines are parallel.  
   **Conclusion:** The lines are coplanar.

   - **Conclusion:** The lines are coplanar.  
   - **Hypothesis:** Two lines are parallel.

2. **Writing a Converse of a Conditional:** Write the converse of the following conditional.

   **Conditional:** If \( x = 3 \), then \( a = 9 \).  
   **Conclusion:** \( a = 9 \).  
   **Hypothesis:** \( x = 3 \).  

   **Converse:** If \( a = 9 \), then \( x = 3 \).  

   Evaluating the converse:
   - **If \( x = 3 \), then \( a = 9 \).**  
   - **If \( a = 9 \), then \( x = 3 \).**

   - **If \( x = 3 \), then \( a = 9 \).**  
   - **If \( a = 9 \), then \( x = 3 \).**

   The converse is true.

3. **Finding a Counterexample:** Find a counterexample to show that this conditional is false.  

   **Conditional:** If \( x < 0 \), then \( x^2 > 0 \).  

   **Counterexample:** \( x = -2 \) (negative number); \( x^2 = 4 \) (positive number).  

   - **Hypothesis:** \( x < 0 \)  
   - **Conclusion:** \( x^2 > 0 \)  

   - **Counterexample:** \( x = -2 \) (negative number); \( x^2 = 4 \) (positive number).  

   - **Hypothesis:** \( x < 0 \)  
   - **Conclusion:** \( x^2 > 0 \)  

   The converse is false.

**Writing a Biconditional:** Consider the true conditional statement. Write its converse. Then determine the truth value of each.

**Conditional:** If \( x = 2 \), then \( a = 9 \).  

- **Conditional:** If \( x = 2 \), then \( a = 9 \).  
- **Converse:** If \( a = 9 \), then \( x = 2 \).  

- **Conclusion:** If \( x = 2 \), then \( a = 9 \).  
- **Conclusion:** If \( a = 9 \), then \( x = 2 \).
Lesson 2-2

Deductive Reasoning

Lesson Objectives:
- Use the Law of Detachment
- Use the Law of Syllogism

Vocabulary and Key Concepts

Law of Detachment
If a conditional is true and its hypothesis is true, then its conclusion is true.
In symbolic form: If \( p \rightarrow q \) is a true statement and \( p \) is true, then \( q \) is true.

Law of Syllogism
If \( p \rightarrow q \) and \( q \rightarrow r \) are true statements, then \( p \rightarrow r \) is a true statement.

Deductive reasoning is a process of reasoning logically from given facts to a conclusion.

Examples
4. Using the Law of Detachment: A gardener knows that if it rains, the garden will be wet. It is raining. What condition can be made?
   - Because the hypothesis is true, the gardener can conclude that the garden will be wet.

5. Using the Law of Detachment: For the given statement, what can you conclude?
   - Given: \( A \) is acute, \( m\angle A < 90^\circ \).
   - Conclusion: \( A \) is acute, \( m\angle A < 90^\circ \).
   - \( A \) is acute, \( m\angle A < 90^\circ \).

Quick Check
1. Suppose that a mechanic begins work on a car and finds that the car will not start. Can the mechanic conclude that the car has a dead battery? Explain.
   - No, there could be other things wrong with the car, such as a faulty starter.

2. If Vladimir Nuñez is a pitcher, then he pitches a complete game two days in a row. Vladimir Nuñez is a pitcher. On Monday, he pitches a complete game. What can you conclude?
   - Answers may vary. Sample: Vladimir Nuñez should not pitch a complete game on Tuesday.

Geometry lessons 2-3

Daily Notetaking Guide
Quick Check.

Fill in each missing reason.

44. If a number ends in 4, then it is divisible by 2.

b. bisects rivers.

rivers.

If a river is less than 2300 miles long, it is not one of the world’s ten longest rivers.

The Volga River is in Europe.

The Volga River is less than 2300 miles long.

Conclusion:

The Volga River is not one of the world’s ten longest rivers.

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Lesson 2-5 Proving Angles Congruent

Lesson Objectives
✓ Prove and apply theorems about angles

Vocabulary and Key Concepts

1. 
2. 
3. 

Theorem 2-1: Vertical Angles Theorem
Vertical angles are congruent.

Theorem 2-2: Congruent Supplements Theorem
If two angles are supplements of the same angle (or of congruent angles), then the two angles are congruent.

Theorem 2-3: Congruent Complements Theorem
If two angles are complements of the same angle (or of congruent angles), then the two angles are congruent.

Lesson Objectives
✓ Prove and apply theorems about parallel lines

Vocabulary and Key Concepts

1. 
2. 
3. 

Theorem 2-5
Vertical angles are congruent.

Theorem 2-4
All angles are congruent.

Theorem 2-3
If two angles are supplementary, then each is a right angle.

Examples

Using the Vertical Angles Theorem. Find the value of x.

Vertical Angles Theorem: Two angles are complementary angles if the sum of their measures is 90°. The angles are complementary.

Using the Vertical Angles Theorem: Two angles are supplementary angles if the sum of their measures is 180°. The angles are supplementary.

Sides form two pairs of opposite rays.

Adjacent angles

Vertical angles

Two angles are vertical angles if they have a common vertex but no common interior points.

Adjacent angles

Two angles are adjacent angles if they have a common side and a common vertex but no common interior points.

Quick Check

1. Refer to the diagram for Example 1.
   a. Find the measure of the labeled pair of vertical angles.
   b. Find the measure of the other pair of vertical angles.
   c. Check to see that adjacent angles are supplementary.
   2. Recall the proof of Theorem 2-2. Does the size of the angles in the diagram affect the proof? Would the proof change if \( \angle A \) and \( \angle B \) were acute rather than obtuse? Explain.

Lesson 3-1 Properties of Parallel Lines

Lesson Objectives
✓ Identify angles formed by two lines and a transversal
✓ Prove and use properties of parallel lines

Vocabulary and Key Concepts

1. 
2. 
3. 

Postulates 3-1: Corresponding Angles Postulate
If a transversal intersects two parallel lines, then corresponding angles are congruent.

Theorem 3-1: Alternate Interior Angles Theorem
If a transversal intersects two parallel lines, then alternate interior angles are congruent.

Theorem 3-2: Same-Side Interior Angles Theorem
If a transversal intersects two parallel lines, then same-side interior angles are supplementary.

Theorem 3-3: Alternate Exterior Angles Theorem
If a transversal intersects two parallel lines, then alternate exterior angles are congruent.

Theorem 3-4: Same-Side Exterior Angles Theorem
If a transversal intersects two parallel lines, then same-side exterior angles are supplementary.
Lesson 3-2 Proving Lines Parallel

Lesson Objectives
1. Use the diagram at the right. (Lines and planes)
2. Classify angles.
3. Find the value of x.

Vocabulary and Key Concepts
- Postulate 3-2: Converse of the Corresponding Angles Postulate
- Theorem 3-6: Converse of the Alternate Interior Angles Theorem
- Theorem 3-7: Converse of the Same-Side Interior Angles Theorem
- Theorem 3-8: Converse of the Alternate Exterior Angles Theorem
- Theorem 3-9: Converse of the Same-Side Exterior Angles Theorem

Finding Measures of Angles
In the diagram at right, find and classify each angle.
1. \( \angle 1 \) and \( \angle 2 \) are corresponding angles.

Using Algebra to Find Angle Measures
In the diagram at right, find the value of each angle.

Using Postulate 3-2
Use the diagram at the right. Which lines, if any, must be parallel? Explain your answer with a theorem or postulate.

Using Algebra
Find the value of \( x \) for which the lines \( l \) and \( m \) are parallel.

Examples
1. Using Postulate 3-2
   Use the diagram at the right. Which lines, if any, must be parallel? Explain your answer with a theorem or postulate.

2. Using Algebra
   Find the value of \( x \) for which the lines \( l \) and \( m \) are parallel.
Quick Check.

1. Can the picture frame in Example 2 be assembled into a frame with opposite sides parallel? Explain.

   Yes, by Theorem 3-11 and the fact that \( \angle 1 \) is a right angle.

2. From what is given in Example 2, can you also conclude that the transversal is perpendicular to line \( k \)?

   No, by Theorem 3-11 and the fact that \( \angle 1 \) is a right angle.

The transversal is perpendicular to line \( k \).

**Examples**

**Real-World Connection**

A picture frame is assembled as shown. Given the book’s explanation for why the outer edges on opposite sides of the frame are parallel, why must the inner edges on opposite sides be parallel, too?

**Lesson 3-2 Daily Notetaking Guide**

Geometry

**Lesson Objectives**

- Prove two lines are parallel and perpendicular.
- Use exterior angles of triangles.

**Key Concepts**

- **Theorem 3-9:** If two parallel lines are cut by a transversal, then the measures of the acute and obtuse angles on one side of the transversal are equal to each other.

- **Theorem 3-10:** If two lines are parallel, then they are parallel to each other.

- **Theorem 3-11:** If two lines are perpendicular to the same line in a plane, then they are parallel.

- **Theorem 3-12:** The measure of each exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

- **Theorem 3-13:** The measure of each interior angle of a square equals the sum of the measures of its two remote interior angles.

**Vocabulary and Key Concepts**

- **Remote interior angles** are the two nonadjacent interior angles corresponding to each exterior angle of a triangle.

**Parallel and Perpendicular Lines**

- **NAFSTD 2005 Strand:** Geometry
- **Topic:** Relationships Among Geometric Figures
- **Local Standards:**

**Local Standards:**

- **NAEP 2005 Strand:** Geometry
- **Topic:** Relationships Among Geometric Figures

**Vocabulary and Key Concepts**

- **Remote interior angles** are the two nonadjacent interior angles corresponding to each exterior angle of a triangle.
Lesson 3-5
The Polygon Angle-Sum Theorems

Examples:

1. Applying the Triangle-Angle-Sum Theorem
   In triangle \(\triangle ABC\), \(\angle A\) is a right angle, and \(\angle B = 70^\circ\).
   Find the measures of the angles of the triangle.

   \[
   \angle C = 180^\circ - \angle A - \angle B
   \]
   \[
   \angle C = 180^\circ - 90^\circ - 70^\circ = 20^\circ
   \]

   \(\angle A = 90^\circ\), \(\angle B = 70^\circ\), and \(\angle C = 20^\circ\).

2. Applying the Triangle-Angle-Sum Theorem
   Explain what happens to the measure of an interior angle of a triangle.

   As one interior angle of a triangle increases, the other two angles decrease.

   \[m_1 + m_2 + m_3 = 180^\circ\]

   \(m_1\) and \(m_2\) are decreased, and \(m_3\) increases.

3. Applying the Triangle-Angle-Sum Theorem
   Explain what happens to the sum of the measures of the interior and exterior angles of a polygon.

   The sum of the measures of the interior angles of a polygon is \(\sum (n - 2) \times 180^\circ\), where \(n\) is the number of sides of the polygon.

   The sum of the measures of the exterior angles of a polygon is \(360^\circ\).

Classifying Polygons:

- Convex polygon: A polygon with all vertices pointing inwards.
- Concave polygon: A polygon with at least one vertex pointing outwards.

Finding a Polygon Angle Sum

- A polygon with \(n\) sides has a sum of \(n - 2\) interior angles.
- The sum of the measures of the \(n - 2\) interior angles is \(\sum (n - 2) \times 180^\circ\).

Using the Polygon-Angle-Sum Theorem

- Find \(m_1\) in quadrilateral \(ABCD\).

\[
\angle A + \angle B + \angle C + \angle D = 360^\circ
\]

\[
m_1 + m_2 + m_3 + m_4 = 360^\circ
\]

\(m_1\) is a corresponding angle.  

\[
m_1 = 180^\circ - m_2 - m_3 - m_4
\]

\(m_1\) is an exterior angle.

Quick Check:

1. Critical Thinking
   - Describe how you could use either \(\triangle ABC\) or \(\triangle ACD\) to find the value of \(x\) in Example 1.

   For \(\triangle ABC\), \(70^\circ + 70^\circ + x = 180^\circ\). Then, \(140^\circ + x = 180^\circ\).
   \(x = 40^\circ\)

2. a. Find \(w\).
   - \(w = 180^\circ - 140^\circ = 40^\circ\)
   - \(w = 180^\circ - 90^\circ = 90^\circ\)
   - \(w = 180^\circ - 70^\circ = 110^\circ\)
   - \(w = 180^\circ - 50^\circ = 130^\circ\)

   Critical Thinking
   - Is it true that if two acute angles of a triangle are complementary, then the third angle must be a right angle? Explain.

   Yes, because the measures of two complementary angles add to \(90^\circ\), leaving \(90^\circ\) for the third angle.
**Lesson 3-6 Lines in the Coordinate Plane**

**Objectives**
- Graph lines given their equations.
- Write equations of lines.

**Vocabulary**
- Point-slope form: \( y - y_1 = m(x - x_1) \)
- Slope-intercept form: \( y = mx + b \)

**Examples**

### Graph Lines Using Intercepts

1. **Writing an Equation of a Line Given Two Points**

   - **Procedure**
     1. **Step 1**: Find the slope.
        
     - **Step 2**: Use the slope-intercept form to write the equation.

   - **Example**
     - **Problem**: Write an equation of the line that contains the points \((0, 0)\) and \((2, 1)\). Draw the line containing the two points.

   - **Solution**
     - **Step 1**: Find the slope.
       
     - **Step 2**: Use the slope-intercept form to write the equation.

   - **Equation**: \( y = \frac{1}{2}x \)

   - **Graph**: Show the line on the coordinate plane.

2. **Writing an Equation of the Line with Slope \(-1\) that contains the Point \((2, -4)\)**

   - **Equation**: \( y = -x - 2 \)

   - **Graph**: Show the line on the coordinate plane.

3. **Writing an Equation of the Line that contains the Points \((5, 0)\) and \((7, -1)\)**

   - **Equation**: \( y = -\frac{1}{2}x + 3 \)

   - **Graph**: Show the line on the coordinate plane.

---

**Quick Check**

1. **Write an equation of the line with slope \(-3\) that contains the point \((5, 2)\).**

   - **Equation**: \( y = -3x + 17 \)

   - **Graph**: Show the line on the coordinate plane.

2. **Write an equation of the line that contains the points \((1, 2)\) and \((3, 4)\).**

   - **Equation**: \( y = 2x + 0 \)

   - **Graph**: Show the line on the coordinate plane.

---

**Geometry: All-In-One Answers Version A**

**Quick Check**

1. **Graph each equation.**

   - **Equation**: \( y = \frac{1}{2}x + 3 \)

   - **Graph**: Show the line on the coordinate plane.

2. **Classify each polygon by its sides. Identify each as convex or concave.**

   - **Polygon**: Pentagon

   - **Classification**: Convex

   - **Reason**: The sum of the measures of the angles of a given exterior angle is the angle of a polygon's angle because they have equal measures.

**Critical Thinking**

1. **Find \( \angle 5 \) and \( \angle 6 \).**

   - **Solution**: \( \angle 5 = 66^\circ \) and \( \angle 6 = 114^\circ \)

   - **Reason**: The exterior angle is the angle of a given polygon's angle because they have equal measures.

---

**Geometry Lesson 3-5 Daily Notetaking Guide**

**Quick Check**

1. **Graph each equation.**

   - **Equation**: \( y = x + 2 \)

   - **Graph**: Show the line on the coordinate plane.

2. **Critical Thinking**

   - **Reason**: The sum of the measures of the angles of a given exterior angle is the angle of a polygon's angle because they have equal measures.

---

**Geometry Lesson 3-6 Daily Notetaking Guide**

**Quick Check**

1. **Graph each equation.**

   - **Equation**: \( y = 2x - 1 \)

   - **Graph**: Show the line on the coordinate plane.

2. **Find \( \angle 5 \) and \( \angle 6 \).**

   - **Solution**: \( \angle 5 = 66^\circ \) and \( \angle 6 = 114^\circ \)

   - **Reason**: The exterior angle is the angle of a given polygon's angle because they have equal measures.

---

**Geometry Lesson 3-7 Daily Notetaking Guide**

**Quick Check**

1. **Graph each equation.**

   - **Equation**: \( y = -x + 1 \)

   - **Graph**: Show the line on the coordinate plane.

2. **Critical Thinking**

   - **Reason**: The sum of the measures of the angles of a given exterior angle is the angle of a polygon's angle because they have equal measures.
### Lesson 3-7: Slopes of Parallel and Perpendicular Lines

#### Example
1. **Determining Whether Lines are Parallel**
   - Are the lines $y = -5x + 4$ and $x = -3y + 4$ parallel? Explain.
   - The equation $y = -5x + 4$ is in slope-intercept form.
   - The equation $x = -3y + 4$ is not in slope-intercept form.
   - To determine if the lines are parallel, we need to find the slopes.
   - The slope of $y = -5x + 4$ is $-5$.
   - The slope of $x = -3y + 4$ is $-\frac{1}{3}$.
   - Since the slopes are not equal, the lines are not parallel.

2. **Writing Equations for Perpendicular Lines**
   - If two nonvertical lines are perpendicular, the product of their slopes is $-1$.
   - Any two vertical lines are perpendicular.
   - If two nonvertical lines are parallel, their slopes are equal.

3. **Slopes of Perpendicular Lines**
   - The line $y = 3x - 2$ has slope $3$.
   - The line $y = -x + 5$ has slope $-1$.
   - The product of the slopes of perpendicular lines is $-1$.

4. **Relate slope and perpendicular lines**
   - If the slope of one line is $m$, the slope of the line perpendicular to it is $-\frac{1}{m}$.

5. **Relate slope and parallel lines**
   - If the slope of one line is $m$, the slope of the line parallel to it is also $m$.

#### Quick Check
1. Are the lines parallel? Explain.
   - $y = x + 2$ and $2x - y = 5$ have different slopes.
   - Yes, they are parallel.

2. Write an equation for the line parallel to $y = 2x - 3$ that contains $(2, 5)$.
   - The slope of the given line is $2$.
   - The slope of the parallel line is $2$.
   - Using point-slope form, $y - y_1 = m(x - x_1)$, with $(2, 5)$:
     
     $$y - 5 = 2(x - 2)$$

3. **Writing Equations of Parallel Lines**
   - Write an equation in point-slope form for the line parallel to $6x - 3y = 4$ that contains $(-5, 3)$.
   - First, write the line in slope-intercept form:
     
     $$6x - 3y = 4$$
     
     $$-3y = -6x + 4$$
     
     $$y = 2x - \frac{4}{3}$$

   - The slope of the line is $2$.
   - Using point-slope form with $(-5, 3)$:
     
     $$y - 3 = 2(x + 5)$$

4. **Writing Equations of Perpendicular Lines**
   - Write an equation for a line perpendicular to $x - 3y = 6$ and passing through the point $(1, 2)$.
   - The slope of the given line is $\frac{1}{3}$.
   - The slope of the perpendicular line is $-3$.
   - Using point-slope form with $(1, 2)$:
     
     $$y - 2 = -3(x - 1)$$

### Lesson 3-8: Constructing Parallel and Perpendicular Lines

#### Example
1. **Constructing Congruent Angles**
   - Examine the diagram at right. Explain how to construct $\angle A$ congruent to $\angle J$.
   - Construct the angle.

2. **Constructing Lines**
   - Draw a line through the point $L$.
   - Draw a line through the point $N$.
   - The lines are parallel.

3. **Quick Check**
   - If corresponding angles are congruent, the lines are parallel.

#### Quick Check
1. Are lines $k$ and $l$ parallel? Explain.
   - Yes, because the corresponding angles are congruent.

2. **Writing an Equation for the Line Parallel to**
   - Write an equation for a line parallel to $y = -x + 4$ that contains $(-2, 5)$.
   - The slope of the given line is $-1$.
   - The slope of the parallel line is $-1$.
   - Using point-slope form with $(-2, 5)$:
     
     $$y - 5 = -1(x + 2)$$

3. **Writing an Equation for the Line Perpendicular to**
   - Write an equation for a line perpendicular to $2x + y = 3$ that contains $(5, -4)$.
   - First, write the line in slope-intercept form:
     
     $$2x + y = 3$$
     
     $$y = -2x + 3$$

   - The slope of the given line is $-2$.
   - The slope of the perpendicular line is $\frac{1}{2}$.
   - Using point-slope form with $(5, -4)$:
     
     $$y + 4 = \frac{1}{2}(x - 5)$$
Lesson 4-1 Congruent Figures

Name____________________  Class________________  Date________________

Example

1. Constructing a Special Quadrilateral: Construct a quadrilateral with both pairs of opposite sides parallel.
   Step 1: Draw point A and two rays with endpoints at A. Label point B on one ray and point C on the other ray.
   Step 2: Construct a ray parallel to AB through point C.
   Step 3: Construct a ray parallel to AC through point B.
   Step 4: Label point D where the ray parallel to AC intersects the ray parallel to AB. Quadrilateral ABCD has both pairs of opposite sides parallel.

Quick Check

1. Draw two segments. Label their lengths a and d. Construct a quadrilateral with one pair of parallel sides of length a and one of length d.

Vocabulary and Key Concepts

Theorem 4-1

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.

Examples

1. Naming Congruent Parts: ∆ABC ≅ ∆QEF. List the congruent corresponding parts.
   List the corresponding sides and angles in the same order.
   Angles: ∠A = ∠Q, ∠B = ∠R, ∠C = ∠F
   Sides: AB = QR, BC = RF, CA = EF

2. Using Congruency: ∆XYZ ≅ ∆KLM. m∠Y = 67° and m∠M = 48°. Find m∠K.
   Use the Triangle Angle-Sum Theorem and the definition of congruent polygons to find m∠K.
   m∠X + m∠Y + m∠Z = 180°
   m∠X + 67° + 48° = 180°
   m∠X = 65°

Finding Congruent Triangles: Can you conclude that ∆ABC ≅ ∆DEF?
   List corresponding vertices in the same order.
   If ∆ABC ≅ ∆DEF, then ∆ABC ≅ ∆DEF.
   The diagram above shows ∆ABC ≅ ∆DEF, not ∆BCE.
   The statement ∆ABC ≅ ∆DEF is not true.
   Also, ∠CBA ≠ ∠CDE and ∠ACB ≠ ∠CDE.
   Using Theorem 4-1, you can conclude that ∆ABC ≅ ∆DEF.
   Since all of the corresponding sides and angles are congruent, the triangles are congruent. The correct way to state this is ∆ABC ≅ ∆DEF.

Quick Check

1. ∆XYZ ≅ ∆MKL. List the congruent corresponding parts. Use three letters for each angle.
   Sides: XY = MK, YZ = KL, ZX = LM
   Angles: ∠X = ∠M, ∠Y = ∠K, ∠Z = ∠L

2. It is given that ∠XYZ ≅ ∠MKL. If m∠Y = 35°, what is m∠K? Express m∠K = ___.

Corresponding angles of congruent triangles are congruent and have the same measure.

3. Can you conclude that ∆JKL ≅ ∆MNP? Justify your answer.

No. The corresponding sides are not necessarily equal.
Example 3.3: Proving Triangles Congruent

Given: \( \triangle ABC \) and \( \triangle DEF \) with \( \angle A \cong \angle D \), \( \angle B \cong \angle E \), and \( \angle C \cong \angle F \).

Prove: \( \triangle ABC \cong \triangle DEF \)

Solution:
1. \( \angle A \cong \angle D \) (Given)
2. \( \angle B \cong \angle E \) (Given)
3. \( \angle C \cong \angle F \) (Given)

Conclusion: \( \triangle ABC \cong \triangle DEF \) (ASA Congruence Postulate)

Quick Check

1. Given: \( \triangle ABC \) and \( \triangle DEF \) with \( \angle A \cong \angle D \), \( \angle B \cong \angle E \), and \( \angle C \cong \angle F \).

Prove: \( \triangle ABC \cong \triangle DEF \)

2. Given: \( \triangle ABC \) and \( \triangle DEF \) with \( \angle A \cong \angle D \), \( \angle B \cong \angle E \), and \( \angle C \cong \angle F \).

Prove: \( \triangle ABC \cong \triangle DEF \)

3. Given: \( \triangle ABC \) and \( \triangle DEF \) with \( \angle A \cong \angle D \), \( \angle B \cong \angle E \), and \( \angle C \cong \angle F \).

Prove: \( \triangle ABC \cong \triangle DEF \)

4. Given: \( \triangle ABC \) and \( \triangle DEF \) with \( \angle A \cong \angle D \), \( \angle B \cong \angle E \), and \( \angle C \cong \angle F \).

Prove: \( \triangle ABC \cong \triangle DEF \)

Lesson Objective

Prove two triangles congruent using the SSS and SAS Postulates.

Key Concepts

Postulate 4-1: Side-Side-Side (SSS) Postulate
If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent.

Postulate 4-2: Side-Angle-Side (SAS) Postulate
If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.

Example 4.1: Proving Triangles Congruent

Given: \( \triangle ABC \) and \( \triangle DEF \) with \( \angle A \cong \angle D \), \( \angle B \cong \angle E \), and \( \angle C \cong \angle F \).

Prove: \( \triangle ABC \cong \triangle DEF \)

Solution:
1. \( \angle A \cong \angle D \) (Given)
2. \( \angle B \cong \angle E \) (Given)
3. \( \angle C \cong \angle F \) (Given)

Conclusion: \( \triangle ABC \cong \triangle DEF \) (ASA Congruence Postulate)

Quick Check

1. Given: \( \triangle ABC \) and \( \triangle DEF \) with \( \angle A \cong \angle D \), \( \angle B \cong \angle E \), and \( \angle C \cong \angle F \).

Prove: \( \triangle ABC \cong \triangle DEF \)

2. Given: \( \triangle ABC \) and \( \triangle DEF \) with \( \angle A \cong \angle D \), \( \angle B \cong \angle E \), and \( \angle C \cong \angle F \).

Prove: \( \triangle ABC \cong \triangle DEF \)

3. Given: \( \triangle ABC \) and \( \triangle DEF \) with \( \angle A \cong \angle D \), \( \angle B \cong \angle E \), and \( \angle C \cong \angle F \).

Prove: \( \triangle ABC \cong \triangle DEF \)

4. Given: \( \triangle ABC \) and \( \triangle DEF \) with \( \angle A \cong \angle D \), \( \angle B \cong \angle E \), and \( \angle C \cong \angle F \).

Prove: \( \triangle ABC \cong \triangle DEF \)
Lesson Objective

When two triangles are congruent, you can form congruence statements about three pairs of corresponding angles and the corresponding sides are congruent. By the Reflexive Property, the congruence statements that remain to be proved are ∠DGE ≅ ∠AGB and DE ≅ AG.

Quick Check

1. Write a proof.
   Given: \( \angle Q = \angle R \)
   Prove: \( \triangle QPS \cong \triangle RPS \)
   It is given that \( \angle Q = \angle R \). The Reflexive Property of Congruence, \( \triangle QPS \cong \triangle RPS \) by ASA.

2. Recall Example 2. About how wide was the river if the officer paced off the distance to be the width of the river? What conditions must hold for that to be true?

   The officer then paced off the distance to this spot and declared that this distance is about the width of the river!

   The conditions needed for the officer's method to work are right angles; the officer makes right angles with the ground, and the ground must be level or flat.
Vocabulary and Key Concepts.

Lesson Objective: Use the diagram from Example 3. The path around the garden is made up of rectangles and equilateral triangles.

Examples.

Quick Check.

Using Isosceles Triangles

Example

Quick Check

Using Algebra

Examples

Quick Check

Using the Isosceles Triangle Theorems

Examples

Quick Check

Using Isosceles Triangles

Example

Quick Check

Using the Isosceles Triangle Theorems

Examples

Quick Check
Geometry: All-In-One Answers Version A (continued)

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Lesson 4-7

Using Corresponding Parts of Congruent Triangles

Lesson Objectives
- Identify congruent overlapping triangles
- Prove two triangles congruent by first proving two other triangles congruent
- Write a two-column proof using Two Pairs of Triangles

Examples

1. Identifying Common Parts

- Identify the overlapping triangle
- Parts of sides that are shared by \( \Delta ABD \) and \( \Delta ABC \)

2. Using Common Parts

- Write a plan for a proof that includes using Two Pairs of Triangles
- Use the given to prove that the triangles are congruent

Quick Check

1. The diagram shows three triangles from the scaffolding that workers used when they repaired and cleaned the Statue of Liberty.
   - Name the common side in \( \Delta ACD \) and \( \Delta ABC \)
   - Name another part of a triangle that shares a common side and name the common side

Answers may vary. Sample: \( \overline{AC} \) and \( \overline{AB} \)

Local Standards: ____________________________________

Topic: Relationships Among Geometric Figures

NAEP 2005 Strand: ____________________

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Write a two-column proof.

Given: \( \angle H \cong \angle K \)

Prove: \( \angle A \cong \angle C \)

Statement | Reason
--- | ---
1. \( \angle H \cong \angle K \) | Given
2. \( \angle HAC \cong \angle KBC \) | Vertical Angles Theorem
3. \( \angle A \cong \angle C \) | Substitution

Write a paragraph proof.

Identifying Parallel Segments

Two segments are cut by transversal \( t \) as \( \overline{AM} \parallel \overline{BN} \) and \( \overline{BC} \parallel \overline{AM} \). By the Triangle Midsegment Theorem, \( \overline{MN} = \overline{AB} \).

Applying the Triangle Midsegment Theorem

Dean plans to measure the length of the lake, as shown in the figure. He knows that \( \overline{AC} \parallel \overline{BN} \) and \( \overline{BC} \parallel \overline{AM} \). By the Triangle Midsegment Theorem, \( \overline{MN} = \overline{AB} \).

To find the length of the lake, Dean starts at the middle of the left side of the lake and paces straight along the left side of the lake. After the lake ends, he counts the number of strides \( (236) \). Dean has paced along the middle of one side of a triangle. Dean paces from the midpoint of one side of the lake to the middle of the other end of the lake. He counts the number of his strides \( (236) \). Dean finds the midsegment of the second side by pacing the number of strides back toward the second side. He paces \( (250) \) strides. From the midpoint of the second side of the triangle, Dean returns to the midpoint of the first side, counting the number of strides \( (236) \). Dean has paced a triangle. He has also formed a new segment \( (\overline{MN}) \) in a triangle whose third side is the length of the lake.

By the Triangle Midsegment Theorem, the segment connecting the two midpoints is \( \overline{MN} \). The distance across the lake is \( 128 \). So, Dean multiplies the length of the midsegment by \( 2 \) to find the length of the lake.
Lesson 5-2

Bisectors in Triangles

Lesson Objective
Identify properties of perpendicular bisectors, angle bisectors, medians, and altitudes of a triangle.

Vocabulary and Key Concepts
- **Theorem 5-2: Perpendicular Bisector Theorem**
  - If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.
- **Theorem 5-3: Converse of the Perpendicular Bisector Theorem**
  - If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.
- **Theorem 5-4: Angle Bisector Theorem**
  - If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.
- **Theorem 5-5: Converse of the Angle Bisector Theorem**
  - If a point is equidistant from the sides of the angle, then it is on the angle bisector.

Quick Check

2. a. Use the information given in the diagram. \( \overline{CA} \) is the perpendicular bisector of \( \overline{DB} \). Find \( x \).

Example 1

Applying the Perpendicular Bisector Theorem: Use the map of Washington, D.C. Describe the set of points that are equidistant from the Lincoln Memorial and the Capitol.

The Theorem of the Perpendicular Bisector states: If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

A point that is equidistant from the Lincoln Memorial and the Capitol must be on the perpendicular bisector of the segment whose endpoints are the Lincoln Memorial and the Capitol.

Example 2

Using the Angle Bisector Theorem

Find \( x \) in the diagram above.

\[ 7x - 37 = 2x + 5 \]

Subtract \( 2x \) from each side.

\[ 5x = 42 \]

Divide each side by 5.

\[ x = 8.4 \]

Using the Angle Bisector Theorem

Find \( x \) and \( y \) in each diagram.

Quick Check

1. a. Use the information given in the diagram. \( \angle A \) is the angle bisector of \( \angle DBC \). Find \( \overline{AE} \).

Lesson 5-3

Concurrent Lines, Medians, and Altitudes

Lesson Objective
Identify properties of perpendicular bisectors, angle bisectors, medians, and altitudes of a triangle.

Vocabulary and Key Concepts
- **Theorem 5-6**: The angle bisectors of a triangle are concurrent at a point that is equidistant from the sides.
- **Theorem 5-7**: The medians of a triangle are concurrent at a point that is two-thirds the distance from each vertex to the midpoint of the opposite side.
- **Theorem 5-8**: The perpendicular bisectors of a triangle are concurrent at a point equidistant from the vertices.
- **Theorem 5-9**: The line that contains the altitudes of a triangle is concurrent.

Examples

Example 1

Applying the Angle Bisector Theorem: Use the map of Washington, D.C. Describe the set of points that are equidistant from the Lincoln Memorial and the Capitol.

The point where the angle bisectors are concurrent is the circumcenter of the circle.

Example 2

Using the Parallel Line Theorem

Find \( x \) and \( y \) in each diagram.

Quick Check

1. a. Use the information given in the diagram. \( \overline{DE} \) is the angle bisector of \( \overline{EF} \). Find \( \overline{DE} \).

Lesson 5-3

Concurrent Lines, Medians, and Altitudes

Lesson Objective
Identify properties of perpendicular bisectors, angle bisectors, medians, and altitudes of a triangle.

Vocabulary and Key Concepts
- **Theorem 5-6**: The angle bisectors of a triangle are concurrent at a point that is equidistant from the sides.
- **Theorem 5-7**: The medians of a triangle are concurrent at a point that is two-thirds the distance from each vertex to the midpoint of the opposite side.
- **Theorem 5-8**: The perpendicular bisectors of a triangle are concurrent at a point equidistant from the vertices.
- **Theorem 5-9**: The line that contains the altitudes of a triangle is concurrent.

Examples

Example 1

Applying the Angle Bisector Theorem: Use the map of Washington, D.C. Describe the set of points that are equidistant from the Lincoln Memorial and the Capitol.

The point where the angle bisectors are concurrent is the circumcenter of the circle.

Example 2

Using the Parallel Line Theorem

Find \( x \) and \( y \) in each diagram.

Quick Check

1. a. Use the information given in the diagram. \( \overline{DE} \) is the angle bisector of \( \overline{EF} \). Find \( \overline{DE} \).
Quick Check.

Using the diagram in Example 3, find...

1. The towns of Adamsville, Brooksville, and Cartersville want to build a library that is equidistant from the three towns. Show on the diagram where they should build the library. Explain.

2. The vertices of Adamsville, Brooksville, and Cartersville are connected by the segments shown below. Which of these segments is the perpendicular bisector of the segment connecting Adamsville and Brooksville?

3. Using the diagram in Example 3, find $WX$. Check that $WM = MW = WX$.

4. Is $\triangle XYZ$ acute, right, or obtuse? Explain.

Vocabulary and Key Concepts

- Acute Triangle
- Right Triangle
- Obtuse Triangle

- Incircle
- Circumcircle

- Median
- Altitude
- Bisector
- Incenter
- Centroid
- Orthocenter
- Circumcenter

- Theorem 5-6: All the perpendicular bisectors of the sides of a triangle are concurrent.

- Theorem 5-7 states that the perpendicular bisector of a segment is the set of all points equidistant from the endpoints.

- The city planners should find the point of concurrency of the perpendicular bisectors of the triangle formed by the three roads and locate the fountain there.

- Finding Lengths of Medians

- Finding the Circumcenter

- Finding the Incenter

- Finding the Orthocenter

- Finding the Centroid

Lesson Objectives

- Write the negation of a statement and the converse and contrapositive of a conditional statement.

- Use indirect reasoning.

Vocabulary and Key Concepts

- Negation, Inverse, and Contrapositive Statements

- Symbolic Form

- You Read It

- Conditional

- Negation of $p$

- Inverse

- Contrapositive

- Writing an Indirect Proof

Step 1: State an assumption (false) and the conclusion (true).

Step 2: State that the assumption leads to a contradiction.

Step 3: Conclude that the assumption must be false and that the conclusion must be true.

The negation of a statement $p$ has the opposite truth value from that of the original statement.

The converse of a conditional statement is not always true.

The contrapositive of a conditional statement is always true.

A conditional and its contrapositive always have the same truth value.

Equivalent statements have the same truth value.

An indirect proof is a proof involving indirect reasoning.
Lesson 5-4 Daily Notetaking Guide

Key Concepts:

- **Inverse**: If you don't stand for something, you'll fall for anything.
- **Contrapositive**: If you don't stand for something, you'll fall for anything.

**Identifying Contradictions**

Identify the two statements that contradict each other.

1. If $P$, $Q$, and $R$ are collinear, then $m\angle PQR = 90$.
2. If $P$, $Q$, and $R$ are collinear, then $m\angle PQR = 60$.
3. If three distinct points are collinear, then they form a straight line, so $m\angle PQR = 180$.

**Steps**

- **Step 1**: Assume the opposite of what you want to prove.
- **Step 2**: If $\angle A$ and $\angle B$ are supplementary, then $m\angle A + m\angle B = 180$.
- **Step 3**: This contradiction proves the Triangle Angle-Sum Theorem, which states that $m\angle A + m\angle B + m\angle C = 180$.

**Quick Check**

a. The measure of $\angle XYZ$ is not more than 70.

b. Today is not Tuesday.

Today is Tuesday.

Lesson 5-5 Inequalities in Triangles

Key Concepts:

- **Comparison Property of Inequality**

\[ a > b \implies a + c > b + c \]

- **Corollary to the Triangle Exterior Angle Theorem**

The measure of an exterior angle of a triangle is greater than the measure of each of its remote interior angles.

**Theorem 5-10**

If two sides of a triangle are not congruent, then the larger angle lies opposite the longer side.

**Theorem 5-11**

If two angles of a triangle are not congruent, then the longer side is opposite the greater angle.

**Theorem 5-12: Triangle Inequality Theorem**

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

\[ LM + LN > LN + MN \]

\[ LN + MN > LM \]

\[ LM + MN > LN \]
Applying the Corollary

The Corollary to the Exterior Angle Theorem states that the measure of an exterior angle of a triangle is greater than the measures of each of its remote interior angles. The remote interior angles are \( \angle 2 \) and \( \angle 5 \). Since \( \angle 4 \) and \( \angle 5 \) are angles formed by transversals \( \overline{SQ} \) and \( \overline{QR} \), you can conclude that \( \angle 4 \) is greater than \( \angle 2 \). From this Corresponding Angles Postulate. Thus, \( m\angle 2 = m\angle 5 \) and \( m\angle 2 > m\angle 4 \).

The Triangle Inequality Theorem: The sum of the lengths of any two sides of a triangle is greater than the length of the third side. Solve these inequalities.

- \( FH + FI > FI \)
- \( FH + FI > HI \)
- \( HI + FI > FH \)

The Triangle Inequality Theorem must be true for \( FH, FI, \) and \( HI \) to form a triangle. Therefore, \( FH, FI, \) and \( HI \) must be longer than \( 0 \) in and shorter than \( 26 \) in.

Quick Check

1. Use the diagram in Example 1. If \( \triangle XYZ \) is not an equilateral triangle, list the angles of \( \triangle XYZ \) in order from smallest to largest.

2. List the angles of \( \triangle ABC \) in order from smallest to largest.

3. Can a triangle have sides with the given lengths? Explain.
   - a. 2 m, 6 m, 9 m
   - b. 4 m, 6 m, and 9 m
   - c. 4, 9, and 14 m

4. A triangle has sides of lengths 3 m, 4 m, and 6 m. Describe the possible lengths of the third side.

Using the Properties of Special Quadrilaterals

- A quadrilateral is a parallelogram if both pairs of opposite sides are parallel.
- A quadrilateral is a parallelogram if both pairs of opposite sides are congruent.
- A quadrilateral is a parallelogram if one pair of opposite sides is parallel and the other pair of opposite sides is congruent.
- A quadrilateral is a parallelogram if it has a pair of parallel sides and a pair of congruent opposite angles.

Classifying by Coordinate Methods

- Quadrilateral \( \overline{FGHI} \) is not \( \overline{GH} \) because its slopes are parallel.
- Quadrilateral \( \overline{GH} \) not parallel to \( \overline{FI} \) because their slopes are equal.
- One pair of opposite sides is parallel, so \( \overline{FI} \parallel \overline{GH} \) by definition of parallel line.
- \( \overline{FI} \parallel \overline{GH} \) and \( \overline{FI} \parallel \overline{GH} \) by definition of parallel line.
- \( \overline{FI} \parallel \overline{GH} \) and \( \overline{FI} \parallel \overline{GH} \) by definition of parallel line.

Lesson 6-1

Classifying Quadrilaterals

- A quadrilateral is a trapezoid if it has exactly one pair of parallel sides.
- A quadrilateral is an isosceles trapezoid if it has one pair of parallel sides and the nonparallel sides are congruent.
- A quadrilateral is an isosceles trapezoid if it has one pair of parallel sides and the nonparallel sides are congruent.
- A quadrilateral is a quadrilateral whose nonparallel sides are congruent.
- A quadrilateral is a quadrilateral whose nonparallel sides are congruent.
- A quadrilateral is a quadrilateral whose nonparallel sides are congruent.
- A quadrilateral is a quadrilateral whose nonparallel sides are congruent.
- A quadrilateral is a quadrilateral whose nonparallel sides are congruent.
Quick Check

1. a. Graph quadrilateral \( \text{ABCD} \) with vertices \((-3, 3), (2, 4), (3, -1), \) and \((-2, -3)\).
   
2. Classify \( \text{ABCD} \) as many ways as possible.
   
   - \( \text{ABCD} \) is a quadrilateral because it has four sides.
   - \( \text{ABCD} \) is a parallelogram because both pairs of opposite sides are parallel.
   - \( \text{ABCD} \) is a trapezoid because one pair of opposite sides are parallel.
   - \( \text{ABCD} \) is a rectangle because it has four right angles and its opposite sides are congruent.
   - \( \text{ABCD} \) is a square because it has four right angles and its sides are all congruent.

3. Which name gives the most information about \( \text{ABCD} \)? Explain.
   
   - The name \( \text{ABCD} \) gives the most information about the quadrilateral.

4. Find the values of the variables in the rhombus. Then find the measure of \( \angle H \).
   
   - \( x = 110 \)
   - \( y = 53 \)
   - \( z = 53 \)
   - \( m \angle H = 2 \times 53 = 106 \)

5. Find the values of the variables in the trapezoid. Then find the measure of \( \angle B \).
   
   - \( x = 100 \)
   - \( y = 100 \)
   - \( m \angle B = 2 \times 100 = 200 \)

6. Classify the quadrilateral with vertices \( \text{ABCD} \) and \( \text{EFGH} \). Then find the measure of \( \angle G \).
   
   - \( \text{ABCD} \) and \( \text{EFGH} \) are both parallelograms.
   - \( m \angle G = 2 \times 100 = 200 \)

Lesson 6-2

Properties of Parallelograms

- Opposite sides of a parallelogram are congruent.
- Opposite angles of a parallelogram are congruent.
- The diagonals of a parallelogram bisect each other.
- Consecutive angles of a parallelogram are supplementary.

Quick Check

1. Find the values of \( a \) and \( b \) in \( \text{ABCD} \).
   
   - \( a = 16 \)
   - \( b = 16 \)

2. In the figure, \( \text{DF} \parallel \text{EF}, \text{DF} \parallel \text{EF} \). If \( \text{DF} = \text{EF} \), find \( \ell \).
   
   - \( \ell = 2.5 \)

3. Find the value of \( c \) in \( \text{ABCD} \).
   
   - \( c = 3 \)

4. Find the length of the side that is \( x \) in \( \text{EF} \).
   
   - \( x = \) \text{side length \( \text{EF} \)}
Lesson 6-3

Proving That a Quadrilateral is a Parallelogram

Lesson Objective

Key Concepts

- Theorem 6-1: If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
- Theorem 6-2: If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
- Theorem 6-3: If the diagonals of a parallelogram bisect each other, then the quadrilateral is a parallelogram.
- Theorem 6-4: If the diagonals of a parallelogram are congruent, then the quadrilateral is a parallelogram.
- Theorem 6-5: If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.

Examples

1. Finding Values for Parallelograms

Find values for $x$ and $y$ for which $ABCD$ is a parallelogram.

- Given: $ABCD$ is a parallelogram.
- Write and solve two equations to find values for $x$ and $y$ for which the diagonals bisect each other.

   - Diagonals of parallelograms bisect each other.
   - Collect the variables on one side.
   - Solve.

2. If $x = 18$ and $y = 10$, then $ABCD$ is a parallelogram.

Lesson 6-4

Special Parallelograms

Lesson Objective

Key Concepts

- Rhombuses
  - Theorem 6-8: Each diagonal of a rhombus bisects two angles of the rhombus.
  - Theorem 6-9: If one diagonal of a parallelogram bisects two angles of the parallelogram, then the parallelogram is a rhombus.

- Rectangles
  - Theorem 6-10: The diagonals of a rectangle are congruent.

- Parallelograms
  - Theorem 6-11: If one diagonal of a parallelogram bisects two angles of the parallelogram, then the parallelogram is a rectangle.
  - Theorem 6-12: If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rectangle.
  - Theorem 6-13: If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.
  - Theorem 6-14: If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rectangle.
Examples

1. Finding Angle Measures: Find the measures of the numbered angles in the rhombus.

   Theorem 6-9 states that each diagonal of a rhombus bisects two adjacent angles of the rhombus, so \( m \angle 1 = 70 \).

   Theorem 6-10 states that the diagonals of a rhombus are perpendicular.

   Because the angles formed by the diagonals all must have measure 90, \( \angle 3 \) and \( \angle 4 \) must be complementary.

   Because \( m \angle ABD = 70 \), \( m \angle 3 + m \angle 4 = 90 \).

   Finally, because \( \angle 2 = \angle 3 \), the Isosceles Triangle Theorem allows you to conclude that \( m \angle 2 = 65 \).

2. Finding Diagonal Length: One diagonal of a rectangle has length \( x + 2 \).

   The other diagonal has length \( 2x + 3 \). Because the four angles formed by the diagonals all must have measure 90, \( x + 2 \) and \( 2x + 3 \) must be complementary.

   Solve the equation \( (x + 2) + (2x + 3) = 90 \) to find \( x = 11 \).

   Therefore, the length of each diagonal is \( 26 \).

Lesson 6-5 Quiz Check

1. Find the measures of the numbered angles in the trapezoid.

   \( m \angle 1 = 38 \), \( m \angle 2 = 56 \), \( m \angle 3 = 70 \), \( m \angle 4 = 68 \).

Geometric Theorems and Proofs

Vocabulary and Key Concepts

Trapezoids

- Theorem 6-15: The base angles of an isosceles trapezoid are congruent.
- Theorem 6-16: The diagonals of an isosceles trapezoid are congruent.

Kites

- Theorem 6-17: The diagonals of a kite are perpendicular.

Quick Check

1. In the isosceles trapezoid, \( m \angle 5 = 70 \) \( \implies m \angle A = m \angle B = 70 \).

2. Find the length of each diagonal.

3. \( m \angle 1 = 38 \), \( m \angle 2 = 56 \), \( m \angle 3 = 70 \), \( m \angle 4 = 68 \).

4. So \( \angle 1 + \angle 2 + \angle 3 + \angle 4 = 360 \).

5. Because \( \angle 2 = \angle 3 \), the Isosceles Triangle Theorem allows you to conclude that \( \angle 2 = 65 \).

6. The length of each diagonal is \( 26 \).

Quick Check

1. Because the base angles of an isosceles trapezoid are congruent, \( m \angle A = m \angle B \).

2. So \( AC = BD \).

3. Another way to find the measures of each acute angle is to divide the difference between 180 and the measure of the vertex angle by 2.

4. Because \( \angle A = \angle B \), the two angles that share a leg are supplementary, so \( m \angle A + m \angle B = 180 \).

5. Because the bases of a trapezoid are parallel, the two angles that share a leg are supplementary, so \( m \angle A + m \angle B = 180 - 75 = 105 \).
Quick Check.

1. Find the coordinates of the missing vertices of the kite RDUQ.
   \[ \begin{array}{l}
   R(3, 3) \\
   D(2, 0) \\
   U(0, 3) \\
   Q(0, 0)
   \end{array} \]

2. The middle ring of the piece of ceiling shown is made from congruent isosceles triangles. Do not use any new variables.

3. Find \( \angle ABC \) and \( \angle ADC \) for the kite ABCD.

Examples.

Example 1: Finding Angle Measures in Kites

Find \( \angle ABC \) and \( \angle ADC \) for the kite ABCD.

Solution:

1. \( \angle ABC \) is a right angle because it is a right triangle.
2. \( \angle ADC \) is an acute angle because it is an isosceles triangle.

Example 2: Proving Congruency

Show that \( \triangle TVS \) is a parallelogram by proving pairs of opposite sides congruent.

Solution:

1. \( TS = VS \) and \( TV = VS \)
2. \( \triangle TVS \) is a parallelogram.
Examples.

Write both measurements in the same units.

a.

The ratio of the length of the scale model to the length of the car is \( \text{scale model length} : \text{car length} \).

b.

Finding Ratios

A scale model of a car is 4 in. long. The actual car is 15 ft long.

a. Find the ratio of the length of the scale model to the length of the car.

To find the ratio, we can write the lengths as a fraction and simplify.

\[
\frac{\text{scale model length}}{\text{car length}} = \frac{4 \text{ in.}}{15 \text{ ft}}
\]

b. Find and compare the slopes of \( \overline{XY} \) and \( \overline{YZ} \).

The slopes of \( \overline{XY} \) and \( \overline{YZ} \) can be compared by finding the change in the y-values divided by the change in the x-values.

\[
\text{slope of } \overline{XY} = \frac{\text{change in y}}{\text{change in x}} = \frac{2 - 0}{1 - 0} = 2
\]

\[
\text{slope of } \overline{YZ} = \frac{\text{change in y}}{\text{change in x}} = \frac{4 - 2}{2 - 1} = 2
\]

Both slopes are equal, so the lines are parallel.

c. Find and compare the length of \( MN \) and \( RA \).

The lengths of \( MN \) and \( RA \) can be compared by finding the distance between the endpoints.

\[
MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(2 - 0)^2 + (3 - 0)^2} = \sqrt{4 + 9} = \sqrt{13}
\]

\[
RA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4 - 2)^2 + (b - a)^2} = \sqrt{4 + (b - a)^2}
\]

The lengths of \( MN \) and \( RA \) are not equal, so they are not parallel.

d. In parts (b) and (c), how does placing a line along the x-axis help?

Placing a line along the x-axis allows calculating lengths by subtracting x-values.

Quick Check.

1. Using Coordinate Geometry Use coordinate geometry to prove that the quadrilateral formed by connecting the midpoints of the sides of \( ABCD \) is a parallelogram.

Draw quadrilateral \( ABCD \) by connecting the midpoints of \( X, Y, Z, \) and \( W \) with \( X(0, 0), Y(2, 0), Z(2, 3), \) and \( W(0, 3) \).

From Lesson 6-6, you know that \( \overline{XY} \) is a parallelogram.

To show that \( \overline{XY} \) is a parallelogram, find the lengths of its diagonals, and then compare them to show that they are congruent.

\[
\overline{XY} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(2 - 0)^2 + (0 - 0)^2} = \sqrt{4}
\]

\[
\overline{YW} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0 - 2)^2 + (3 - 0)^2} = \sqrt{13}
\]

Since \( \overline{XY} \neq \overline{YW} \), \( \overline{XY} \) is not a parallelogram.

2. Write two expressions that are equivalent to \( \frac{m}{n} \).

Answers may vary. Examples: \( \frac{5}{10} = \frac{1}{2} = \frac{5}{10} \)

Local Standards: ____________________________________

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Lesson 7-2
Similar Polygons

Lesson Objectives
 Identify similar polygons
 Apply similar polygons

NAEP 2005 Strand: Geometry and Measurement

Topic: Transformation of Shapes and Preservation of Proportion; Measuring Physical Attributes

Local Standards: ____________________________________

Vocabulary

Similar figures have the same shape but not necessarily the same size. Two polygons are similar if corresponding angles are congruent and corresponding sides are proportional.

The mathematical symbol for similarity is \(\sim\).

The similarity ratio is the ratio of the lengths of corresponding sides of similar figures.

A golden rectangle is a rectangle that can be divided into a square and a rectangle that is similar to the original rectangle.

The golden ratio is the ratio of the length to the width of any golden rectangle, about 1.618:1.

Example

Understanding Similarity \(\triangle ABC \sim \triangle XYZ\).

Complete each statement.

a. \(m\angle A = \angle X\)

b. \(\frac{BC}{XY} = \frac{AB}{XZ}\)

Because corresponding angles are congruent.

The similarity ratio is 4:1.

The shorter side of the larger rectangle is 12 in.

The similarity ratio is 4:1.

The shorter side is 40 in.

Find the longer side.

Corresponding sides of the two parallelograms are proportional.

The shorter side is 35 in.

Find the longer side.

Corresponding sides of the two parallelograms are proportional.

Corresponding angles of the two parallelograms are congruent.

Although corresponding triangles are not similar because the corresponding angles are not congruent.

Using Similar Figures \(\triangle ABC \sim \triangle XYZ\), find the value of \(x\).

Because \(\triangle ABC \sim \triangle XYZ\), you can write and solve a proportion.

Substitute.

Solve for \(x\).

\(x\) = 20

Using the Golden Ratio The dimensions of a rectangular tabletop are in the Golden Ratio. The shorter side is 40 in. Find the longer side.

40 \(\sim\) 20

Write a proportion using the Golden Ratio.

Property

The table is about 160 in. long.
Lesson 7-3: Proving Triangles Similar

Vocabulary and Key Concepts

**Postulate 7-1: Angle-Angle Similarity (AA~) Postulate**
If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

**Theorem 7-1: Side-Angle-Side Similarity (SAS~) Theorem**
If an angle of one triangle is congruent to an angle of a second triangle, and the sides including the two angles are proportional, then the triangles are similar.

**Theorem 7-2: Side-Side-Side Similarity (SSS~) Theorem**
If the corresponding sides of two triangles are proportional, then the triangles are similar.

**Example:**
1. Using the AA~ Postulate: Explain why the triangles are similar. Write a similarity statement.

**Examples:**
2. Explain why the triangles at the right must be similar. Write a similarity statement.

**Examples:**
3. In sunlight, a cactus casts a 9-ft shadow. At the same time, a person 6 ft tall casts a 4-ft shadow. Use similar triangles to find the height of the cactus.

**Examples:**
4. The light reflects off the mirror at the same angle at which it hits the mirror, so \( \angle JM = \angle JRM \) and therefore \( \triangle JOM \) and \( \triangle JRM \) are similar. Write a similarity statement.

**Examples:**
5. Use similar triangles to find the height of the tree.

**Examples:**
6. Explain why the triangles \( \triangle VYZ \) and \( \triangle VWX \) must be similar. Write a similarity statement.

**Examples:**
7. The tree is feet tall.

**Examples:**
8. The geometric mean of 3 and 12 is \( \sqrt{3 	imes 12} = \sqrt{36} = 6 \).

**Examples:**
9. The geometric mean of two positive numbers \( a \) and \( b \) is the positive number \( x \) such that \( \frac{a}{x} = \frac{x}{b} \). Find the geometric mean of 3 and 12.

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Quick Check.

1. Solve for \( x \) and \( y \).

- Find the geometric mean of 15 and 20.
- Maria's ball is \( y \) yd from the cup, and Gabriel's ball is \( x \) yd from the cup.

Using the Triangle-Angle-Bisector Theorem

- Find the distance between Maria's ball and Gabriel's ball.

Using the Side-Splitter Theorem

- If three parallel lines intersect two transversals, then the segments intercepted on the transversals are proportional.

Examples

1. Using the Side-Splitter Theorem: Find \( y \).
2. Using the Triangle-Angle-Bisector Theorem: Find the value of \( x \).

Quick Check

1. Find the geometric mean of 15 and 20.
2. Find the distance between Maria's ball and Gabriel's ball.

Lesson 7-5

Proportions in Triangles

- If three parallel lines intersect two transversals, then the segments intercepted on the transversals are proportional.

Key Concepts

Theorem 7-4: Side-Splitter Theorem

- If a line is parallel to one side of a triangle and intersects the other two sides, then it divides those sides proportionally.

Theorem 7-5: Triangle-Angle-Bisector Theorem

- If a ray bisects an angle of a triangle, then it divides the opposite side into two segments that are proportional to the other two sides of the triangle.

Corollary to Theorem 7-4

- If three parallel lines intersect two transversals, then the segments intercepted on the transversals are proportional.

Examples

1. Using the Side-Splitter Theorem: Find \( y \).

Quick Check

1. Using the Side-Splitter Theorem to find the value of \( s \).
2. Solve for \( x \) and \( y \).

Michelle drove her ball 192 yd straight toward the cup. Her brother Gabriel drove his ball straight 240 yd, but not toward the cup. The diagram shows the results. Find \( x \) and \( y \), their remaining distances from the cup.

Use Corollary 2 of Theorem 7-3 to solve for \( x \).

Maria's ball is \( y \) yd from the cup, and Gabriel's ball is \( x \) yd from the cup.

Recall Example 7. Find the distance between Maria's ball and Gabriel's ball.

Use Corollary 2 of Theorem 7-3 to solve for \( x \).

Use Corollary 2 of Theorem 5-3 to solve for \( y \).

Simplify.

Find the positive square root.

Substitute.

Write a proportion.

Cross-Product Property

Substitute.

Solve for \( y \).

Solve for \( x \).

Solve for \( y \).

Solve for \( x \).

Solve for \( y \).

Solve for \( x \).

Substitute.

Solve for \( y \).

Substitute.

Solve for \( y \).

Substitute.

Solve for \( x \).

Substitute.

Solve for \( y \).

Substitute.

Solve for \( y \).

Substitute.

Solve for \( x \).

Substitute.

Solve for \( y \).

Substitute.

Solve for \( y \).

Substitute.

Solve for \( x \).

Substitute.

Solve for \( y \).

Substitute.
Example

Quick Check
1. A right triangle has a hypotenuse of length 25 and a leg of length 10. Find the length of the other leg. Use the Pythagorean Theorem because they are uncertain whether that satisfy $a^2 + b^2 = c^2$, the triangle is right triangle.

2. Is this triangle a right triangle? Simplify.

Example

Quick Check
2. The hypotenuse of a right triangle has length 12. One leg has length 6. Find the length of the other leg. Leave your answer in simplest radical form.

Using Simpler Radical Form Find the value of $b$. Leave your answer in simplest radical form.

Example

Quick Check
3. A triangle has sides of lengths 18, 48, and 50. Is the triangle a right triangle? Classify the triangle by its angles.

Lesson Objectives

Lesson 8-1 Daily Notetaking Guide

Lesson 8-1 Daily Notetaking Guide

Lesson 8-1 Daily Notetaking Guide

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Lesson 8-2 Special Right Triangles

Key Concepts

Example 3 Finding the Length of the Hypotenuse. Find the value of the variable. Use the 45°-45°-90° Triangle Theorem to find the hypotenuse:

\[ \sqrt{2} \cdot x = 2x \]

Simplify.

\[ x = \frac{2}{\sqrt{2}} \]

\[ x = \sqrt{2} \cdot \frac{\sqrt{2}}{\sqrt{2}} \]

\[ x = \sqrt{2} \]

The length of the hypotenuse is \( \sqrt{2} \).

Example 4 Applying the 30°-60°-90° Triangle Theorem: A rhombus-shaped garden has a perimeter of 100 ft and a 60° angle. Find the area of the garden to the nearest square foot.

Because a rhombus has four sides of equal length, each side is \( \frac{100}{4} = 25 \) ft.

Because a rhombus is also a parallelogram, you can use the area formula \( A = bh \). Draw the altitude \( h \), and then solve for \( h \).

The height of the longer leg of the right triangle. To find the height \( h \), you can use the properties of 30°-60°-90° triangles. Then apply the area formula:

\[ 25 \cdot h = \text{shorter leg} \cdot \text{hypotenuse} \]

\[ x = \frac{25}{\sqrt{3}} \]

\[ h = \frac{25}{\sqrt{3}} \]

To the nearest square foot, the area is 375 ft².

Quick Check

3. Find the value of each variable.

\[ \tan \theta = \frac{2}{3} \]

4. A rhombus has 10-in. sides, two of which meet to form the indicated angle. Find the area of each rhombus. (Hint: Use a special right triangle to find heights.)

\[ a) \theta = 30^\circ \]

\[ \text{area} = \frac{1}{2} \cdot 10 \cdot 5 \sqrt{3} \]

\[ \text{area} = 25 \sqrt{3} \text{ in}^2 \]

\[ b) \theta = 60^\circ \]

\[ \text{area} = \frac{1}{2} \cdot 10 \cdot 5 \]

\[ \text{area} = 25 \text{ in}^2 \]
1. Using a Tangent Ratio: To measure the height of a tree, Alma walked 125 ft from the tree and measured a 32° angle from the ground to the top of the tree. Estimate the height of the tree.

\[ \tan 32° = \frac{\text{height}}{125} \]

Use the tangent ratio.

Solve for the height.

The tree is about \( 47 \) ft tall.

2. Using the Inverse of Tangent: Find \( m\angle R \) to the nearest degree.

\[ \tan R = \frac{41}{32} \]

Use the inverse tangent ratio.

So \( m\angle R \approx 52° \).

3. Writing Sine and Cosine Ratios: Use the triangle to find \( \sin \angle A \) and \( \cos \angle A \). Write your answers in simplest terms.

\[ \sin \angle A = \frac{\text{opposite}}{\text{hypotenuse}} \]

\[ \cos \angle A = \frac{\text{adjacent}}{\text{hypotenuse}} \]

4. Using the Inverse of Sine and Cosine: A right triangle has a leg 1.5 units long and hypotenuse 4.0 units long. Find the measures of its acute angles to the nearest degree.

\[ \sin \angle B = \frac{1.5}{4.0} \]

Use the sine ratio.

\[ \angle B \approx 22° \] (Round to the nearest degree.)

5. Using the Cosine Ratio: A 20-ft wire supporting a flagpole forms a 37° angle with the flagpole. To the nearest foot, how high is the flagpole?

\[ \cos 37° = \frac{\text{height}}{20} \]

Use the cosine ratio.

Solve for height.

The flagpole is about 16 ft tall.

6. Using the Inverse of Cosine and Sine: A right triangle has a leg 1.5 units long and hypotenuse 4.0 units long. Find the measures of its acute angles to the nearest degree.

\[ \cos \angle A = \frac{1.5}{4.0} \]

Use the inverse cosine function to find \( m\angle A \).

\[ \angle A \approx 67.98° \] (Round to the nearest degree.

To find \( m\angle B \), use the fact that the acute angles of a right triangle are complementary.
Name __________________________________________ Class __________________________ Date __________

Lesson 8-5

Angles of Elevation and Depression

**Lesson Objective**
- Use angles of elevation and depression to solve problems.

**NAEP 2005 Strand:** Measurement

**Topic:** Measuring Physical Attributes

**Local Standards:**

**Vocabulary:**
- **Angle of Elevation:** The angle formed by a horizontal line and the line of sight to an object above the horizontal line.
- **Angle of Depression:** The angle formed by a horizontal line and the line of sight to an object below the horizontal line.

**Example 1:**

**Identifying Angles of Elevation and Depression.** Describe \( \angle 1 \) and \( \angle 2 \) as they relate to the situation shown.

- One side of the angle of depression is a horizontal line. \( \angle 2 \) is the angle of depression from the building to the person on the ground.
- One side of the angle of elevation is a horizontal line. \( \angle 1 \) is the angle of elevation from the person on the ground to the building.

**Example 2:**

**Surveys.** A surveyor stands 200 ft from a building to measure its height using a 5-ft-tall theodolite. The angle of elevation to the top of the building is \( 32^\circ \). How tall is the building? Use a diagram to represent the situation.

- The building is about \( 6.5 \) ft tall.
- The building is about \( 6.5 \) ft tall.

**Example 3:**

**Aviation.** An airplane flying 3500 ft above the ground begins a \( 2^\circ \) descent to land at the airport. How many miles from the airport is the airplane when it starts its descent? (Note: The angle is not drawn to scale.)

- The airplane is about \( 13 \) miles from the airport when it starts its descent.
Lesson 8-6

Vectors

Lesson Objectives
- Describe vectors
- Apply problems that involve vector addition

Vocabulary and Key Concepts

Adding Vectors
For \( \mathbf{v} = (v_1, v_2) \) and \( \mathbf{w} = (w_1, w_2) \):
\[ \mathbf{v} + \mathbf{w} = (v_1 + w_1, v_2 + w_2) \]

A vector is any quantity with magnitude (size) and direction.
The magnitude of a vector is the size or length.
The initial point of a vector is the point at which it starts.
The terminal point of a vector is the point at which it ends.
A resultant vector is the sum of other vectors.
The terminal point of a vector is the point used as the initial point.
The initial point of a vector is the point used as the terminal point.
The magnitude of a vector is the distance from its initial point to its terminal point.
A vector can be represented with an ordered pair.

Quick Check.
1. Write the sum of the two vectors \( \mathbf{v} = (2, 3) \) and \( \mathbf{w} = (1, -2) \) as an ordered pair.
2. Describe the vector at the right as an ordered pair. Give the coordinates to the nearest tenth.

Examples
1. **Describing a Vector:** Describe \( \mathbf{v} \) as an ordered pair.
   - Draw a diagram for the situation.
   - To find the distance sailed, use the Distance Formula.
   - \( d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \)
   - The boat sailed 12 mi east and 9 mi south.
     - The boat sailed about 15 mi at about 41\(^\circ\) south of E.

2. **Describing a Vector Direction:** A boat sailed 12 mi east and 9 mi south.
   - The boat sailed about 15 mi at about 41\(^\circ\) south of E.
   - The trip can be described by the vector \((12,-9)\).
   - The direction the boat sails is the angle that the vector forms with the x-axis.
   - The magnitude of vector \( \mathbf{v} \) = distance sailed.
   - The angle \( \theta \) of vector \( \mathbf{v} \) is the angle that the vector forms with the x-axis.

3. **Identifying Isometries:** An isometry is a transformation that maps all points the same distance and in the same direction.
   - A translation is a transformation that maps all points the same distance and in the same direction.
   - An isometry is a transformation in which the preimage and the image are congruent.
   - A composition of transformations is a combination of two or more transformations.

Local Standards: ____________________________________

Lesson Objectives
- Describe vectors
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Local Standards: ____________________________________

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Local Standards: ____________________________________
Lesson 9-1 Daily Notetaking Guide

**Name:** ______________________  **Class:** ______________________  **Date:** ________________

### Geometry: All-In-One Answers Version A

#### Naming Images and Corresponding Parts

- In the diagram, \( \triangle XYZ \) is an image of \( \triangle ABC \).

#### Quick Check

1. Does the transformation appear to be an isometry? Explain.

2. In the diagram, \( \triangle ABC \) is an image of \( \triangle XYZ \). Describe the translation that maps \( \triangle ABC \) to \( \triangle XYZ \).

3. Use the rule \((x, y) \rightarrow (x + 6, y - 3)\) to translate \( \triangle ABC \) to \( \triangle XYZ \). Identify the preimage and image vertices.

#### Finding a Translation Image

- **Example:** Find the image of \((1, 2)\) under the translation \((x, y) \rightarrow (x + 6, y - 3)\).

#### Finding a Translation Image

- **Example:** Find the image of \((x, y) \rightarrow (x + 6, y - 3)\) under the translation \((x, y) \rightarrow (x - 2, y + 5)\).

#### Finding a Translation Image

- **Example:** Find the image of \((x, y) \rightarrow (x + 6, y - 3)\) under the translation \((x, y) \rightarrow (x - 2, y + 5)\).

#### Writing a Rule to Describe a Translation

- **Example:** Write the rule that maps \( \triangle ABC \) to \( \triangle XYZ \).

#### Adding Translations

- **Example:** Tritt rides his bicycle 3 blocks north and 5 blocks east of a pharmacy to deliver a prescription. Then he rides 4 blocks south and 8 blocks west to make a second delivery. How many blocks is he now from the pharmacy?

### Lesson 9-2

#### Reflections

- **Lesson Objectives:**
  - Find reflection images of figures

- **NADP 2003 Strand:** Geometry

- **Topic:** Transformation of Shapes and Preservation of Properties

#### Vocabulary

- A reflection in line \( r \) is a transformation such that if \( A \) is on line \( r \), then the image of \( A \) is \( B \), and if a point \( B \) is not on line \( r \), then its image \( B' \) is the point such that \( \angle rAB = \angle rB'A \).

#### Example

- **Finding Reflection Images:** If point \( Q(-1.7) \) is reflected across the line \( x = y \), what are the coordinates of the reflection image?

#### Quick Check

- **Example:** What are the coordinates of the image of \( Q \) if the reflection line is the perpendicular bisector of \( QF \) and \( F \) is at \((1, 1)\)?

- **Example:** What are the coordinates of the image of \( Q \) if the reflection line is the perpendicular bisector of \( QF \) and \( F \) is at \((1, 1)\)?

#### Writing a Rule to Describe a Translation

- **Example:** Write a rule to describe the translation \( \triangle ABC \rightarrow \triangle A'B'C' \).

- **Example:** You can use any point on line \( r \) to find the image of \( A \) under the reflection in line \( r \).

- **Example:** The rule represents a ride of 4 blocks south and 8 blocks west to make a second delivery. How many blocks is he now from the pharmacy?

- **Example:** Tritt rides his bicycle 3 blocks north and 5 blocks east of a pharmacy to deliver a prescription. Then he rides 4 blocks south and 8 blocks west to make a second delivery. How many blocks is he now from the pharmacy?

- **Example:** Tritt rides his bicycle 3 blocks north and 5 blocks east of a pharmacy to deliver a prescription. Then he rides 4 blocks south and 8 blocks west to make a second delivery. How many blocks is he now from the pharmacy?
Name_____________________________________ Class____________________________ Date ________________

Quick Check.

2. ΔABC has vertices (3, 4), (0, 3), and (2, 3). Draw ΔABC and its reflection image in the line \( y = 4 \).

3. In Example 1, what kind of triangle is \( ∆MPN \)? Imagine the image of point \( P \) reflected across line \( y = 4 \). What can you say about \( ∆MPN \) and \( ∆MP'N' \)?

Quick Check.

1. Draw the image of \( ∆LOM \) for a 90° rotation about point \( R \). Label the vertices of the image.

2. Regular pentagon \( ∆PENTA \) is divided into 5 congruent triangles. Name the image of \( P \) for a 144° rotation about point \( R \).

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Lesson 9-4 Symmetry

Vocabulary
A figure has symmetry if there is no isometry that maps the figure onto itself.
A figure has reflectional symmetry if there is a reflection that maps the figure onto itself.
A figure has rotational symmetry if it is its own image for some rotation of 180° or less.
A figure has point symmetry if it has 180° rotational symmetry.

Line of Symmetry
The letter V does not have rotational symmetry because it must be rotated yes; 180° before it is its own image.
The letter H is its own image after one half-turn, so it has rotational symmetry.
The heart-shaped figure has rotational symmetry with an angle of rotation yes.
The nut has lines of symmetry.

Example 1: Identifying Lines of Symmetry
Draw all lines of symmetry for the isosceles trapezoid.
The figure also has line of symmetry.

Example 2: Identifying Rotational Symmetry
Judging from appearance, do the letters V and H have rotational symmetry? If so, give an angle of rotation.
The letter V does not have rotational symmetry because it must be rotated yes; 180° before it is its own image.
The letter H is its own image after one half-turn, so it has rotational symmetry with a 180° angle of rotation.

Example 3: Finding Symmetry
A nut holds a bolt in place. Some nuts have square faces, like the top view shown below. Tell whether the nut has rotational symmetry and/or reflectional symmetry. Draw all lines of symmetry.
The nut has a square outline with a circular opening. The square and circle are congruent.
The nut is its own image after one quarter-turn, so it has rotational symmetry.
The nut has line of symmetry.

Local Standards: ____________________________________

Transformation of Shapes and Preservation of Properties
Topic: Geometry
NAEP 2005 Strand: Geometry

Lesson 9-5 Dilations

Vocabulary
A dilation is a transformation with center \( C \) and scale factor \( k \) for which the following are true:
- The image of \( C \) is \( C' \) (that is, \( C' = C \)).
- For any point \( P \) in \( 

Example 1: Finding a Scale Factor
Circle \( A \) with 5 cm diameter and center \( C \) is a dilation of concentric circle \( B \) with 8-cm diameter. Describe the dilation.
The circles are concentric, so the dilation has center yes.
Because the diameter of the dilation image is smaller, the dilation is a reduction.

Example 2: Finding a Scale Factor
The dilation is a reduction with center yes; yes; yes.
Lesson 9-6 Compositions of Reflections

**Vocabulary and Key Concepts**

- **Theorem 9-1**: A translation or rotation is a composition of two reflections.
- **Theorem 9-2**: A composition of reflections across two parallel lines is a translation.
- **Theorem 9-3**: A composition of reflections across two intersecting lines is a rotation.

**Example**

**Composition of Reflections in Intersecting Lines**

- The letter D is reflected in line \(x\) and then in line \(y\). Describe the resulting rotation.

1. Find the image of \(D\) through another reflection across line \(y\), then translate the image to line \(x\), and then to line \(y\).

- **Solution**

  The composition of two reflections across intersecting lines is a rotation.

  The center of rotation is the point where the line intersects, and the angle is twice the sum of the angles between the line and the center of rotation.

- **Finding a Glide Reflection Image**

  The letter D is reflected in line \(x\) and then in line \(y\), and then in line \(z\). Find the image of \(D\) for a glide reflection.

- **Solution**

  The glide reflection is the composition of a glide translation and a reflection across a line parallel to the direction of translation.

- **Graphing Dilation Images**

  The vertices of the reduction image of \(\triangle ABC\) are \((6, 2), (2, 4),\) and \((3, 1)\).

- **Solution**

  The vertices of the reduction image of \(\triangle ABC\) are \((0.75a - 1.5b + 6, 0.75c - 1.5d + 12)\).

- **Quick Check**

  1. Quadrilateral \(JKLM\) is a dilation image of quadrilateral \(ABC\). Describe the dilation.

- **Solution**

  The dilation is a reduction with center \((0, 0)\) and scale factor \(\frac{1}{2}\).

- **Quick Check**

  2. The height of a tractor-trailer truck is 4.2 m. The scale factor for a model truck is 1:24. Find the height of the model to the nearest centimeter.

- **Solution**

  The height of the model is 0.175 m or 17.5 cm.
Quick Check

1. a. Reflect the letter $R$ across $a$ and then $b$. Describe the resulting rotation.

   - The result is a clockwise rotation about point $P$ through an angle of $2\pi/3$.

2. Two parallel lines $e$ and $m$. Draw $R$ between $e$ and $m$. Find the image of $R$ after a reflection across line $e$ and then across line $m$. Describe the resulting translation.

   - The result is a translation twice the distance between $e$ and $m$.

   - b. Would the result of part (a) be the same if you reflected $R$ first, and then translated it? Explain.

   - Answers may vary. The result is a translation twice the distance between $e$ and $m$.

Vocabulary and Key Concepts

- Translational symmetry is the type of symmetry for which there is a translation that maps a figure onto itself.
- Glide reflectional symmetry is the type of symmetry for which there is a glide reflection that maps a figure onto itself.

Examples

1. Determining Figures That Will Tessellate

   - Determine whether a regular 15-gon tessellates a plane. Explain.
   - Because the figures in a tessellation do not overlap or leave gaps, the sum of the measures of the angles around any vertex must be $360\degree$. Check to see whether the measure of an angle of a regular $n$-gon is a factor of $360\degree$.
   - a. $180\degree$ and $30\degree$.
   - Use the formula for the measure of an angle of a regular $n$-gon.
   - $180\degree - \frac{180\degree \times (n-2)}{n}$
   - Simplify.
   - Because $180\degree$ divides evenly into $360\degree$, a regular 15-gon will tessellate a plane.
### Name ________________________ Class __________________ Date ____________

#### Lesson 10-1

**Areas of Parallelograms and Triangles**

**Lesson Objectives**
- Find the area of a parallelogram.
- Find the area of a triangle.

**Vocabulary and Key Concepts**

**Theorem 10-1: Area of a Rectangle**
The area of a rectangle is the product of its base and height.

**Theorem 10-2: Area of a Parallelogram**
The area of a parallelogram is the product of a base and the corresponding altitude.

**Theorem 10-3: Area of a Triangle**
The area of a triangle is half the product of its base and the corresponding altitude.

### Examples

**Finding a Missing Dimension**

- **A parallelogram has 9-in. and 18-in. sides.** The height corresponding to the 9-in. base is 15 in. Find the height corresponding to the 18-in. base.

\[
A = bh
\]

- Find the height corresponding to the 18-in. base.

**Finding the Area of a Triangle**

- **A triangle has an area of \( 120 \) square inches.** If the base of the triangle is 18 in., find its height.

\[
A = \frac{1}{2}bh
\]

- Find the height.

**Finding the Area of a Parallelogram**

- **A parallelogram has 9-in. and 18-in. sides.** The height corresponding to the 9-in. base is 15 in. Find the height corresponding to the 18-in. base.

\[
A = bh
\]

- Find the height corresponding to the 18-in. base.

### Quick Check

- **A parallelogram has sides 15 cm and 16 cm.** The height corresponding to a 15 cm base is 8 cm. Find the height corresponding to an 18 cm base.

\[
A = bh
\]

- Find the height corresponding to the 18 cm base.

### Lesson Objectives
- Find the area of a trapezoid.
- Find the area of a rhombus or a kite.

**Incorporating the Area of a Trapezoid**

- **A trapezoid is 10 in. long with bases 5 in. and 7 in.** Find the area of the trapezoid.

\[
A = \frac{1}{2}(b_1 + b_2)h
\]

- Find the area.

### Quick Check

- **A trapezoid is 10 in. long with bases 5 in. and 7 in.** Find the area of the trapezoid.

\[
A = \frac{1}{2}(b_1 + b_2)h
\]

- Find the area.

---

### Name ________________________ Class __________________ Date ____________

#### Lesson 10-2

**Areas of Trapezoids, Rhombuses, and Kites**

**Lesson Objectives**
- Find the area of a trapezoid.
- Find the area of a rhombus or a kite.

**Vocabulary and Key Concepts**

**Theorem 10-4: Area of a Trapezoid**
The area of a trapezoid is \( \frac{1}{2} \) the product of the height and the sum of the bases.

**Theorem 10-5: Area of a Rhombus or a Kite**
The area of a rhombus or a kite is \( \frac{1}{2} \) the product of its diagonals.

### Examples

**Applying the Area of a Trapezoid**

- **A car window is shaped like the trapezoid shown.** Find the area of the window.

\[
A = \frac{1}{2}(b_1 + b_2)h
\]

- Find the area of the window.

### Quick Check

- **A car window is shaped like the trapezoid shown.** Find the area of the window.

\[
A = \frac{1}{2}(b_1 + b_2)h
\]

- Find the area of the window.

---

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In Example 2, suppose and change so that

Because opposite sides of rectangle $ABXD$ are congruent, $\overline{DX} = \overline{XH} = 11.0$ ft.
By the Pythagorean Theorem, $\overline{AB} = \overline{BX}$, so $\overline{BX} = \sqrt{11^2 - 5^2} = \sqrt{121 - 25} = \sqrt{96} = 4\sqrt{6}$.
Taking the square root, $\overline{DX} = 11.0$ ft. You may remember that 5, 12, 13 is a Pythagorean triple.

The area of trapezoid $ABCD$ is $A = \frac{1}{2}(11 + 13) \times 5 = 60$ square feet.

Quick Check:

1. Find the area of a trapezoid with height 7 cm and bases 12 cm and 15 cm.
   
   $64.5 \text{ cm}^2$

2. In Example 2, suppose $\overline{AB}$ and $\overline{XH}$ change so that $m\angle C = 80$ while bases and angle $A$ and $B$ are unchanged. Find the area of trapezoid $ABCD$.
   
   $105.0\text{ ft}^2$ or $116.9\text{ ft}^2$

Examples:

Finding the Area of a Kite

Find the area of kite $XYZW$.

First draw an altitude from vertex $A$ to $\overline{XY}$ or $\overline{XW}$, creating a right triangle.

Substitute for $b = 13.0$ ft and $h = 3.0$ ft.

Simplify.

The area of kite $XYZW$ is $21.7\text{ ft}^2$.

Finding the Area of a Rhombus

Find the area of a rhombus whose sides are 15.0 ft.

First draw the diagonals of the rhombus, forming a rectangle.

Divide the right triangle formed by one diagonal and a side into two congruent triangles. The base of each is 7.5 ft and the height of each is 12.0 ft.

Simplify.

The area of the rhombus is $120.0\text{ ft}^2$.

Finding the Area of a Regular Polygon

A library is in the shape of a regular octagon. Each side is 18.0 ft. The radius of the octagon is $15.6\text{ ft}$.

The center of a regular polygon is the center of the circumscribed circle.

The radius of a regular polygon is the distance from the center to a vertex.

The apothem of a regular polygon is the perpendicular distance from the center to a side.

To find the area formula $A = \frac{1}{2}(p \times a)$, you need to find $p$ and $a$.

Step 1: Find the perimeter $p$.

Substitute for $n = 8$ and $a = 18.0$ ft. Simplify.

$p = 115.2\text{ ft}$

Step 2: Find the apothem $a$.

Substitute for $n = 8$ and $a = 15.6$ ft. Simplify.

$a = 6.9\text{ ft}$

Step 3: Find the area $A$.

Substitute for $n = 8$, $a = 6.9\text{ ft}$, and $p = 115.2\text{ ft}$. Simplify.

$A = 297.5\text{ ft}^2$

The area of the regular octagon is $297.5\text{ ft}^2$.

Applying Theorem 10.6

Find the area of an equilateral triangle with an apothem $a = 6\text{ cm}$. Leave your answer in simplest radical form.

This equilateral triangle can be divided into two right triangles, each with sides $a$, $\frac{a}{2}$, and $\frac{a}{2} + a$. The area of each right triangle is $\frac{1}{2} \times \frac{a}{2} \times a = \frac{a^2}{4}$.

The area of the equilateral triangle is $3 \times \frac{a^2}{4} = \frac{3a^2}{4}$.

The area of the equilateral triangle is $108\text{ cm}^2$.
Quick Check

1. At the right, a portion of a regular octagon has radii and an apothem drawn. Find the measure of each numbered angle.

2. Find the area of a regular pentagon with 11.6-cm sides and an 8-cm apothem.

3. The side of a regular hexagon is 16 ft. Find the area of the hexagon.

4. Benita plants the same crop in two rectangular fields. Each dimension of the larger field is 3.5 times the dimension of the smaller field. Seeding the smaller field costs $8. How much money does seeding the larger field cost?

Finding Ratios in Similar Figures
The triangles at the right are similar. Find the ratio (larger to smaller) of their perimeters and of their areas.

Finding Areas Using Similar Figures
All regular octagons are similar. Find the area of the smaller octagon.

Finding Similarity and Perimeter Ratios
Two similar polygons have corresponding sides in the ratio 5 : 7.

a. Find the ratio of their perimeters.

b. Find the ratio of their areas.

A. The side of a regular hexagon is 16 ft. Find the area of the hexagon.

B. The triangles at the right are similar. Find the ratio (larger to smaller) of their perimeters and of their areas.

C. The similarity ratio is 5 : 3, so the ratio of the areas of the fields is $(\frac{5}{3})^2 : (\frac{3}{5})^2$.

Because seeding the smaller field costs $8, seeding 12.25 times as much costs $98.

Seeding the larger field costs $6.94.

The corresponding sides of two similar parallelograms are in the ratio $a : b$.

The area of the larger parallelogram is 96 in.$^2$. Find the area of the smaller parallelogram.

X. The similarity ratio of the dimensions of two similar pieces of window glass is 3 : 5. The smaller piece costs $2.50. What should be the cost of the larger piece?

Lesson Objective
Find the perimeters and areas of similar figures.

NAEP 2005 Strand: Measurement and Number Properties

Topics: Systems of Measurement; Ratios and Proportional Reasoning

Local Standards: ____________________________________

Measurement and Number Properties

Systems of Measurement; Ratios and Proportional Reasoning

Examples.

Example 1: Perimeters and Areas of Similar Figures
If the similarity ratio of two similar figures is $a : b$, then

(1) the ratio of their perimeters is $a : b$ and

(2) the ratio of their areas is $a^2 : b^2$.

Example 2: Finding Ratios in Similar Figures
The scale factor of the triangles at the right is 3. Find the ratio of their perimeters and their areas.

Example 3: Finding Areas Using Similar Figures
The ratio of the areas of two similar rectangles is 1875 ft.$^2$ and 135 ft.$^2$. Find the ratio of their corresponding sides.

Example 4: Finding Similarity and Perimeter Ratios
The areas of two similar rectangles are 1875 ft.$^2$ and 135 ft.$^2$. Find the ratio of their corresponding sides.
Lesson 10-6 Circles and Arcs

Name_____________________________________ Class____________________________ Date________________

Vocabulary and Key Concepts.

Theorem 10-9: Circumference of a Circle

\[ C = 2\pi r \]

The ratio of the circumference of a circle to its diameter is \( \pi \). Hence, \( A = \pi \) is the formula.

Example 1: Finding Area

The radius of a garden is 18 feet. Find the area of the garden.

1. **Finding Area**
   - The garden is a circle.
   - The area of a circle is \( A = \pi r^2 \).
   - Substitute the radius \( r = 18 \) feet into the formula.
   - \( A = \pi (18)^2 \)
   - \( A = 324\pi \) ft²

Example 2: Finding Area

A triangular park has two sides that measure 200 ft and 300 ft and form a 65° angle. Find the area of the park to the nearest hundred square feet.

1. **Finding Area**
   - The park is a triangle.
   - The area of a triangle is \( A = \frac{1}{2}bh \).
   - Substitute the base \( b = 200 \) ft and the height \( h = 180 \) ft (calculated using the Pythagorean theorem).
   - \( A = \frac{1}{2} \times 200 \times 180 \)
   - \( A = 18,000 \) sq ft

Quick Check

1. Find circumference and arc length
   - \( C = 2\pi r \)
   - \( A = \frac{1}{2} \times r \times 2\pi r \)

2. Congruent circles are circles that have the same measure and are in the same circle or in congruent circles.

3. Adjacent arcs are arcs of the same circle that have exactly one point in common.

4. Concentric circles are circles that lie in the same plane and have the same center.

5. Arc length is a fraction of a circle's circumference.

6. Congruent arcs are arcs that have the same measure and are in the same circle or in congruent circles.

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Lesson Objectives:
- Find the measure of a central angle and arc
- Find circumferences and arc length

Vocabulary and Key Concepts:
- Postulate 10-1: Arc Addition Postulate
  - The measure of the arc formed by two adjacent arcs is the sum of the measures of the two arcs.
  - \( m \text{ARC} = m \text{ARC}_1 + m \text{ARC}_2 \)

- Theorems:
  1. **Theorem 10-10: Arc Length**
     - The length of an arc of a circle is the product of the measure of the angle that intercepts the arc and the circumference of the circle.
     - \( m \text{ARC} \times \text{CIRCULAR COORDINATE} = \text{ARC LENGTH} \)

A circle is the set of all points equidistant from a given point called the center.

A segment that has one endpoint at the center and the other endpoint on the circle is a radius.

A diameter is a segment that contains the center of a circle and has both endpoints on the circle.

A central angle is an angle whose vertex is the center of the circle.

Circumference of a circle is the distance around the circle.

The measure of an angle whose vertex is the center of the circle is the measure of its corresponding central angle.

\( \theta \) is the measure of a central angle.

\( \text{CIRCULAR COORDINATE} = \frac{\text{ARC LENGTH}}{\text{CIRCUMFERENCE}} \times \text{CIRCUMFERENCE} \)

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Lesson 10-5 Trigonometry and Area

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Lesson Objectives:
- Find the area of a regular polygon using trigonometry
- Find the area of a triangle using trigonometry

Vocabulary and Key Concepts:
- The area of a triangle is half the product of the length of two sides and the sine of the included angle.
- The area of a triangle is \( A = \frac{1}{2}bc \sin C \).

Example 1: Finding Area

The area of the pentagon is regular, \( A = \frac{1}{2}bc \sin C \).

- \( A = \frac{1}{2} \times 200 \times 300 \times \sin 65° \)
- \( A \approx 27,200 \) sq ft

Example 2: Finding Area

Find the area of the park to the nearest hundred square feet.

- \( A = \frac{1}{2} \times 200 \times 180 \)
- \( A = 18,000 \) sq ft

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Lesson 10-4 Geometry: All-In-One Answers Version A (continued)
Examples

1. Applying Circumference
A circular swimming pool is 16 ft in diameter. What length of fencing material is needed? Round your answer to the nearest whole number.

- The diameter of the pool is 16 ft, so the fencing will be 16 ft.
- The fencing will be enclosed in a circular fence 4 ft from the pool. What length of fencing material is needed?
- The fencing material is needed.
- About 24 ft of fencing material is needed.

2. Finding Arc Length
Find the length of \( \text{arc} \, \overline{AB} \) of a circle with a radius of 12 ft and a central angle of 30°.

- The length of \( \text{arc} \, \overline{AB} \) is 21.2 ft.

3. Finding the Area of a Sector of a Circle
Find the area of sector \( \overline{ACB} \) in circle \( A \) in terms of \( \pi \).

- The area of sector \( \overline{ACB} \) is \( 108 \pi \) ft².

Quick Check

1. How much more pizza is in a 14-in. diameter pizza than in a 12-in. pizza?
   - About 45 in²

2. Critical Thinking
A circle has a diameter of 20 cm. What is the area of a sector bounded by a 28° major arc? Round your answer to the nearest tenth.

- The area of the sector is 51.2 cm².
Lesson 10-8

Geometric Probability

Lesson Objective

√ Use segment and area models to find the probability of events.

NAP 2005 Strand: Data Analysis and Probability

Topic: Probability

Local Standards:

Vocabulary

Geometric probability is a model in which you let points represent outcomes.

Example

Finding Probability Using Segments

A gnat lands at random on the edge of the ruler below. Find the probability that the gnat lands on a point between 2 and 10.

The length of the segment between 2 and 10 is 8.

The length of the ruler is 12.

Quick Check

Example

A point on is selected at random. What is the probability that it is a point on ?

To win a prize, you must toss a quarter. If you win, you receive $32. Outline the probability that you expect to win a prize? Explain.

The probability that a dart landing randomly in the square does not land within the circle is about 80.4%.

Quick Check

Example

An edge is a flat surface of a polyhedron in the shape of a polygon.

Find the area of the region between the square and the circle.

A vertex is a point where three or more edges intersect.

The probability that a dart landing randomly within the square does not land within the circle is about 96.9%.

Critical Thinking

Example

Use Example 5. Suppose you toss 100 quarters. Would you expect to win a prize? Explain.

Yes, theoretically you should win 32.6 times out of 100.
Lesson 11-2 Surface Areas of Prisms and Cylinders

Vocabulary and Key Concepts

1. **Using Euler’s Formula**
   - Use Euler’s Formula to find the number of edges on a polyhedron.
   - List the vertices, edges, and faces of the polyhedron.
   - The formula is: $V - E + F = 2$
   - A solid with 6 faces and 8 vertices has 12 edges.

Quick Check

1. List the vertices, edges, and faces of the polyhedron.
   - $V = 8$, $E = 14$, $F = 9$
   - Vertices: $A$, $B$, $C$, $D$, $E$, $F$, $G$, $H$
   - Faces: $ABC$, $ABD$, $ABE$, $ABF$, $ABG$, $ABH$, $ACD$, $ACE$, $ACF$, $ACH$, $ADE$, $ADF$, $ADG$, $ADE$, $AEF$, $AFG$, $AGH$, $BDE$, $BDF$, $BDG$, $BGE$, $BGF$, $BGH$, $CD$, $CDE$, $CDF$, $CFE$, $CGE$, $CGF$, $CGH$, $DHE$, $DHF$, $DGH$, $EFG$, $EGH$, $EFH$.

2. Use Euler’s Formula to find the number of edges on a polyhedron with eight triangular faces.
   - Since the polyhedron has 8 vertices and 8 faces, it must have 12 edges according to Euler’s Formula: $V - E + F = 2$.

Lesson Objectives

- Find the surface area of a prism
- Find the surface area of a cylinder

Theorem 11-1: Lateral and Surface Area of a Prism

- The lateral area of a right prism is the product of the perimeter of the base and the height.
- The surface area of a right prism is the sum of the lateral area and the area of the bases.

Theorem 11-2: Lateral and Surface Area of a Cylinder

- The lateral area of a right cylinder is the product of the circumference of the base and the height of the cylinder.
- The surface area of a right cylinder is the sum of the lateral area and the area of the bases.

Examples

1. **Verify Euler’s Formula**
   - Verify Euler’s Formula for the two-dimensional net of the solid in Example 1.
   - Count the regions: $P = 6$
   - Count the vertices: $V = 4$
   - Count the segments: $E = 8$
   - Verify: $4 - 8 + 6 = 2$.

Quick Check

A. The figure at the right is a trapezoidal prism.
   - Verify Euler’s Formula $P = V - E + 2$ for the prism.
   - Possible answer: $10 - 6 + 4 = 8$

B. Draw a net for the prism.
   - Possible answer:

C. Verify Euler’s Formula $P = V - E + 1$ for your two-dimensional net.
   - Possible answer: $8 - 5 + 4 = 7$
**Lesson Objectives**

- The surface area of a pyramid is
- The lateral area of a pyramid is
- The slant height of a regular pyramid is
- The altitude of a pyramid or a cone is

A regular pyramid is a pyramid whose base is a regular polygon and whose lateral faces are congruent isosceles triangles.

**Examples**

1. **Finding Surface Areas of Cylinders**
   - A company ships cornmeal and barley in cylindrical containers. The diameter of the base of the 4-in.-high cornmeal container is 6 in. The diameter of the base of the 4-in.-high barley container is 8 in. Which container has the greater surface area?

   - Find the surface area of each container. Remember that $r = \frac{d}{2}$.

   - **Cornmeal Container**
     - L.A. = $\pi r^2$ and L.A. = 56.25 in$^2$ for $r = 3$ in.
     - S.A. = $2\pi r^2 + \pi r h$ and S.A. = 75.4 in$^2$ for $h = 6$ in.

   - **Barley Container**
     - L.A. = $\pi r^2$ and L.A. = 100.5 in$^2$ for $r = 4$ in.
     - S.A. = $2\pi r^2 + \pi r h$ and S.A. = 120.7 in$^2$ for $h = 6$ in.

Because 75.4 in$^2 >$ 60 in$^2$, the barley container has the greater surface area.

2. **Finding Surface Area of a Pyramid**

   - Find the surface area of a square pyramid with base edges 7.5 in. and slant height 12.8 in.

   - The perimeter of the base is 24 in, so you can find the lateral area.

   - L.A. = $\frac{1}{2} pl$ and L.A. = 84 in$^2$ for $p = 24$ in. and $l = 12.8$ in.

   - S.A. = L.A. + B.

   - S.A. = 90 in$^2$ + 56.25 in$^2$ = 146.25 in$^2$.

**Quick Check**

1. The surface area and lateral area of the prism. $821$ cm$^2$; about 659 cm$^2$

2. Find the surface area of a cylinder with height 10 cm and radius 5 cm in terms of $\pi$.

   - L.A. = $2\pi rh$ and L.A. = $2\pi (5)(10)$ in$^2$ or $100\pi$ in$^2$.

3. The company in Example 2 wants to make a label to cover the cornmeal container. The label will cover the container all the way around, but will not cover any part of the top or bottom. What is the area of the label to the nearest tenth of a square inch?

   - The surface area of a square is $s^2$ where $s = $ the length of the side.

   - Use the diagram of the base.

   - Use 30°-60°-90° triangles to find the shorter leg.

   - Use the formula for surface area.

   - S.A. = $2s^2 + \sqrt{3}s^2$ (because $s^2 + \sqrt{3}s^2 = 18^2$)

   - S.A. = $306.7$ in$^2$.
Geometry: All-In-One Answers Version A (continued)

Lesson 11-4 Volumes of Prisms and Cylinders

Lesson Objectives

- Find the volume of a prism
- Find the volume of a cylinder

Vocabulary and Key Concepts

Theorem 11-5: Cavalieri’s Principle
If two space figures have the same height and the same cross sectional area at every level, then they have the same volume.

Theorem 11-6: Volume of a Prism
The volume of a prism is the product of the area of a base and the height of the prism.

Volume of a cylinder is the product of the area of the base and the height of the cylinder.

Examples

Finding Volume of a Cylinder
Find the volume of the cylinder in terms of π.

Leave your answer in terms of π.

V = πr²h

Area of the base = πr²

Volume of the cylinder = πr²h

Find the volume of a cylinder given its radius and height.

V = πr²h

Example

Volume of a Cylinder

Radius = 3 m

Height = 4 m

V = π(3)²(4) = 36π m³

Finding Volume of a Triangular Prism
Find the volume of the triangular prism.

The prism is a triangular prism with triangular bases. The base of the triangular prism is a right triangle where one leg is the base and the other leg is the height. Use the Pythagorean Theorem to find the length of the other leg.

The area of the base is A = bh, where b is the base and h is the height.

Volume of the triangular prism = Bh

Example

Finding Volume of a Composite Figure
Find the volume of the composite space figure.

You can use three rectangular prisms to find the volume:

Volume of Prism I = 80
Volume of Prism II = 90
Volume of Prism III = 100

State the volumes:

Volume of Prism I = 80
Volume of Prism II = 90
Volume of Prism III = 100

State the sum:

Total volume = 270

The volume of the composite space figure is 270 m³.

Quick Check

1. Find the volume of the triangular prism.

2. The cylinder shown is oblique.

   a. Find its volume in terms of π.

   b. Find its volume to the nearest tenth of a cubic meter.

3. Find the volume of the composite space figure.

   a. Find its volume in terms of π.

   b. Find its volume to the nearest tenth of a cubic meter.

   The volume of the cylinder is 36π m³.
Lesson 11-5
Volumes of Pyramids and Cones

Key Concepts

Theorem 11-8: Volume of a Pyramid
The volume of a pyramid is one third the product of the base area of the base and the height of the pyramid.

Theorem 11-9: Volume of a Cone
The volume of a cone is one third the product of the area of the base and the height of the cone.

Examples

1. Finding Volume of a Pyramid
Find the volume of a square pyramid with base edges 24 m and slant height 17 m.

Step 1: Find the height of the pyramid.

Step 2: Use the formula for the volume of a pyramid.

The volume of the square pyramid is 960 m³.

2. Finding Volume of an Oblique Cone
Find the volume of the oblique cone in terms of \( \pi \) and also rounded as indicated.

The volume of the cone is \( 1280 \pi \) cubic inches, or \( 7812.6 \) cubic inches.

Lesson 11-6
Surface Areas and Volumes of Spheres

Key Concepts

Theorem 11-10: Surface Area of a Sphere
The surface area of a sphere is four times the product of \( \pi \) and the square of the radius of the sphere.

Theorem 11-11: Volume of a Sphere
The volume of a sphere is four thirds the product of \( \pi \) and the cube of the radius of the sphere.

A sphere is the set of all points in space equidistant from a given point.

The center of a sphere is the given point from which all points on the sphere are equidistant.

A great circle is the intersection of a plane and a sphere containing the center of the sphere.

A great circle divides a sphere into two congruent hemispheres.

The circumference of a sphere is the circumference of its great circle.
Vocabulary and Key Concepts.

Identifying Similar Solids

The similarity ratio of two similar solids is \( \frac{a}{b} \) and \( \frac{c}{d} \) when two similar solids have the same shape and all of their corresponding parts are proportional.

The similarity ratio of two similar solids is the ratio of their corresponding linear dimensions.

Finding Similar Solids Ratio

Example

Identifying Similar Solids: Are the two solids similar? If so, give the similarity ratio.

Both figures have the same shape. Check that the ratios of the corresponding dimensions are equal.

The ratio of the radii is 4

and the ratio of the heights is 2

The cones are similar because \( 4 \times 5 = 2 \times 10 \).

Finding the Similarity Ratio

Two similar square pyramids have volumes of 48 cm³ and 162 cm³. The surface area of the larger pyramid is 135 cm².

Find the surface area of the smaller pyramid.

Step 1: Find the similarity ratio.

\( \frac{S_2}{S_1} = \frac{135}{48} \)

Simplify.

Step 2: Use the similarity ratio to find the surface area \( S_1 \) of the smaller pyramid.

\( S_2 \) is the surface area of the larger pyramid.

\( S_1 = \frac{S_2}{\text{similarity ratio}} \)

Simplify.

Finding Surface Area

Find the surface area of a sphere with 1.

Step 1: Use the radius to find the surface area.

\( S = \frac{4}{3} \pi r^2 \)

Substitute \( r = \frac{1}{2} \) in. for \( r \).

Step 2: Simplify.

To the nearest whole number, the surface area of the rubber ball is 63 in².

Finding Volume

Find the volume to the nearest cubic inch of a sphere with diameter 60 in.

The volume of a sphere is \( V = \frac{4}{3} \pi r^3 \).

Step 1: Substitute for \( r = \frac{d}{2} \) in. for \( r \).

Step 2: Simplify.

The volume of the sphere is 4500 \( \text{in}^3 \).

Finding Volume to the Nearest Whole Number

The volume of the sphere is 4500 \( \text{in}^3 \). Give your answer in two ways, in terms of \( \pi \) and rounded to the nearest whole number.

Using a Similarity Ratio

Two similar square pyramids have volumes of 48 \( \text{cm}^3 \) and 162 \( \text{cm}^3 \). The surface area of the larger pyramid is 135 \( \text{cm}^2 \).

Find the surface area of the smaller pyramid.

Step 1: Find the similarity ratio.

Step 2: Use the similarity ratio to find the surface area \( S_1 \) of the smaller pyramid.

The surface area of the smaller pyramid is 30 \( \text{cm}^2 \).
Quick Check

1. Are the two cylinders similar? If so, give the similarity ratio.

2. Find the similarity ratio of two similar prisms with surface areas 144 m² and 324 m².

3. Are the two cylinders similar? If so, give the similarity ratio.

4. The volumes of two similar solids are 128 m³ and 250 m³. The surface area of the larger solid is 250 m². What is the surface area of the smaller solid?

Lesson 12-1 Tangent Lines

Lesson Objectives
- Use the relationship between a radius and a tangent
- Use the relationship between two tangents from one point

Vocabulary and Key Concepts

Theorem 12-1
If a line is tangent to a circle, then the line is perpendicular to the radius drawn to the point of tangency.

Theorem 12-2
If a line in the plane of a circle is perpendicular to a radius at its endpoint on the circle, then the line is tangent to the circle.

Theorem 12-3
The two segments tangent to a circle from a point outside the circle are congruent.

Examples

1. Finding a Tangent Line: \( O \) has radius 5. Point \( P \) is outside \( O \) such that \( PO = 12 \), and point \( A \) is in \( O \) such that \( PA = 13 \). Explain why \( \overline{PA} \) is not tangent to \( O \) at \( A \).

2. Circles Inscribed in Polygons: \( C \) is inscribed in quadrilateral \( XYZW \). Find the perimeter of \( XYZW \).

Quick Check

1. \( \overline{EF} \) is tangent to \( \odot O \). Find the value of \( s \).
Quick Check
1. A belt fits tightly around two circular pulleys, as shown. Find the distance between the centers of the pulleys.

\[ \text{distance} = \sqrt{14^2 + 14^2} \]

about 19.8 in.

2. In Example 3, if \( OA = 4 \), \( AP = 7 \), and \( OP = 8 \) in \( \triangle OAP \) tangent to \( \odot O \) at \( A \)? Explain.

No; 42

3. The radius of \( \odot O \) is inscribed in \( \triangle PQR \). \( \angle PQR \) is a right angle. Find \( QR \).

18 in.

4. If \( \odot O \) is in a circle, what can you conclude?

A chord is a segment whose endpoints are on a circle.

Lesson 12-2 Chords and Arcs

Lesson Objectives
- Within a circle or in congruent circles
- Congruent central angles have congruent chords.
- Congruent chords have congruent central angles.
- Congruent arcs have congruent central angles.
- Congruent arcs have congruent chords.
- Congruent chords have congruent arcs.

Vocabulary and Key Concepts
- Chord: a segment whose endpoints are on a circle.
- Central angle: an angle whose vertex is the center of the circle.
- Arc: a portion of the circle.
- Minor arc: an arc whose measure is less than 180 degrees.
- Major arc: an arc whose measure is greater than 180 degrees.
- Unequal arcs: arcs with different measures.

Examples

1. Using Theorem 12-4: In the diagram, \( \odot O \) is the definition of a perpendicular bisector.

2. Using Theorem 12-5: Find \( AP \).

3. Using Theorem 12-6: Find \( PS \).

4. Using Theorem 12-7: Find \( PQ \).

5. If \( \odot O \) is a circle, what can you conclude?

A chord is a segment whose endpoints are on a circle.

Quick Check
1. Given that \( \overline{PC} \parallel \overline{PD} \) in the circles, what can you conclude?

(1) Congruent central angles have congruent chords.
(2) Congruent chords have congruent central angles.
(3) Congruent arcs have congruent central angles.

2. Find the value of \( x \) in the circle at the right.

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3. Use the circle at the right.

a. Find the length of the chord to the nearest unit.

b. Find the distance from the midpoint of the chord to the midpoint of its minor arc.

About 11

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Lesson 12-3

**Inscribed Angles**

**Lesson Objectives:**
1. Find the measure of an inscribed angle.
2. Find the measure of an angle formed by a tangent and a chord.
3. Use the properties of tangents to find angles.

**Vocabulary and Key Concepts**

**Theorem 12-9: Inscribed Angle Theorem**
The measure of an inscribed angle is half the measure of its intercepted arc.

**Theorem 12-10**
The measure of an angle formed by a tangent and a chord is half the measure of the intercepted arc.

**Corollaries to the Inscribed Angle Theorem**
1. Two inscribed angles that intercept the same arc are congruent.
2. An angle inscribed in a semicircle is a right angle.
3. The opposite angles of a quadrilateral inscribed in a circle are supplementary.

**Examples**

1. Using the Inscribed Angle Theorem: Find the value of $x$ and $y$.

   - **Exercise A:**
     - **Example 1:**
       - **Solution:**
         - **Diagram:**
         - **Calculation:**
           - **Inscribed Angle Theorem:**
             - **Substitution:**
               - **Result:**

   - **Exercise B:**
     - **Example 2:**
       - **Solution:**
         - **Diagram:**
         - **Calculation:**
           - **Inscribed Angle Theorem:**
             - **Substitution:**
               - **Result:**

   - **Exercise C:**
     - **Example 3:**
       - **Solution:**
         - **Diagram:**
         - **Calculation:**
           - **Inscribed Angle Theorem:**
             - **Substitution:**
               - **Result:**

**Lesson 12-4

**Angle Measures and Segment Lengths**

**Lesson Objectives:**
1. Find the measure of angles formed by chords, secants, and tangents.
2. Find the lengths of segments associated with circles.

**Vocabulary and Key Concepts**

**Theorem 12-11**
The measure of an angle formed by two lines that intersect inside a circle is half the sum of the measures of the intercepted arcs.

**Theorem 12-12**
For a given point and circle, the product of the lengths of the two segments from the point to the circle is constant along any line through the point and circle.
**Lesson 12-5**

**Key Concepts**

1. **Finding Segment Lengths**
   - A train travels from point A to point B along the arc of a circle with a radius of 125 ft. Find the shortest distance from point A to point B.
   - The perpendicular bisector of the chord contains the center of the circle. Because the radius is 125 ft, the diameter is 250 ft. The length of the other segment along the diameter is 200 ft. Then the segment along the arc is 50 ft.

2. **Using the Center and a Point on a Circle**
   - The center of a circular range is at \((5, 8)\) and the radius is 15 ft.
   - The length of the arc of the "flying saucer" in Example 1, an equilateral triangle, is 8 in. long. Each side of the triangle is a chord to an arc of a circle. The basis of a design of a rotor for a Wankel engine is an equilateral triangle. Each side of the triangle is a chord to an arc of a circle. The length of the arc is 8 in. long.

3. **Writing the Equation of a Circle**
   - A diagram locates a radio tower at \((3, 5)\) and the range of the tower.
   - A diagram locates a radio tower at \((13, 5)\) and the range of the tower.
   - The center of a circle is located at \((3, 5)\) and the radius is 6 units.
   - The center of a circle is located at \((13, 5)\) and the radius is 6 units.

4. **Applying the Equation of a Circle**
   - A diagram locates a radio tower at \((10, 12)\) on a coordinate grid where each unit represents 1 mi. The radio tower’s range is 80 mi. Find an equation that describes the position and range of the tower.
   - The center of a circle is located at \((10, 12)\) and the range is 80 mi.
   - The center of a circle is located at \((10, 12)\) and the range is 80 mi.

5. **Graphing a Circle Given Its Equation**
   - Find the center and radius of the circle with equation \((x - 4)^2 + (y - 5)^2 = 25\). Then graph the circle.
   - The center is \((4, 5)\) and the radius is 5 units.

6. **Writing the Standard Equation of a Circle**
   - Find the equation of a circle with center \((3, 5)\) and radius 6 units.
   - Find the equation of a circle with center \((13, 5)\) and radius 6 units.
   - Find the equation of a circle with center \((3, 5)\) and radius 6 units.
   - Find the equation of a circle with center \((13, 5)\) and radius 6 units.

7. **Critical Thinking**
   - To photograph a 100° arc of the "flying saucer" in Example 1, should you move toward or away from the object? What angle should the tangent form?

8. **Arc Measures**
   - An advertising agency wants a frontal photo of a "flying saucer" (see Example 1) at an amusement park. The photographer stands at the vertex of the angle formed by tangents to the "flying saucer." What is the measure of the arc that will be in the foreground? In the diagram, the photographer stands at point \(T\). The arc \(AY\) is an arc of the "flying saucer" in Example 1.
   - Let \(m\angle TXY = \alpha\).
   - Then \(m\angle TXY = 360° - 2\alpha\).
   - What is the measure of the arc that will be in the foreground?

**Quick Check**

1. Find the value of \(w\).
   - Then \(m\angle TXY = 360° - 2\alpha\).
   - Substitute to find \(\alpha\).
   - Then find the standard equation of the circle with center \((5, 8)\) and radius 12 ft.
   - Substitute to find \(r\).

2. **Critical Thinking**
   - To photograph a 100° arc of the "flying saucer" in Example 1, should you move toward or away from the object? What angle should the tangent form?

3. Find the value of \(x\) to the nearest tenth.
   - Use standard form.
   - Substitute to find \(r\).
   - Substitute to find \(r\).

4. **Arc Lengths**
   - The basis of a design of a rotor for a Wankel engine is an equilateral triangle. Each side of the triangle is a chord to an arc of a circle. The length of the arc is 8 in. long. Each side of the triangle is a chord to an arc of a circle. The basis of a design of a rotor for a Wankel engine is an equilateral triangle. Each side of the triangle is a chord to an arc of a circle.

**Steps for Finding Arc Measures**

1. Find the length of the arc.
2. Use standard form.
3. Substitute to find \(r\).
4. Substitute to find \(r\).

**Example**

- **Finding Arc Measures**
  - An advertising agency wants a frontal photo of a "flying saucer" (see Example 1) at an amusement park. The photographer stands at the vertex of the angle formed by tangents to the "flying saucer." What is the measure of the arc that will be in the foreground? In the diagram, the photographer stands at point \(T\). The arc \(AY\) is an arc of the "flying saucer" in Example 1.
  - Let \(m\angle TXY = \alpha\).
  - Then \(m\angle TXY = 360° - 2\alpha\).
  - What is the measure of the arc that will be in the foreground?
3. Find the center and radius of the circle with equation 
\[(x - 2)^2 + (y - 3)^2 = 100.\]
Then graph the circle.

4. When you make a call on a cellular phone, a tower receives the call. In the diagram, imagine plane \(P\) is a tower and plane \(Q\) is a cellular telephone tower. Write an equation that describes the positions and ranges of towers \(O\) and \(Q\) that are 4 cm from plane \(M\). The set of points in a plane that are 1.5 cm from plane \(M\) is a circle with center \(O\) and radius 1.5 cm. The set of points in the interior of \(O\) that are 1.5 cm from plane \(M\) is a circle with center \(O\) and radius 1.5 cm. The set of points exterior to \(O\) that are 1.5 cm from plane \(M\) is a circle with center \(O\) and radius 1.5 cm. The locus of points in a plane parallel to and equidistant from each of two parallel lines is a line midway between the two parallel lines, equidistant from each. The locus of points in a plane parallel to, but not equidistant from, each of two parallel lines is a plane midway between the two parallel lines, parallel to and equidistant from each.
Chapter 1

Practice 1-1
1. 47, 53  2. 42, 54  3. –64, 128  4. Sample: 2 or 3  5. 6 or 8  6. Y or A  7. any hexagon  8. a 168.75° angle  9. 34  10. Sample: The farther out you go, the closer the ratio gets to a number that is approximately 0.618.

Guided Problem Solving 1-1
1. The pattern is easier to visualize.  2. The graph will go up.  3. Use Years for the horizontal axis.  4. Use Number of Stations for the vertical axis.  5. increasing  6. greater
7. Yes. Since the number of stations increases steadily from 1950 to 2000, we can be confident that the number of stations in 2010 will be greater than in 2000.  8. Patterns are necessary to reach a conclusion through inductive reasoning.
9. (any list of numbers without a pattern would apply) 2, 435; 16,439; 16,454; 3,765; 210,564

Practice 1-2
1. They represent three-dimensional objects on a two-dimensional surface.
2. The graph will go up.  3. Use Years for the horizontal axis.
4. Use Number of Stations for the vertical axis.
5. increasing  6. greater
7. Yes. Since the number of stations increases steadily from 1950 to 2000, we can be confident that the number of stations in 2010 will be greater than in 2000.
8. Patterns are necessary to reach a conclusion through inductive reasoning.
9. (any list of numbers without a pattern would apply) 2, 435; 16,439; 16,454; 3,765; 210,564

Guided Problem Solving 1-2
1. They represent three-dimensional objects on a two-dimensional surface.  2. nine  3. See the figure in 4, below.
4.  5. Yes. It is similar to the foundation drawing, except there are no numbers.
6. no
7.  8. Yes.
9. (any list of numbers without a pattern would apply) 2, 435; 16,439; 16,454; 3,765; 210,564

Practice 1-3
1. any two of the following: $ABD, DBC, CBE, ABE, ECD, ADE, ACE, ACD$
2. yes  3. no
4. yes  5. no
6. yes  7. yes  8. yes  9. G
10. $LM$  11. the empty set
12. $KP$  13. Sample: plane $ABD$
14. $AB$  15. no
16. yes  17. the empty set  18. no
19. yes  20. yes

Guided Problem Solving 1-3
1. Collinear points lie on the same line.  2. Answers may vary. (Some people might note that the $y$-coordinate of two of the points is the same so that the third point must have the same $y$-coordinate to be collinear. Since it does not, the points are not collinear.)
3. horizontal  4. No.  5. No.  6. All points must have the same $y$-coordinate, –3.
7. No.  8. $1, \frac{1}{2}$

Practice 1-4
1. true  2. false  3. false  4. false  5. $JK$, $HG$  6. any three of the following pairs: $EF$ and $FH$; $EF$ and $GK$; $HG$ and $IE$, $HG$ and $FK$, $JK$ and $EH$; $JK$ and $FG$, $EF$ and $FG$; $EH$ and $FK$; $JE$ and $KG$, $EH$ and $KG$, $JH$ and $KF$; $JH$ and $GE$
7. planes $A$ and $B$  8. planes $A$ and $C$
9. Sample: \( \overrightarrow{EG} \) 10. \( \overrightarrow{EF} \) and \( \overrightarrow{ED} \) or \( \overrightarrow{EG} \) and \( \overrightarrow{ED} \) 11. \( FE, FD \) 12. yes 13. Sample:  

14. Sample:

Guided Problem Solving 1-4
1. Opposite rays are two collinear rays with the same endpoint. 2. a line 3–4. See graph in Exercise 5 answer. 5. Answers may vary. Sample: (0,0) (Answers will be coordinates \((x, y)\), where \(y = \frac{2}{3}x, x < 2\).)

Practice 1-5
1. 4 2. 12 3. 20 4. 6 5. 22 6. -3, 4 7. no 8. -2 9. 11 10. 29 11. 29

Guided Problem Solving 1-5
1. \( AD \equiv DC \) 2. \( AD = DC \) 3. Segment Addition Postulate. 4. Since \( AD = DC, AC = 2(AD) \). 5. \( AC = 2(12) = 24 \) 6. \( y = 15 \) 7. \( DC = AD = 12 \) 8. Answers may vary. 9. \( ED = 11, DB = 11, EB = 22 \)

Practice 1-6
1. any three of the following: \( \angle O, \angle MOP, \angle POM, \angle 1 \) 2. \( \angle AOB \) 3. \( \angle EOC \) 4. \( \angle DOC \) 5. 51 6. 90 7. 141 8. 68 9. \( \angle ABD, \angle DBE, \angle EBF, \angle DBF, \angle FBC \) 10. \( \angle ABE, \angle EBC \)

Guided Problem Solving 1-6
1. Angle Addition Postulate 2. supplementary angles 3. \( m\angle RQS + m\angle TQS = 180 \) 4. \( (2x + 4) + (6x + 20) = 180 \) 5. \( x = 19.5 \) 6. \( m\angle RQS = 43; m\angle TQS = 137 \) 7. The sum of the angle measures should be 180; \( m\angle RQS + m\angle TQS = 43 + 137 = 180 \). 8a. \( x = 11 \) 8b. \( m\angle AOB = 17; m\angle COB = 73 \)

Practice 1-7
1. 

2. 

3. 

4. 

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5. \( \angle 2 \equiv \angle 1 \)

6. \( 2 \cdot \angle 2 \)

7. \( 2 \cdot \angle X \)

9. true  
10. false  
11. false  
12. true

**Guided Problem Solving 1-7**

1. \( \angle DBC \equiv \angle ABC \)
2. complementary angles
3. \( \angle CBD \)
4. \( m \angle CBD = m \angle CBA = 41 \)
5. \( m \angle ABD = m \angle CBA + m \angle CBD = 41 + 41 = 82 \)
6. \( m \angle ABE + m \angle CBA = 90 \)
   \( m \angle ABE + 41 = 90 \)
   \( m \angle ABE = 49 \)
7. \( m \angle DBF = m \angle ABE = 49 \)
8. Answers may vary. Sample: The sum of the measures of the complementary angles should be 90 and the sum of the measures of the supplementary angles should be 180.
9. \( m \angle CBD = 21, m \angle FBD = 69, m \angle CBA = 21, \) and \( m \angle EBA = 69 \)

**Practice 1-8**

1.-5.

6. \( \angle DBF \equiv \angle ABE = 49 \)
7. \( \angle X \)
8. Answers may vary. Sample: The sum of the measures of the complementary angles should be 90 and the sum of the measures of the supplementary angles should be 180.
9. \( m \angle CBD = 21, m \angle FBD = 69, m \angle CBA = 21, \) and \( m \angle EBA = 69 \)

**Guided Problem Solving 1-8**

1. Distance Formula  
2. The distance \( d \) between two points \( A(x_1, y_1) \) to \( B(x_2, y_2) \) is \( d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \)
3. No; the differences are opposites but the squares of the differences are the same.
4. \( XY = \sqrt{(5 - (-6))^2 + (-2 - 9)^2} \)
5. To the nearest tenth, \( XY = 15.6 \) units.  
6. To the nearest tenth, \( XZ = 12.0 \) units.  
7. \( Z \) is closer to \( X \).  
8. The results are the same; e.g., \( XY = \sqrt{(-6 - 5)^2 + (9 - (-2))^2} = \sqrt{242} \), or about 15.6 units, as before.  
9. \( YZ = \sqrt{(17 - (-6))^2 + (-3 - 9)^2} = \sqrt{673} \); to the nearest tenth, \( YZ = 25.9 \) units. To the nearest tenth, \( XY + YZ + XZ = 53.5 \) units.
Practice 1-9
1. 792 in.²  2. 2.4 m²  3. 16π  4. 7.8π  5. 26 cm; 42 cm²
6. 29 in.; 42 in.²  7. 30 m; 99 m²  8. 26; 22  9. 30; 44
10. 156.25π  11. 10,000π  12. 36  13. 26; 13

Guided Problem Solving 1-9
1. six  2. It is a two-dimensional pattern you can fold to form a three-dimensional object. 3. rectangles
4.

5. 208 in.²  6. They are equal. 7. 208 in.²  8. Answers will vary. Sample; 2(4 • 6) + 2(4 • 8) + 2(6 • 8); the results are the same, 208 in.²  9. 6(7²) = 294 in.²

1A: Graphic Organizer
1. Tools of Geometry  2. Answers may vary. Sample: patterns and inductive reasoning; measuring segments and angles; basic constructions; and the coordinate plane  3. Check students’ work.

1B: Reading Comprehension
1. Answer may vary. Sample: \( AB \parallel CD, EF \parallel GH \), \( JK \parallel TM, \angle JKL = \angle TKM, m\angle JKF + m\angle FJK = 180° \), \( \angle HKB \equiv \angle KMD \). 2. Points A, M, and S are collinear. 3. \( AB, HT \), and \( LN \) intersect at point M.

1C: Reading/Writing Math Symbols
1. Line \( BC \) is parallel to line \( MN \). 2. Line \( CD \)
3. Line segment \( GH \)  4. Ray \( AB \)  5. The length of segment \( XY \) is greater than the length of segment \( ST \).
6. \( MN = XY \)  7. \( GH = 2(KL) \)  8. \( ST \perp UV \)
9. plane \( ABC \parallel \) plane \( XYZ \)  10. \( AB \parallel DE \)

1D: Visual Vocabulary Practice

1E: Vocabulary Check
Net: A two-dimensional pattern that you can fold to form a three-dimensional figure.
Conjecture: A conclusion reached using inductive reasoning.
Collinear points: Points that lie on the same line.
Midpoint: A point that divides a line segment into two congruent segments.
Postulate: An accepted statement of fact.

1F: Vocabulary Review Puzzle
Chapter 2

Practice 2-1
1. Sample: It is 12:00 noon on a rainy day. 2. Sample: 6 3. If you are strong, then you drink milk. 4. If a rectangle is a square, then it has four sides the same length. 5. If \( x = 26 \), then \( x - 4 = 22 \); true. 6. If \( m \) is positive, then \( m^2 \) is positive; true. 7. If lines are parallel, then their slopes are equal; true. 8. Hypothesis: If you like to shop; conclusion: Visit Pigeon Forge outlets in Tennessee. 9. Hypothesis: If you visit Pigeon Forge outlets, then you like to shop. 10. Drinking Sustain makes you train harder and run faster. 11. If you drink Sustain, then you will train harder and run faster. 12. If you train harder and run faster, then you drink Sustain.

Guided Problem Solving 2-1
1. Hypothesis: \( x \) is an integer divisible by 3. 2. Conclusion: \( x^2 \) is an integer divisible by 3. 3. Yes, it is true. Since 3 is a factor of \( x \), it must be a factor of \( x \cdot x = x^2 \). 4. If \( x^2 \) is an integer divisible by 3 then \( x \) is an integer divisible by 3. 5. The converse is false. Counterexamples may vary. Let \( x^2 = 3 \). Then \( x = \sqrt{3} \), which is not an integer and is not divisible by 3. 6. No. The conditional is true, so there is no such counterexample. 7. No. By definition, a general statement is false if a counterexample can be provided. 8. If \( 5x + 3 = 23 \), then \( x = 4 \). The original statement and the converse are both true.

Practice 2-2
1. Two angles have the same measure if and only if they are congruent. 2. The converse, “If \( |n| = 17 \), then \( n = 17 \),” is not true. 3. If a whole number is a multiple of 5, then its last digit is either 0 or 5. If a whole number has a last digit of 0 or 5, then it is a multiple of 5. 4. If two lines are perpendicular, then the lines form four right angles. If two lines form four right angles, then the lines are perpendicular. 5. Sample: Other vehicles, such as trucks, fit this description. 6. Sample: Baseball also fits this definition. 7. Sample: Pleasing, smooth, and rigid all are too vague. 8. yes 9. no 10. yes

Guided Problem Solving 2-2
1. A good definition is clearly understood, precise, and reversible. 2. \( \angle 3 \) and \( \angle 4 \), \( \angle 5 \) and \( \angle 6 \). 3. No 4. They are not supplementary. 5. A linear pair has a common vertex, shares a common side, and is supplementary. 6. yes 7. yes; yes 8. linear pairs: \( \angle 1 \) and \( \angle 2 \), \( \angle 3 \) and \( \angle 4 \); not linear pairs: \( \angle 1 \) and \( \angle 3 \), \( \angle 1 \) and \( \angle 4 \), \( \angle 2 \) and \( \angle 3 \), \( \angle 2 \) and \( \angle 4 \). 9. They are adjacent complementary angles.

Practice 2-3
1. Football practice is canceled for Monday. 2. \( \triangle DEF \) is a right triangle. 3. If two lines are not parallel, then they intersect at a point. 4. If you vacation at the beach, then you like Florida. 5. Tamika lives in Nebraska. 6. not possible 7. It is not freezing outside. 8. Shannon lives in the smallest state in the United States.

Guided Problem Solving 2-3
1. conditional; hypothesis  2. Yes  3. Beth will go. 4. Anita, Beth, Aisha, Ramon  5. No; only two students went. 6. Beth, Aisha, Ramon; no—only two went. 7. Aisha, Ramon 8. The answer is reasonable. It is not possible for another pair to go to the concert. 9. Ramon

Practice 2-4
1. \( UT = MN \)  2. \( y = 51 \)  3. \( PMQ \)  4. Addition; Subtraction Property of Equality; Multiplication Property of Equality; Division Property of Equality 5. Substitution 6. Substitution 7. Symmetric Property of Congruence 8. Definition of Complementary Angles; 90; Substitution; 3x, Simplify; 3x, 84, Subtraction Property of Equality; 28, Division Property of Equality

Guided Problem Solving 2-4
1. Angle Addition Postulate 2. Substitution Property of Equality; Simplify; Addition Property of Equality; Division Property of Equality 3. 40 4. yes; yes 5. 13; 13

Practice 2-5
1. 30 2. 15 3. 6 4. \( m \angle A = 135; m \angle B = 45 \) 5. \( m \angle A = 10; m \angle B = 80 \) 6. \( m \angle PMO = 55 \) 7. \( m \angle BWC = m \angle CWD, m \angle AWB + m \angle BWC = 180; m \angle CWD + m \angle DWA = 180; m \angle AWB = m \angle AWD \)

Guided Problem Solving 2-5
1. 90 2. See graph in Exercise 5 answer. 3. on the positive y-axis 4. Answers may vary. \( B \) can be any point on the positive y-axis. Sample: \( B(0, 3) \).

Guided Problem Solving 2-5
1. 90 2. See graph in Exercise 5 answer. 3. on the positive y-axis 4. Answers may vary. \( B \) can be any point on the positive y-axis. Sample: \( B(0, 3) \). 5. 6. They are adjacent complementary angles. 7. Answers may vary. \( C \) can be any point on the line \( y = -\frac{1}{2}x, x > 0 \). Sample: \( C(3, -1) \). 8. a right angle 9. Yes; their sum corresponds to the right angle formed by the positive x-axis and the positive y-axis. 10. Answers may vary. \( D \) can be any point on the negative x-axis, sample: \( D(-4, 0) \).
2A: Graphic Organizer
1. Reasoning and Proof 2. Answers may vary. Sample: conditional statements; writing biconditionals; converses; and using the Law of Detachment 3. Check students’ work.

2B: Reading Comprehension
1. 42 degrees 2. 38 degrees 3. b

2C: Reading/Writing Math Symbols
1. Segment $MN$ is congruent to segment $PQ$. 2. If $p$, then $q$. 3. The length of $MN$ is equal to the length of $PQ$. 4. Angle $XQV$ is congruent to angle $RDC$. 5. If $q$, then $p$. 6. The measure of angle $XQV$ is equal to the measure of angle $RDC$. 7. $p$ if and only if $q$. 8. $a \rightarrow b$ 9. $AB = MN$ 10. $m\angle XYZ = m\angle RPS$ 11. $b \rightarrow a$ 12. $AB \equiv MN$ 13. $a \leftrightarrow b$ 14. $\angle XYZ \equiv \angle RPS$

2D: Visual Vocabulary Practice

2E: Vocabulary Check
Truth value: “True” or “false” according to whether the statement is true or false, respectively
Hypothesis: The part that follows if in an if-then statement.
Biconditional: The combination of a conditional statement and its converse; it contains the words “if and only if.”
Conclusion: The part of an if-then statement that follows then.
Conditional: An if-then statement.

2F: Vocabulary Review Puzzle

Guided Problem Solving 3-1
1. The top and bottom sides are parallel, and the left and right sides are parallel. 2. The two diagonals are transversals, and also each side of the parallelogram is a transversal for the two sides adjacent to it. 3. Corresponding angles, interior and exterior angles are formed. 4. $v$, $w$ and $y$; By the Alternate Interior Angles Theorem, $v = 42$, $w = 25$ and $x = 76$. 5. Answers may vary. Possible answer: By the Same-Side Interior Angles Theorem, $(w + 42) + (y + 76) = 180$. Since $w = 25$, $y = 37$. (The two $y$’s are equal by Theorem 3-1.) 6. $w = 25$, $y = 37$, $v = 42$, $x = 76$; yes 7. $v = 42$, $w = 35$, $x = 57$, $y = 46$

Chapter 3

Practice 3-1
1. corresponding angles 2. alternate interior angles 3. same-side interior angles 4. $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 8$, $\angle 4$ and $\angle 7$ 5. $\angle 4$ and $\angle 6$, $\angle 3$ and $\angle 5$ 6. $\angle 4$ and $\angle 5$, $\angle 3$ and $\angle 6$ 7. $m\angle 1 = 100$, alternate interior angles; $m\angle 2 = 100$, corresponding angles or vertical angles 8. $m\angle 1 = 135$, corresponding angles; $m\angle 2 = 135$, vertical angles 9. $x = 103; 77^\circ$, $103^\circ$ 10. $x = 30; 85^\circ$, $85^\circ$
Practice 3-2
1.  \( l \) and \( m \), Converse of Same-Side Interior Angles Theorem
2.  none  3.  \( BC \) and \( AD \), Converse of Same-Side Interior Angles Theorem  4.  \( BH \) and \( CT \), Converse of Corresponding Angles Postulate  5.  43  6.  90  7.  38  8.  100

Guided Problem Solving 3-2
1.  \( \ell \) and \( m \)  2.  transversals  3.  \( x \)  4.  the angles measuring 19\( \degree \) and 17\( \degree \)  5.  180\( \degree \)  6.  17\( x \)  7.  180 - 19\( x \) = 17\( x \) or 19\( x \)+17\( x \) = 180  8.  \( x = 5 \)  9.  With \( x = 5 \), 19\( x \) = 95 and 17\( x \) = 85.  10.  \( x = 6 \)

Practice 3-3
1.  True. Every avenue will be parallel to Founders Avenue, and therefore every avenue will be perpendicular to any street that is parallel to Center City Boulevard.  2.  True. The fact that one intersection is perpendicular, plus the fact that every street belongs to one of two groups of parallel streets, is enough to guarantee that all intersections are perpendicular.  3.  Not necessarily true. If there are more than three avenues and more than three boulevards, there will be some blocks bordered by neither Center City Boulevard nor Founders Avenue.  4.  \( a \perp e \)  5.  \( a \parallel e \)  6.  \( a \parallel e \)  7.  \( a \parallel e \)  8.  If the number of \( \perp \) statements is even, then \( \perp_1 \parallel \perp_n \). If it is odd, then \( \perp_1 \perp \perp_n \).

Guided Problem Solving 3-3
1.  supplementary angles  2.  right angle  3.  Any one of the following: Postulate 3-1, or Theorem 3-1, 3-2, 3-3 or 3-4  4.  90  5.  It is congruent; Postulate 3-1  6.  90  7.  \( a \perp e \)  8.  It is true for any line parallel to \( b \).  9.  Yes. The point is that a transversal cannot be perpendicular to just one of two parallel lines. It has to be perpendicular to both, or else to neither.

Practice 3-4
1.  125  2.  143  3.  129  4.  136  5.  \( x = 35; y = 145; z = 25 \)  6.  \( v = 118; w = 37; t = 62 \)  7.  50  8.  88  9.  \( m\angle 1 = 33; m\angle 2 = 52 \)  10.  right scalene  11.  obtuse isosceles  12.  equiangular equilateral

Guided Problem Solving 3-4
1.  three  2.  180  3.  right triangle  4.  \( z = 90 \); Because it is given in the figure that \( BD \perp AC \).  5.  Theorem 3-12, the Triangle Angle-Sum Theorem  6.  \( x = 38 \)  7.  \( y = 36 \)  8.  \( \triangle ABD \) is a 36-54-90 right triangle. \( \triangle BCD \) is a 38-52-90 right triangle.  9.  74  10.  \( \triangle ABC \) is a 52-54-74 acute triangle.  11.  Yes, all three are acute angles, with \( \angle ABC \) visibly larger than \( \angle A \) and \( \angle C \).  12.  \( \angle BCD \)

Practice 3-5
1.  \( x = 120; y = 60 \)  2.  \( n = 51\frac{1}{2} \)  3.  \( a = 108; b = 72 \)  4.  109  5.  133  6.  129  7.  30  8.  150  9.  6  10.  5  11.  \( \triangle BDE \)  12.  \( \angle FAE \)  13.  \( \angle FAE \) and \( \angle BAE \)  14.  \( \triangle ABC \)

Guided Problem Solving 3-5
1.  A theater stage, consisting of a large platform surrounding a smaller platform. The shapes in the bottom part of the figure may represent a ramp for actors to enter and exit.  2.  The measures of angles 1 and 2.  3.  octagon  4.  \( (8 - 2)180 = 1080 \) degrees  5.  135  6.  45  7.  Yes, angle 1 is an obtuse angle and angle 2 is an acute angle.

Practice 3-6
1.  \( y = \frac{1}{3}x - 7 \)  2.  \( y = -2x + 12 \)  3.  \( y = \frac{4}{3}x - 2 \)  4.  \( y = 4x - 13 \)

6.

7.

8.  \( y = x + 4 \)  9.  \( y = \frac{1}{2}x - 3 \)  10.  \( y = \frac{1}{2}x - \frac{1}{2} \)  11.  \( y = -6x + 45 \)  12.  \( y = -11; x = 2 \)  13.  \( y = 2; x = 0 \)  14.  \( (-2, 0) \)  15.  \( (0, -1) \)  16.  \( (-2, 0) \)  17.  \( (0, -1) \)
Guided Problem Solving 3-7

1. 

2. $m_1 \cdot m_2 = -1$

3. $\overrightarrow{GH} \parallel \overrightarrow{GK}$

4. Sides $\overrightarrow{GH}$ and $\overrightarrow{GK}$ are not perpendicular.

5. $\triangle GHK$ has no pair of perpendicular sides. It is not a right triangle.

6. $\angle H G K$; approximately 80

7. Slope of $\overrightarrow{LM} = \frac{7}{2}$ and slope of $\overrightarrow{LN} = -\frac{2}{7}$. The product of the slopes is $-1$, so $\overrightarrow{LM}$ and $\overrightarrow{LN}$ are perpendicular.

Practice 3-8

1. 2.

3. Sample:

4. 

Guided Problem Solving 3-6

1. 

2. $m = \frac{y_2 - y_1}{x_2 - x_1}$

3. $y - y_1 = m(x - x_1)$

4. Slope of $\overrightarrow{AB} = \frac{5}{2}$, slope of $\overrightarrow{BC} = -\frac{5}{2}$. The absolute values of the slopes are the same, but one slope is positive and the other is negative.

5. Point-slope form: $y - 0 = \frac{5}{2}(x - 0)$

6. Point-slope form: $y - 5 = -\frac{5}{2}(x - 2)$ or $y - 0 = -\frac{5}{2}(x - 4)$

7. Of line $\overrightarrow{AB}$: $(0, 0)$

8. $\triangle ABC$ appears to be an isosceles triangle, which is consistent with a horizontal base and two remaining sides having slopes of equal magnitude and opposite sign.

9. Slope = 0; $y = 0$; $y$-intercept = $(0, 0)$ just as for line $\overrightarrow{AB}$ (they intersect on the $y$-axis).

Practice 3-7

1. neither; $3 \neq \frac{1}{3}$, $3 \cdot \frac{1}{3} \neq -1$

2. perpendicular;

$\frac{1}{2} \cdot -2 = -1$

3. parallel; $-\frac{2}{3} = -\frac{2}{3}$

4. perpendicular;

$y = 2$ is a horizontal line, $x = 0$ is a vertical line

5. perpendicular; $-1 \cdot 1 = -1$

6. neither; $\frac{1}{2} \neq -\frac{5}{2}$

7. neither; $\frac{1}{2} \neq \frac{9}{2}$, $\frac{4}{7} \neq -1$

8. parallel;

$\frac{1}{2} \cdot -2 \neq -\frac{5}{2}$

9. $y = \frac{2}{3}x$

10. $y = 2x - 4$
Guided Problem Solving 3-8
1. a line segment of length \(c\)
2. Construct a quadrilateral with one pair of parallel sides of length \(c\), and then examine the other pair.  
3. The procedure is given on p. 181 of the text.
4. Adjust the compass to exactly span line segment \(c\), end to end. Then tighten down the compass adjustment as necessary.
5. They appear to be both congruent and parallel.  
6. The answers to Step 7 are confirmed.

3A: Graphic Organizer
1. Parallel and Perpendicular Lines  
2. Answers may vary. Sample: properties of parallel lines; finding the measures of angles in triangles; classifying polygons; and graphing lines  
3. Check students’ work.

3B: Reading Comprehension
1. 14 spaces  
2. 6 spaces  
3. 60°  
4. corresponding  
5. $7000; $480  
6. the width of the stalls, 10 ft  
7. b

3C: Reading/Writing Math Symbols
1. \(m \perp n\)
2. \(m \angle 1 + m \angle 2 = 180\)
3. \(\overrightarrow{AB} \parallel \overrightarrow{CD}\)
4. \(m \angle MNP + m \angle MNQ = 90\)
5. \(\angle 3 \equiv \angle EFD\)
6. Line 1 is parallel to line 2.  
7. The measure of angle \(ABC\) is equal to the measure of angle \(XYZ\).  
8. Line \(AB\) is perpendicular to line \(DF\).  
9. Angle \(ABC\) and angle \(ABD\) are complementary.  
10. Angle 2 is a right angle, or the measure of angle 2 is 90°.  
11. Sample answer:
\(\overrightarrow{CB}, m \angle BAF = m \angle GFA\)

3D: Visual Vocabulary Practice/High-Use Academic Words
1. property
2. conclusion
3. describe
4. formula
5. measure
6. approximate
7. compare
8. contradiction
9. pattern

3E: Vocabulary Check
Transversal: A line that intersects two coplanar lines in two points.
Alternate interior angles: Nonadjacent interior angles that lie on opposite sides of the transversal.
Same-side interior angles: Interior angles that lie on the same side of a transversal between two lines.
Corresponding angles: Angles that lie on the same side of a transversal between two lines in corresponding positions.
Flow proof: A convincing argument that uses deductive reasoning, in which arrows show the logical connections between the statements.

3F: Vocabulary Review
### Chapter 4

#### Practice 4-1

1. \( m\angle 1 = 110; m\angle 2 = 120 \)
2. \( m\angle 3 = 90; m\angle 4 = 135 \)
3. \( \overline{CA} \cong \overline{TS}, \overline{AT} \cong \overline{ST}, \overline{CT} \cong \overline{TJ} \)
4. \( \angle C \cong \angle J \)
5. Yes; \( \triangle GHJ \cong \triangle IHJ \) by Theorem 4-1 and the Reflexive Property of \( \cong \). Therefore, \( \triangle GHJ \cong \triangle IHJ \) by the definition of \( \cong \) triangles.
6. No; \( \triangle QRS \cong \triangle TSV \) because vertical angles are congruent, and \( \triangle QRS \cong \triangle TSV \) by Theorem 4-1, but none of the sides are necessarily congruent. 7a. Given 7b. Vertical angles are \( \cong \). 7c. Theorem 4-1 7d. Given 7e. Definition of \( \cong \) triangles

#### Guided Problem Solving 4-1

1. Right triangles 2. \( m\angle A = 45, m\angle B = m\angle L = 90 \), and \( AB = 4 \) in. 3. \( \triangle ABC \) is congruent to \( \triangle KLM \) means corresponding sides and angles are congruent. 4. \( x \) and \( t \)
5. \( m\angle K = m\angle M = 45 \)
6. \( m\angle A = m\angle B \)

#### Guided Problem Solving 4-2

1. \( \triangle ADB \cong \triangle CDB \) by SAS 2. not possible 3. \( \triangle TUS \cong \triangle XWY \) by SSA. not possible 5. \( \triangle DEC \cong \triangle GHF \) by SAS 6. \( \triangle PRN \cong \triangle PRQ \) by SSA 7. \( \angle C \cong \angle B \)
8. \( \overline{AB} \cong \overline{BC} \)
9. \( \angle A \cong \angle B \)
10a. Given 11a. Reflexive Property of Congruence 11c. ASA Postulate

#### Guided Problem Solving 4-3

1. \( \overline{TSO} \) and \( \overline{ST} \) bisect \( \angle ISO \). 2. Prove whatever additional facts can be proven about \( \triangle ISP \) and \( \triangle OSP \), based on the given information. 3. \( \overline{TS} \cong \overline{SO} \)
4. \( \triangle ISP \cong \triangle PSO \)
5. \( \triangle ISP \cong \triangle OSP \) by Postulate 4-2, the Side-Angle-Side (SAS) Postulate 7. It does not matter. The Side-Angle-Side Postulate applies whether or not they are collinear.
8. It does follow; because \( \triangle ISP \cong \triangle OSP \) and because \( \overline{TP} \) and \( \overline{PQ} \) are corresponding parts.

#### Practice 4-3

1. not possible 2. ASA Postulate 3. AAS Theorem 4. ASA Postulate 5. not possible 6. AAS Theorem 7. Statements Reasons
1. \( \angle K \cong \angle M \)
2. \( \angle JLK \cong \angle PLM \)
3. \( \triangle KJM \cong \triangle PML \)
8. \( \overline{BC} \cong \overline{EF} \)
9. \( \angle KJH \cong \angle HKG \) or \( \angle KJH \cong \angle HGK \)

#### Guided Problem Solving 4-3

1. Corresponding angles and alternate interior angles. 2. \( \angle EAB \) and \( \angle DBC \)
3. \( \angle EBA \) and \( \angle DCB \)
4. \( \angle EAB \) and \( \angle DBC \)
5. \( \angle EAB \equiv \angle DBC, \overline{AE} \equiv \overline{BD}, \) and \( \angle E \equiv \angle D \)
6. \( \angle EAB \equiv \angle DBC \) by Postulate 4-3, the Angle-Side-Angle (ASA) Postulate 7. Yes; Theorem 4-2, the Angle-Side-Angle (AAS) Theorem; \( \angle EBA \equiv \angle DCB \).
8. No, because now there is no way to demonstrate a second pair of congruent sides, nor a second pair of congruent angles.

#### Practice 4-4

1. \( \overline{BD} \) is a common side, so \( \triangle ADB \cong \triangle CDB \) by SAS, and \( \angle A \equiv \angle C \) by CPCTC. 2. \( \overline{FH} \) is a common side, so \( \triangle FHE \cong \triangle HFG \) by ASA, and \( \overline{HE} \equiv \overline{FG} \) by CPCTC.
3. \( \overline{QS} \) is a common side, so \( \triangle QTS \cong \triangle SRQ \) by AAS. \( \angle QST \equiv \angle QSR \) by CPCTC. 4. \( \angle ZAY \) and \( \angle CAB \) are vertical angles, so \( \angle ABC \equiv \angle AYZ \) by ASA, \( \angle ZA \equiv \angle AC \) by CPCTC. 5. \( \angle HKJ \) and \( \angle LJM \) are vertical angles, so \( \angle HKJ \cong \angle LJM \) by CPCTC. 6. \( \overline{PR} \) is a common side, so \( \triangle PRN \cong \triangle RQP \) by SSS, and \( \overline{PQ} \equiv \overline{PR} \) by CPCTC. 7. Yes; Theorem 4-1 7d. Given 8. \( \triangle ABC \) and \( \triangle ECD \) are vertical angles. Then, show \( \angle ABC \equiv \angle EDC \) by ASA. Last, show \( \angle A \equiv \angle E \) by CPCTC.

#### Guided Problem Solving 4-4

1. A compass with a fixed setting was used to draw two circular arcs, both centered at point \( P \) but crossing \( \ell \) in different locations, which were labeled \( A \) and \( B \). The compass was used again, with a wider setting, to draw two intersecting circular arcs, one centered at \( A \) and one at \( B \). The point at which the new arcs intersected was labeled \( C \). Finally, line \( \overline{CP} \) was drawn. 2. Find equal lengths or distances and explain why \( \overline{CP} \) is perpendicular to \( \ell \). 3. \( \triangle AC \) and \( \triangle BC \)
4. \( AP = PB \) and \( AC = BC \)
5. \( \triangle APC \equiv \triangle BPC \) by Postulate 4-1, the Side-Side-Side (SSS) Postulate 6. \( \triangle APC \equiv \triangle BPC \) by CPCTC. 7. Since \( \triangle APC \equiv \triangle BPC \), \( m\angle APC = m\angle BPC \) and \( m\angle APC + m\angle BPC = 180 \), it follows that \( m\angle APC = m\angle BPC = 90 \). 8. From the definition of perpendicular and the fact that \( m\angle APC = m\angle BPC = 90 \), the distances do not matter, so long as \( AP = PB \) and \( AC = BC \). That is what is required in order that \( \triangle APC \equiv \triangle BPC \). 9. Draw a line, and use the construction technique of the problem to construct a second line perpendicular to the first. Then do the same thing again to construct a third line perpendicular to the second line. The first and third lines will be parallel, by Theorem 3-10.

#### Practice 4-5

1. \( x = 35; y = 35 \)
2. \( x = 80; y = 90 \)
3. \( t = 150 \)
4. \( x = 55; y = 70; \angle z = 125 \)
5. \( x = 6 \)
6. \( z = 120 \)
7. \( \overline{AD} \equiv \angle A \)
8. \( \overline{AT} \equiv \angle B \)
9. \( \overline{AB} \equiv \angle AJB \)
10. \( \overline{AD} \equiv \angle AJB \)
11. \( \overline{AD} \)
12. \( x = 70; y = 55 \)

#### Guided Problem Solving 4-5

1. One angle is obtuse. The other two angles are acute and congruent. 2. Highlight an obtuse isosceles angle and find its angle measures, then find all the other angle measures represented in the figure.
3. Possible answer:

4. 60  5. 30, because the measure of each base angle is half the measure of an angle of the equilateral triangle. 6. 120° because the sum of the angles of the highlighted triangle must equal 180°.
7. The other measures are 90° and 150°. Examples:

8. Yes; $3 \times 120 = 360$. 9. Answers may vary.

### Practice 4-6

**Statements**

1. $\overline{AB} \perp \overline{BC}, \overline{ED} \perp \overline{FE}$
2. $\angle B, \angle E$ are right $\angle$s.
3. $\overline{AC} \cong \overline{FD}, \overline{AB} \cong \overline{ED}$
4. $\triangle ABC \cong \triangle DEF$

**Reasons**

1. Given
2. Perpendicular lines form right $\angle$s.
3. Given
4. HL Theorem

**Practice 4-7**

1. $\triangle ZWX \cong \triangle YXW; SAS$
2. $\triangle LNP \cong \triangle LMO; SAS$
3. $\triangle ADF \cong \triangle BGE; SAS$
4. $F \equiv G$

5. common angle: $\angle L$

**Guided Problem Solving 4-7**

1. The figure, a list of parallel and perpendicular pairs of sides, and one known angle measure, namely $m\angle A = 56$
2. nine 3. They are congruent and have equal measures.
4. $m\angle A = m\angle 1 = m\angle 2 = 56$
5. $m\angle 4 = 90$
6. $m\angle 3 = 34$
7. $m\angle DCE = 56$
8. $m\angle 5 = 22$
9. $m\angle FCG = 90$
10. $m\angle 6 = 34$
11. $m\angle 7 = 34$
12. $m\angle 9 = 112$
13. $m\angle FIC = 180 - (m\angle 2 + m\angle 3) = 90$
14. $m\angle DHC = m\angle A = 90; m\angle FJC = 180 - m\angle 9 = 68;
   m\angle BIG = m\angle FIC = 90$

### 4A: Graphic Organizer

1. Congruent Triangles  2. Answers may vary. Sample: congruent figures; triangle congruence by SSS, SAS, ASA, and AAS; proving parts of triangles congruent; the Isosceles Triangle Theorem  3. Check students’ work.

### 4B: Reading Comprehension

1. Yes. Using the Isosceles Triangle Theorem, $\angle W \cong \angle Y$. It is given that $\overline{WX} \cong \overline{XY}$ and $\overline{WU} \cong \overline{YV}$. Therefore $\triangle WUX \cong \triangle YVX$ by SAS. 2. There is not enough information. You need to know if $\overline{AC} \cong \overline{EC}$, if $\angle A \cong \angle E$, or if $\angle B \cong \angle D$. 3. a
4C: Reading/Writing Math Symbols
1. Angle-Angle-Side  2. triangle XYZ  3. angle PQR
4. line segment BD  5. line ST  6. ray WX  7. hypotenuse-leg
8. line 3  9. angle 6  10. Angle-Side-Angle

4D: Visual Vocabulary Practice
1. theorem  2. congruent polygons  3. base angle of an isosceles triangle
4. CPCTC  5. postulate  6. vertex angle of an isosceles triangle
7. corollary  8. base of an isosceles triangle  9. legs of an isosceles triangle

4E: Vocabulary Check
Angle: Formed by two rays with the same endpoint.
Congruent angles: Angles that have the same measure.
Congruent segments: Segments that have the same length.
Corresponding polygons: Polygons that have corresponding sides congruent and corresponding angles congruent.
CPCTC: An abbreviation for “corresponding parts of congruent triangles are congruent.”

4F: Vocabulary Review Puzzle
1. postulate  2. hypotenuse  3. angle  4. vertex  5. side
6. leg  7. perpendicular  8. polygon  9. supplementary
10. parallel  11. corresponding

Chapter 5

Practice 5-1
1a. 8 cm  1b. 16 cm  1c. 14 cm  2a. 9.5 cm  2b. 17.5 cm
2c. 14.5 cm  3. 17  4. 7  5. 42  6. 16.5  7a. 18  7b. 61
8. \( \overline{PR} \parallel \overline{YZ}, \overline{PO} \parallel \overline{XZ}, \overline{XY} \parallel \overline{RO} \)

Guided Problem Solving 5-1
1. 30 units  2. The three sides of the large triangle are each bisected by intersections with the two line segments lying in the interior of the large triangle. 3. the value of \( x \). 4. They are called midsegments. 5. They are parallel, and the side labeled 30 is half the length of the side labeled 6. \( x = 60 \) 7. Yes; the side labeled x appears to be about twice as long as the side labeled 30. 8. No. Those lengths are not fixed by the given information. (The triangle could be vertically stretched or shrunk without changing the lengths of the labeled sides.) All one can say is that the midsegment is half as long as the side it is parallel to.

Practice 5-2
1. \( WY \) is the perpendicular bisector of \( \overline{XZ} \). 2. 4  3. 9
4. right triangle  5. 5  6. 17  7. isosceles triangle  8. 3.5
9. 21  10. right triangle  11. \( \overline{JP} \) is the bisector of \( \angle LJK \).
12. 9  13. 45  14. 14  15. right isosceles triangle

2. See answer to Step 1, above. 3. See answer to Step 1, above. 4. Plot a point and explain why it lies on the bisector of the angle at the origin. 5. line \( \ell: y = -\frac{3}{4}x + \frac{25}{2} \), line m:
6. \( x = 10 \) 6. \( C(10,5) \) 7. \( CA = CB = 5 \); yes 8. Theorem 5-5, the Converse of the Angle Bisector Theorem 9. \( m \angle AOC = \angle BOC = 27 \) 10. Draw \( \ell, m, \) and C, then draw \( OC \). Since \( OA = OB = 10 \), it follows that \( \triangle AOC \equiv \triangle OBC \), by HL. Then \( CA \equiv CB \) and \( \angle AOC \equiv \angle BOC \) by CPCTC.

Practice 5-3
1. (–2, 2)  2. (4, 0)  3. altitude  4. median
5. perpendicular bisector  6. angle bisector
7a. (2, 0)  7b. (–2, –2)  8a. (0, 0)  8b. (3, –4)

Guided Problem Solving 5-3
1. the figure and a proof with some parts left blank 2. Fill in the blanks. 3. \( \overline{AB} \) 4. Theorem 5-2, the Perpendicular Bisector Theorem 5. \( BC; \overline{XC} \) 6. the Transitive Property of Equality 7. Perpendicular Bisector. (This converse is Theorem 5-3.) 8. The point of the proof is to demonstrate that \( n \) runs through point \( X \). It would not be appropriate to show that fact as already given in the figure. 9. Nothing essential would change. Point \( X \) would lie outside \( \triangle ABC \) (below \( \overline{BC} \)), but the proof would run just the same.

Practice 5-4
1. I and \( \text{III} \)  2. I and \( \text{II} \)  3. The angle measure is not 65.
4. Tina does not have her driver’s license. 5. The figure does not have eight sides. 6. \( \triangle ABC \) is congruent to \( \triangle XYZ \)
7a. If you do not live in Toronto, then you do not live in Canada; false. 7b. If you do not live in Canada, then you do not live in Toronto; true. 8. Assume that \( m \angle A \neq m \angle B \).
9. Assume that \( LM \) does not intersect \( NO \). 10. Assume that it is not sunny outside. 11. Assume that \( m \angle A \geq 90 \). This means that \( m \angle A + m \angle C \geq 180 \). This, in turn, means that the sum of the angles of \( \triangle ABC \) exceeds 180, which contradicts the Triangle Angle-Sum Theorem. So the assumption that \( m \angle A \geq 90 \) must be incorrect. Therefore, \( m \angle A < 90 \).

Guided Problem Solving 5-4
1. Ice is forming on the sidewalk in front of Toni’s house.
2. Use indirect reasoning to show that the temperature of the sidewalk surface must be 32°F or lower. 3. The temperature...
the sidewalk in front of Toni’s house is greater than 32°F.  
4. Water is liquid (ice does not form) above 32°F.  
5. There is no ice forming on the sidewalk in front of Toni’s house.  
6. The result from step 5 contradicts the information identified as given in step 1.  
7. The temperature of the sidewalk in front of Toni’s house is less than or equal to 32°F.  
8. If the temperature is above 32°F, water remains liquid. This is reliably true. Adding salt will cause water to remain liquid even below 32°F. This is not reliably true. Converse: If water remains liquid, the temperature is above 32°F, water remains liquid. This is not reliably true. Adding salt will cause water to remain liquid even below 32°F.

9. Suppose two people are each the world’s tallest person. Call them person A and person B. Then person A would be taller than everyone else, including B, but by the same token B would be taller than A. It is a contradiction for two people each to be taller than the other. So it is impossible for two people each to be the World’s Tallest Person.

Practice 5-5
1. \( \angle M, \angle N \)  
2. \( \angle C, \angle D \)  
3. \( \angle R, \angle P \)  
4. \( \angle A, \angle T \)
5. yes; 4 + 7 > 8, 7 + 8 > 4, 4 + 4 > 7  
6. no; 6 + 10 < 17  
7. yes; 4 + 4 > 4  
8. yes; 11 + 12 > 13, 12 + 13 > 11, 13 + 11 > 12  
9. no; 18 + 20 ≥ 40  
10. no; 1.2 + 2.6 < 4.9  
11. \( BO, BT, TO \)  
12. \( RS, ST, RT \)
13. \( \angle D, \angle S, \angle A \)  
14. \( \angle N, \angle S, \angle J \)  
15. \( 3 < x < 11 \)
16. \( 8 < x < 26 \)  
17. \( 0 < x < 10 \)  
18. \( 9 < x < 31 \)

Guided Problem Solving 5-5
1.  
2. The side opposite the larger included angle is greater than the side opposite the smaller included angle.  
3. The angle opposite the larger side is greater than the angle opposite the smaller side.  
4. The opposite sides each have a length of nearly the sum of the other two side lengths.  
5. The opposite sides are the same length. They are corresponding parts of triangles that are congruent by SAS.

5A: Graphic Organizer
1. Relationships Within Triangles  
2. Answers may vary. Sample: midsegments of triangles; bisectors in triangles; concurrent lines, medians, and altitudes; and inverses, contrapositives, and indirect reasoning  
3. Check students’ work.

5B: Reading Comprehension
1. The width of the tar pit is 10 meters.  
2. b

5C: Reading/Writing Math Symbols
1. L  
2. F  
3. O  
4. G  
5. A  
6. I  
7. M  
8. H  
9. E  
10. K  
11. B  
12. D  
13. N  
14. J  
15. C

5D: Visual Vocabulary Practice
1. median  
2. negation  
3. circumcenter  
4. contrapositive  
5. centroid  
6. equivalent statements  
7. incenter  
8. inverse  
9. altitude

5E: Vocabulary Check
Midpoint: A point that divides a line segment into two congruent segments.  
Midsegment of a triangle: The segment that joins the midpoints of two sides of a triangle.  
Proof: A convincing argument that uses deductive reasoning.  
Coordinate proof: A proof in which a figure is drawn on a coordinate plane and the formulas for slope, midpoint, and distance are used to prove properties of the figure.  
Distance from a point to a line: The length of the perpendicular segment from the point to the line.

5F: Vocabulary Review
1. altitude  
2. line  
3. median  
4. negation  
5. contrapositive  
6. incenter  
7. orthocenter  
8. slope-intercept  
9. exterior  
10. obtuse  
11. alternate interior  
12. centroid  
13. equivalent  
14. right  
15. parallel

Chapter 6

Practice 6-1
1. parallelogram  
2. rectangle  
3. quadrilateral  
4. kite, quadrilateral  
5. trapezoid, isosceles trapezoid, quadrilateral  
6. square, rectangle, parallelogram, rhombus, quadrilateral  
7. \( x = 7; AB = BD = DC = CA = 11 \)  
8. \( f = 5; g = 11; FG = GH = HI = IF = 17 \)  
9. parallelogram  
10. kite

Guided Problem Solving 6-1
1. a labeled figure, which shows an isosceles trapezoid  
2. The nonparallel sides are congruent.  
3. The measures of the angles and the lengths of the sides  
4. \( m_\angle G = c \)  
5. \( c + (4c - 20) = 180 \)  
6. \( 40 \)  
7. \( m_\angle D = m_\angle G = 40 \)  
8. \( a - 4 = 11 \)  
9. 15  
10. \( DE = FG = 11, EF = 15, DG = 32 \)  
11. \( 40 + 40 + 140 + 140 = 360 = (4 - 2)180 \)  
12. \( m_\angle D = m_\angle G = 39, m_\angle E = m_\angle F = 141 \)

Practice 6-2
1. 15  
2. 32  
3. 7  
4. 12  
5. 9  
6. 8  
7. 100  
8. 40; 140; 40  
9. 113; 45; 22  
10. 115; 15; 50  
11. 55; 105; 55  
12. 32; 98; 50  
13. 16  
14. 35  
15. 28
Guided Problem Solving 6-2  
1. the ratio of two different angle measures in a parallelogram  
2. The consecutive angles are supplementary.  
3.  

\[ \text{9x} \]

4. the measures of the angles  
5. The angles are supplementary angles, because they are consecutive.  
6. \( x + 9x = 180 \)  
7. 18 and 162  
8. No; the lengths of the sides are irrelevant in this problem.  
9. 30 and 150  

Practice 6-3  
1. no  
2. yes  
3. yes  
4. yes  
5. \( x = 2; y = 3 \)  
6. \( x = 64; y = 10 \)  
7. \( x = 8 \); the figure is a \( \square \) because both pairs of opposite sides are congruent.  
8. \( x = 25 \); the figure is a \( \square \) because the congruent opposite sides are \( \| \) by the Converse of the Alternate Interior Angles Theorem.  
9. No; the congruent opposite sides do not have to be \( \| \).  
10. No; the figure could be a trapezoid.  
11. Yes; both pairs of opposite sides are congruent.  
12. Yes; both pairs of opposite sides are \( \| \) by the converse of the Alternate Interior Angles Theorem.  
13. No; only one pair of opposite sides is congruent.  
14. Yes; one pair of opposite sides is both congruent and \( \| \).  

Guided Problem Solving 6-3  
1. a labeled figure, which shows a quadrilateral that appears to be a parallelogram  
2. The consecutive angles are supplementary.  
3. find values for \( x \) and \( y \) which make the quadrilateral a parallelogram  
4. \( m\angle A + m\angle D = 180 \), so that \( \angle A \) and \( \angle D \) meet the requirements for same-side interior angles on the transversal of two parallel lines (Theorems 3-2 and 3-6).  
5. \( \angle B \equiv \angle D \), by Theorem 6-2.  
6. \( 3x + 10 + 5y = 180; 8x + 5 = 5y \)  
7. \( x = 15, y = 25 \)  
8. \( m\angle A = m\angle C = 55 \) and \( m\angle B = m\angle D = 125 \), which matches the appearance of the figure.  
9. \( (3x + 10) + (8x + 5) = 180; yes \)  

Practice 6-4  
1a. rhombus  
1b. 72; 54; 74; 22a. rectangle  
2b. 37; 53; 106; 4a. rhombus  
3b. 60; 30; 60; 30  
4b. 22; 68; 68; 90  
5. Possible; opposite angles are congruent in a parallelogram.  
6. Impossible; if the diagonals are perpendicular, then the parallelogram should be a rhombus, but the sides are not of equal length.  
7. \( x = 7; HJ = 7; IK = 7 \)  
8. \( x = 6; HJ = 25; IK = 25 \)  
9a. 90; 90; 29; 29  
9b. 288 cm\(^2\)  
10a. 38; 90; 38  
10b. 260 m\(^2\)  

Guided Problem Solving 6-4  
1. A labeled figure, which shows a parallelogram. One angle is a right angle, and two adjacent sides are congruent. Algebraic expressions are given for the lengths of three line segments.  
2. diagonals  
3. Find the values of \( x \) and \( y \).  
4. It is a square.  
5. congruent; bisect  
6. \( 4x - y + 1 = (2x - 1) + (3y + 5); 2x - 1 = 3y + 5 \)  
7. \( x = \frac{7}{2}; y = 3 \)  
8. It was not necessary to know \( AB \equiv AD \), but it was necessary to know \( m\angle B = 90 \).  
9. The key fact, which enables the use of Theorem 6-11 in addition to Theorem 6-3, is that \( ABCD \) is a rectangle. It does not matter whether all four sides are congruent.  

Practice 6-5  
1. 118; 62  
2. 59; 121  
3. 96; 84  
4. 101; 79  
5. \( x = 4 \)  
6. \( x = 1 \)  
7. 105.5; 105.5  
8. 118; 118  
9. 90; 63; 63  
10. 107; 107  
11. \( x = 8 \)  
12. \( x = 7 \)  

Guided Problem Solving 6-5  
1. Isosceles trapezoid \( ABCD \) with \( AB \equiv CD \) and \( \angle BAD = \angle D \). \( AB \equiv DC \) is Given. \( DC \equiv AE \) because opposite sides of a parallelogram are congruent (Theorem 6-1). \( AB \equiv AE \) is from the Transitive Property of Congruency.  
2. Isosceles; \( \equiv \); because base angles of an isosceles triangle are congruent.  
3. \( \angle 1 \equiv \angle C \) because corresponding angles on a transversal of two parallel lines are congruent.  
4. \( \angle B \equiv \angle C \) by the Transitive Property of Congruency.  
5. \( \angle B \equiv \angle C \) follows by CPCTC and \( \angle BAD \equiv \angle D \) because they are supplements of congruent angles.  

Practice 6-6  
1. \( (1.5a, 2b); a \)  
2. \( (0.5a, 0); a \)  
3. \( (0.5a, b); \sqrt{a^2 + 4b^2} \)  
4. \( 0 \)  
5. 1  
6. \(-\frac{3}{4} \)  
7. \( \frac{2b}{3a} \)  
8. \( \frac{2b}{5}\)  
9. \( E(a, 3b); I(4a, 0) \)  
10. \( D(4a, b); I(3a, 0) \)  
11. \( (4b, -a) \)  
12. \( (-b, 0) \)  

Guided Problem Solving 6-6  
1. a rhombus with coordinates given for two vertices  
2. A rhombus is a parallelogram with four congruent sides.  
3. the coordinates of the other two vertices  
4. They are the diagonals.  
5. They bisect each other.  
6. \( W(2r, 0) \), \( Z(0, -2t) \)  
7. No; neither Theorem 6-3 nor any other theorem or result would apply.  
8. Slope of \( \overline{WX} \) is 0 and slope of \( \overline{YZ} \) is undefined. This confirms Theorem 6-10, which says that the diagonals of a rhombus are perpendicular.  

Practice 6-7  
1a. \( \frac{p}{q} \)  
1b. \( y = mx + b; q = \frac{p}{q}(y) + b; q = b - \frac{p^2}{q^2} \)  
2. \( y = \frac{p}{q}x + q - \frac{p^2}{q} \)  
1c. \( x = r + p \)  
1d. \( y = \frac{p}{q}(r + p) + q - \frac{p^2}{q}; y = \frac{r}{q}x + q - \frac{r^2}{q} \)  
1e. \( \frac{p}{q} \)  
1f. \( \frac{q}{r} \)  
1g. \( y = mx + b; q = \frac{p}{q}(r + p) + q - \frac{p^2}{q}; y = \frac{r}{q}x + q - \frac{r^2}{q} \)  
1h. \( y = \frac{p}{q}(r + p) + q - \frac{p^2}{q}; y = \frac{r}{q}x + q - \frac{r^2}{q}; \)
Guided Problem Solving 6-7
1. kite $DEFG$ with $DE = EF$ with the midpoint of each side identified  
2. A kite is a quadrilateral with two pairs of adjacent sides congruent and no opposite sides congruent. 
3. The midpoints are the vertices of a rectangle. 
4. $D(-2b, 2c)$, $G(0, 0)$  
5. $L(b, a + c)$, $M(b, c)$, $N(-b, c)$, $K(-b, a + c)$  
6. Slope of $KL = $ slope of $NM = 0$, slopes of $KN$ and $LM$ are undefined  
7. Opposite sides are parallel; it is a rectangle.  
8. Adjacent sides are perpendicular.  
9. Right angles  
10. Answers will vary. Example: $a = 3, b = 2, c = 2$ yields the points $D(-4, 4)$, $E(0, 6)$, $F(4, 4)$, $G(0, 0)$ with midpoints at $(-2, 2), (-2, 5), (2, 5)$, and $(2, 2)$. Connecting these midpoints forms a rectangle.  
11. Construct $DF$ and $EG$. Slope of $DF = 0$, so $DF$ is horizontal. Slope of $EG$ is undefined, so $EG$ is vertical.

6A: Graphic Organizer
1. Quadrilaterals  
2. Answers may vary. Sample: classifying quadrilaterals; properties of parallelograms; proving that a quadrilateral is a parallelogram; and special parallelograms  
3. Check students’ work.

6B: Reading Comprehension
1. $QT \parallel SR$, $QR \parallel ST$, $QT \parallel RS$, $QR \parallel TV$  
2. No, it cannot be proven that $\triangle QTV \cong \triangle SRU$ because with the given information, only one side and one angle of the two triangles can be proven to be congruent. Another side or angle is needed. If it were given that $QUSV$ is a parallelogram, then the proof could be made.  
3. All four sides are congruent.  
4. Yes, since $EG \cong EG$ by the Reflexive Property, $\triangle EFG \cong \triangle EHG$ by SSS.  
5. $b$

6C: Reading/ Writing Math Symbols
1. $\times$, $\div$, or $\frac{1}{2}$  
2. $\equiv$, $\geq$, or $\subseteq$  
3. $\frac{1}{2}$, $\frac{3}{4}$

6D: Visual Vocabulary Practice/ High-Use 
Academic Words
1. Solve  
2. Deduce  
3. Equivalent  
4. Indirect  
5. Equal  
6. Analysis  
7. Identify  
8. Convert  
9. Common

6E: Visual Vocabulary Check
Consecutive angles: Angles of a polygon that share a common side.  
Kite: A quadrilateral with two pairs of congruent adjacent sides and no opposite sides congruent.  
Parallelogram: A quadrilateral with two pairs of parallel sides.  
Rhombus: A parallelogram with four congruent sides.  
Trapezoid: A quadrilateral with exactly one pair of parallel sides.

6F: Vocabulary Review Puzzle
Chapter 7

Practice 7-1
18. 12 : 7 19. 8 3 20. 17 13

Guided Problem Solving 7-1
1. ratios 2. 42 42,000,000 or 1 = 1 1,000,000 3. the denominator 4. \( \frac{x}{29,000} = \frac{1}{1,000,000} \) 5. Cross-Product Property 6. 0.029 7. 0.348 8. yes 9. 21.912

Practice 7-2
1. \( \triangle ABC \sim \triangle XYZ \), with similarity ratio 2 : 1 2. Not similar; corresponding sides are not proportional. 3. Not similar; corresponding angles are not congruent. 4. \( \triangle ABC \sim \triangle KMN \), with similarity ratio 4 : 7 5. \( \angle L \), \( \angle O \) 6. NO 7. LO 8. 9. 3.96 ft 10. 3.75 cm 11. \( \frac{2}{3} \) 12. 53 13. 7 \( \frac{1}{2} \) 14. 4 \( \frac{1}{2} \) 15. 37

Guided Problem Solving 7-2
1. equal 2. \( \frac{6.14}{2.61} \) 3. 2.61; 6.14 4. 19.3662; 19.2182 5. no 6. no 7. 2.3706; 2.3525 8. Since the quotients are not equal, the ratios are not equal, and the bills are not similar rectangles. 9. 4.045

Practice 7-3
1. \( \angle AXB \cong \angle RXQ \) because vertical angles are \( \cong \). \( \angle A \cong \angle R \) (Given). Therefore \( \triangle AXB \sim \triangle RXQ \) by the AA ~ Postulate. 2. Because \( \frac{MP}{EF} = \frac{PX}{AW} = \frac{XM}{AL} = \frac{3}{4} \), \( \triangle MPX \sim \triangle LWA \) by the SSS ~ Theorem. 3. \( \angle QMP \cong \angle AMB \) because vertical angles are \( \cong \). Then, because \( \frac{QM}{AM} = \frac{PM}{BM} = \frac{2}{7} \), \( \angle QMP \sim \angle AMB \) by the SAS ~ Theorem. 4. Because \( AX = BX \) and \( CX = RX \), \( \frac{AX}{RX} = \frac{BX}{RX} \). \( \angle AXB \cong \angle CXR \) because vertical angles are \( \cong \). Therefore \( \triangle AXB \sim \triangle CXR \) by the SAS ~ Theorem. 5. \( \frac{15}{2} \) 6. 48 7. \( \frac{20}{7} \) 8. 36 9. 33 ft

Guided Problem Solving 7-3
1. no; N/A 2. yes; \( \overline{WT} \), \( \overline{RS} \) 3. It is a trapezoid. 4. They are congruent. 5. They are parallel. 6. They are congruent. 7. \( \triangle RSZ \) and \( \triangle TWZ \) 8. AA~ or Angle-Angle Similarity Postulate 9. No; there is only one pair of congruent angles. 10. yes; parallelogram, rhombus, rectangle, and square

Practice 7-4
1. 16 2. 8 3. 10\( \sqrt{2} \) 4. 6\( \sqrt{2} \) 5. h 6. y 7. a 8. c 9. \( \frac{9}{2} \) 10. \( x = 6; y = 6\sqrt{3} \) 11. \( x = 4\sqrt{5}; y = \sqrt{55} \) 12. \( 2\sqrt{15} \) in.
Chapter 8

Practice 8-1

1. \( \sqrt{51} \)  2. \( 2\sqrt{65} \)  3. \( 2\sqrt{21} \)  4. \( 18\sqrt{2} \)  5. 46 in.  6. 78 ft  7. 279 cm  8. 19 m  9. acute  10. obtuse  11. right

Guided Problem Solving 8-1

1. the sum of the lengths of the sides  2. Pythagorean Theorem  3. 7 cm  4. 4 cm \( \times \) 3 cm  5. \( c^2 = a^2 + b^2 \)  6. 5  7. 12 cm  8. perimeter of rectangle = 14 cm; yes  9. Answers will vary; example: Draw a 4 cm \( \times \) 3 cm grid, copy the given figure, measure the lengths with a ruler, add them together.  10. 20 cm

Practice 8-2

1. \( x = 2; y = \sqrt{3} \)  2. \( 8\sqrt{2} \)  3. \( 14\sqrt{2} \)  4. \( 2 \)  5. \( x = 15; y = 15\sqrt{3} \)  6. \( 3\sqrt{2} \)  7. 42 cm  8. 10.4 ft, 12 ft  9. \( a = 4; b = 3 \)  10. \( p = 4\sqrt{3}; q = 4\sqrt{3}; r = 8; s = 4\sqrt{6} \)

Guided Problem Solving 8-2

1. 30°-60°-90° triangle  2. \( l \)  3. \( h \)  4. \( \sqrt{3} \)  5. \( \frac{24}{5} \) or \( 8\sqrt{3} \)  6. 2  7. \( \frac{48}{\sqrt{3}} \) or \( 16\sqrt{3} \)  8. 28 ft  9. 0.28 min  10. yes  11. 34 ft

Practice 8-3

1. \( \tan E = \frac{3}{2}; \tan F = \frac{4}{3} \)  2. \( \tan E = \frac{2}{3}; \tan F = \frac{5}{3} \)  3. 12.4  4. 31°  5. 7.1  6. 6.4  7. 26.6  8. 71.6  9. 39  10. 72  11. 39  12. 54

Guided Problem Solving 8-3

1. 2. 180  3. \( m\angle A = 2m\angle X \)  4. 90  5. base: 40 cm, height: 10 cm  6. 4  7. 4  8. 76  9. 152  10. 28  11. yes  12. 46

Practice 8-4

1. \( \sin P = \frac{2\sqrt{10}}{2}; \cos P = \frac{3}{2} \)  2. \( \sin P = \frac{4}{5}; \cos P = \frac{3}{5} \)  3. \( \sin P = \frac{\sqrt{11}}{6}; \cos P = \frac{5}{6} \)  4. \( \sin P = \frac{15}{17}; \cos P = \frac{8}{17} \)  5. 64  6. 11.0  7. 7.0  8. 7.8  9. 53  10. 6.6  11. 11.0  12. 11.5

Guided Problem Solving 8-4

1. The sides are parallel.  2. sine  3. \( \sin 30^\circ = \frac{w}{6} \)  4. 3.0  5. yes  6. cosine  7. \( \cos x^\circ = \frac{3}{4} \)  8. 41  9. Answers may vary. Sample: \( \cos 60^\circ > \frac{3}{6}; \sin 49^\circ > \frac{3}{4} \)  10. 5.2, 2.6

Practice 8-5

1a. angle of depression from the plane to the person  1b. angle of elevation from the person to the plane  1c. angle of depression from the person to the sailboat  1d. angle of elevation from the sailboat to the person  2. 116.6 ft  3. 84.8 ft  4. 46.7 ft  5. 31.2 yd

Guided Problem Solving 8-5

1. \( \angle e = 1; \angle d = \angle d \)  2. congruent  3. \( m\angle e = m\angle d \)  4. \( 7x - 5 = 4(x + 7) \)  5. 11  6. 72  7. 72  8. yes  9. 44, 44

Practice 8-6

1. \( (46.0, 46.0) \)  2. \( (-89.2, -80.3) \)  3. 38.6 mi/h; 31.2° north of east  4. 134.5 m; 42.0° south of west  5. 55° north of east  6. 33° west of north  7a. \( (1, 5) \)  7b.  8a. \((1, -1)\)  8b.

9. Sample: 48°
Guided Problem Solving 8-6
1. Check students’ work. 2. \[ \frac{x}{100}, \frac{y}{100} \]
3. 100 \(\cos 30°\); 100 \(\sin 30°\)
4. 86.6; 50
5. \(\sqrt{H20908} \)
6. \(\sqrt{H20909} \)
7. \(173.2, 0\)
8. 173; due east
9. yes
10. 100; due east

8A: Graphic Organizer
1. Right Triangles and Trigonometry 2. Answers may vary. Sample: the Pythagorean Theorem; special right triangles; the tangent ratio; sine and cosine ratios; angles of elevation and depression; vectors
3. Check students’ work.

8B: Reading Comprehension

8C: Reading/Writing Math Symbols
9. \(\sin^{-1} A = \frac{5}{12}\)
10. \(\triangle ABC \sim \triangle XYZ\)
11. \(m\angle A = 52°\)
12. \(\tan Z = \frac{7}{24}\)

8D: Visual Vocabulary Practice

8E: Vocabulary Check
Obtuse triangle: A triangle with one angle whose measure is between 90 and 180.
Isosceles triangle: A triangle that has at least two congruent sides.
Hypotenuse: The side opposite the right angle in a right triangle.
Right triangle: A triangle that contains one right angle.
Pythagorean triple: A set of three nonzero whole numbers \(a\), \(b\), and \(c\) that satisfy the equation \(a^2 + b^2 = c^2\).

8F: Vocabulary Review Puzzle

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Chapter 9

Practice 9-1
1. No; the triangles are not the same size. 2. Yes; the ovals are the same shape and size. 3a. \( \angle C' \) and \( \angle F' \) 3b. \( CD \) and \( C'D', DE \) and \( D'E', EF \) and \( E'F', CF \) and \( C'F' \) 4. \((x, y) \rightarrow (x - 2, y - 4) \) 5. \((x, y) \rightarrow (x + 4, y - 2) \) 6. \((x, y) \rightarrow (x + 2, y + 2) \) 7. \( W'(-2, 2), X'(-1, 4), Y'(3, 3), Z'(2, 1) \) 8. \( J'(-5, 0), K'(-3, 4), L'(-3, -2) \) 9. \((x, y) \rightarrow (x + 13, y - 13) \) 10. \((x, y) \rightarrow (x, y - 4, y + 2) \) 11a. \( P'(-3, -1) \) 11b. \( P'(0, 8), N'(-5, 2), Q'(2, 3) \)

Guided Problem Solving 9-1
1. the four vertices of a preimage and one of the vertices of the image 2. Graph the image and preimage. 3. \( C(4, 2) \) and \( C'(0, 0) \) 4. \( x = 4, y = 2, x + a = 0, y + b = 0 \) 5. \( a = -4; b = -2; (x, y) \rightarrow (x - 4, y - 2) \) 6. \( A'(-1, 4), B'(1, 3), D'(-2, 1) \)

Guided Problem Solving 9-2
1. a point at the origin, and two reflection lines 2. A reflection is an isometry in which a figure and its image have opposite orientations. 3. the image after two successive reflections

Practice 9-2
1. \((-3, -2) \) 2. \((-2, -3) \) 3. \((-1, -4) \) 4. \((4, -2) \) 5. \((4, -1) \) 6. \((3, -4) \)

Guided Problem Solving 9-2
1. a point at the origin, and two reflection lines 2. A reflection is an isometry in which a figure and its image have opposite orientations. 3. the image after two successive reflections

Practice 9-2
1. \((-3, -2) \) 2. \((-2, -3) \) 3. \((-1, -4) \) 4. \((4, -2) \) 5. \((4, -1) \) 6. \((3, -4) \)
Practice 9-3
1. \( I \), \( I \), \( I \), \( I \), \( G \), \( G \), \( G \), \( G \)
2. \( \overline{GH} \)
3. \( \overline{GH} \)
4. \( \overline{GH} \)
5. \( \overline{GH} \)
6. \( \overline{GH} \)
7. \( \overline{GH} \)
8. \( \overline{GH} \)
9. \( \overline{GH} \)

Guided Problem Solving 9-3
1. The coordinates of point \( A \), and three rotation transformations. It is assumed that the rotations are counterclockwise.
2. Parallelogram, rhombus, square slope of \( \overline{A} = \frac{2}{5} \)
3. \( \overline{A} \)
4. \( \overline{A} \)
5. slope of \( \overline{OB} = \frac{5}{2} \); slope of \( \overline{OC} = \frac{2}{5} \); slope of \( \overline{OD} = \frac{-5}{2} \); the slopes of perpendicular line segments are negative reciprocals.
6. square
7. yes
8. \((2, 7), (7, -2), D(-2, -7)\)

Practice 9-4
1. The helmet has reflectional symmetry.
2. The teapot has reflectional symmetry.
3. The hat has both rotational and reflectional symmetry.
4. \( \overline{A} \)
5. \( \overline{A} \)
6. line symmetry and 72° rotational symmetry
7. line symmetry and 90° rotational symmetry
8. line symmetry and 45° rotational symmetry
9. line symmetry
10. line symmetry and 45° rotational symmetry
11. 180° rotational symmetry
12. \( \overline{A} \)
13. \( \overline{A} \)
Guided Problem Solving 9-4
1. the coordinates of one vertex of a figure that is symmetric about the y-axis
2. Line symmetry is the type of symmetry for which there is a reflection that maps a figure onto itself.
3. the coordinates of another vertex of the figure
4. images (and preimages)
5. reflection across the y-axis
6. (−3, 4) 7. yes 8. (−6, 7)

Practice 9-5
1. 2. 3. yes 4. no 5. no 6. no
7. 
8. 
9. P′(−12, −12), Q′(−6, 0), R′(0, −6) 10. P′(−2, 1), Q′(−1, 0), R′(0, 1)

Guided Problem Solving 9-5
1. A description of a square projected onto a screen by an overhead projector, including the square’s area and the scale factor in relation to the square on the transparency.
2. The scale factor of a dilation is the number that describes the size change from an original figure to its image.
3. the area of the square on the transparency
4. smaller; The scale factor 16 > 1, so the dilation is an enlargement.
5. \( \frac{1}{16} \) 6. \( \frac{1}{256} \)

Being \( \frac{1}{16} \) as high and \( \frac{1}{16} \) as wide, the square on the transparency has \( \frac{1}{16} \times \frac{1}{16} = \frac{1}{256} \) times the area.
7. reflection 8. rotation 9. glide reflection 10. translation

Guided Problem Solving 9-6
1. assorted triangles and a set of coordinate axes
2. a transformation 3. the transformation that maps one
triangle onto another. 4. They are congruent, which confirms that they could be related by one of the isometries listed. Corresponding vertices: E and P, D and Q, C and M.
5. counterclockwise; clockwise. The triangles have opposite orientations, so the isometry must be a reflection or glide reflection. 6. No; there is no reflection line that will work.
7. a glide reflection consisting of a translation \((x, y) \rightarrow (x + 11, y)\) followed by a reflection across line \(y = 0\) (the x-axis) 8. Yes, both are horizontal. 9. 180º rotation with center \((-\frac{1}{2}, 0)\)

Practice 9-7
1. translational symmetry
2. line symmetry, rotational symmetry, translational symmetry, glide reflectional symmetry
3. line symmetry, rotational symmetry, translational symmetry, glide reflectional symmetry
4. rotational symmetry, translational symmetry
5.–6. Samples:
5. 6.
7. yes 8. no 9. yes

Guided Problem Solving 9-7
1. two different-sized equilateral triangles and a trapezoid
2. A tessellation is a repeating pattern of figures that completely covers a plane, without gaps or overlaps.
3. Theorem 9-7 says that every quadrilateral tessellates. If the three polygons can be joined to form a quadrilateral (using each polygon at least once), that quadrilateral can be the basis of a tessellation.
Chapter 10

Practice 10-1
1. 8 2. 60 3. 1.92 4. 0.8125 5. 9 6. 1500 7. 8.75
8. 3.44 9. 30 10. 16 11. 12 12. 48 13. 12

Guided Problem Solving 10-1
1. 2. Check students' work; B(7, 7) 3. a right angle 4. 7
5. OA 6. 7 7. 24.5 8. a square 9. 49
10. 11.

Practice 10-2
1. 48 cm² 2. 784 in² 3. 90 m² 4. 374 ft² 5. 160 cm²
6. 176.25 in² 7. 42.5 square units 8. 226.2 in²

Guided Problem Solving 10-2
1. \( \frac{1}{2}bh \) 2. Check students’ work; \( B(7, 7) \) 3. a right angle
4. 7 5. \( \overrightarrow{OA} \) 6. 7 7. 24.5 8. a square 9. 49 10. \( \frac{1}{2} \)
11. 24.5 12. \( \frac{1}{2}b^2 \)

Practice 10-3
1. \( \frac{1}{2}h(b_1 + b_2) \) 2. 3; 4; 2 3. 9 4. no 5. yes 6. 3

Practice 10-4
1. 4 : 5; 16 : 25 2. 5 : 3; 25 : 9 3. 1 : 2 4. 8 : 3 5. \( \frac{576}{5} \) in²
6. \( \frac{2700}{7} \) cm² 7. 1 : 2 8. 2 : 5 9. 108 ft²

Guided Problem Solving 10-4
1. scale 2. Check students’ work 3. Answers will vary. Let \( a \) be the side opposite the 30° angle, \( b \) be the side opposite the 50° angle, and \( c \) be the side corresponding to 200 yd. Then \( a \approx 20 \) mm, \( b \approx 30 \) mm and \( c \approx 40 \) mm. 4. \( \frac{200 \text{ yd}}{40 \text{ mm}} = \frac{5 \text{ yd}}{1 \text{ mm}} \)
5. ≈ 90 mm 6. ≈ 16 mm 7. ≈ 320 mm² 8. 450 9. 8000 10. yes 11. 16,000

Practice 10-5
1. 174.8 cm² 2. 578 ft² 3. 1250.5 mm² 4. 1131.4 in²
5. 37.6 m² 6. 30.1 m² 7. 55.4 km² 8. 357.6 in²
9. 9.7 mm² 10. 384.0 in² 11. 83.1 m² 12. 65.0 ft²
13. 119.3 ft² 14. 8 m²
Guided Problem Solving 10-5
1. subtract 2. 50 3. \(\frac{1}{2} \text{ap} \) 4. 30 5. 4 6. \(\frac{4}{\tan 30^\circ} \) 7. 166
8. 116 9. yes 10. 151

Practice 10-6
1. 16\(\pi \) 2. 7.8\(\pi \) 3. Samples: \(DF = AB \) 4. Samples: \(FE = FD \) 5. Samples: \(FE , ED \) 6. 32 7. 54 8. 71 9. 50
10. 140 11. 220 12. 130 13. 6\(\pi \) in. 14. 16\(\pi \) cm

Guided Problem Solving 10-6
1. 11; 2. 11; 3; 33 3. circumference; 2. 4; 31 5. 26 6. 41
7. 105 8. 102 9. 3027.15

Practice 10-7
1. 49\(\pi \) 2. 21.2 3. \(\frac{49}{8} \) \(\pi \) 4. \(\frac{49}{8} \pi - 21.2 \) 5. 4\(\pi \) 6. \(\frac{18}{5} \pi \)
7. \(\frac{9}{8} \) \(\pi \) 8. \(\frac{3}{2} \pi \) 9. 6\(\pi \) 10. \(\frac{25}{3} \pi \) 11. 3 12. 2

Guided Problem Solving 10-7
1. \(\frac{mAB}{360} \cdot \pi r^2 \) 2. 45 3. 25 4. 30 5. 27 6. a piece from the outer ring 7. yes 8. a piece from the top tier

Practice 10-8
1. 8.7\% 2. 10.9\% 3. 15.3\% 4. \(\frac{3}{10} \) 5. \(\frac{7}{2} \) 6. 1 7. 4.0\% 8. 37.5\% 9. 20\% 10. 50\% 11. 40\% 12. 33\frac{1}{3}\%

Guided Problem Solving 10-8
1. outcomes 2. event 3. \(\overline{AF} \) 4. \(\overline{FG} \) 5. \(\overline{EG} \) 6. \(\overline{AE} \); Check students’ work. 7. \(\frac{2}{5} \) or about 67\% 8. \(\frac{2}{5} \) or about 67\%; yes 9. \(\frac{3}{5} \) or 60\%

Guided Problem Solving 11-1

Guided Problem Solving 11-1
1. a two-dimensional network 2. Verify Euler’s Formula for the network shown. 3. 4 4. 6; 9 5. 4 + 6 = 9 + 1; This is true. 6. \(F \) becomes 3, \(V \) stays at 6, \(E \) becomes 8. Euler’s Formula becomes 3 + 6 = 8 + 1, which is still true. 7. 4 + 4 = 6 + 2; This is true.

Practice 11-2
1. 840 ft\(^2\) 2. 200.6 ft\(^2\) 3. 44.0 ft\(^2\) 4. 84.8 cm\(^2\)
5a. 320 m\(^2\) 5b. 440 m\(^2\) 5a. 576 in\(^2\) 5b. 684 in\(^2\)
7a. 48 cm\(^2\) 7b. 60 cm\(^2\) 8a. 1500 m\(^2\) 8b. 1800 m\(^2\)
9. 94.5\(\pi \) cm\(^2\) 10. 290\(\pi \) ft\(^2\)

Guided Problem Solving 11-2
1. a picture of a box, with dimensions 2. the amount of cardboard the box is made of, in square inches 3. A front and a back, each 4 in. \(\times \frac{7}{2} \) in.; a top and a bottom, each 4 in. \(\times \) 1 in.; and one narrow side, 1 in. \(\times \frac{7}{2} \) in. 4. Front and back: each 30 in\(^2\); top and bottom: each 4 in\(^2\); one narrow side: \(\frac{7}{2} \) in\(^2\) 5. 75\(\frac{1}{2} \) in\(^2\) 6. 75\(\frac{1}{2} \) in\(^2\) is about half a square foot (1 ft\(^2\) = 144 in\(^2\)), which is reasonable. The answer is an
Guided Problem Solving 11-3
1. a solid consisting of a cylinder and a cone 2. the surface area of the solid 3. The top of the solid is a circular base of the cylinder, \( A_1 = \pi r^2 \). The side of the solid is the lateral area of the cylinder, \( A_2 = 2\pi rh \). The bottom of the solid is the lateral area of the cone, \( A_3 = \pi r \). 4. \( r = 5 \text{ ft}; h = 6 \text{ ft} \); \( \ell = \sqrt{5^2 + 6^2} = 7 \text{ ft} \). 5. Top: \( A_1 = 78.5 \text{ ft}^2 \); side: \( A_2 = 188.5 \text{ ft}^2 \); bottom: 204.2 \( \text{ ft}^2 \). 6. \( 471 \text{ ft}^2 \). 7. About 0.43, or a little less than half. This seems reasonable. 8. \( 628 \text{ ft}^2 \).

Practice 11-4
1. 1131.0 \( m^3 \) 2. 7.1 in.\(^3 \) 3. 1781.3 \( in.\(^3 \) 4. 785.4 \( cm^3 \) 5. 120 in.\(^3 \) 6. 35 \( ft^3 \) 7. 1728 \( ft^3 \) 8. 265 \( in.\(^3 \) 9. 76 \( ft^3 \) 10. 1152 \( m^3 \)

Guided Problem Solving 11-4
1. dimensions for a layer of topsoil, and two options for buying the soil 2. which purchasing option is cheaper 3. \( 1 \frac{1}{2} \text{ in.}^2 \) 4. 1400 \( ft^3 \) 5. 467 bags; \$1167.50 6. 1400 \( ft^3 \approx 52 \text{ yd}^3 \) 7. \$1164 (or slightly less if the topsoil volume is not rounded up to the next whole number) 8. buying in bulk 9. The two costs are quite close. It makes sense that buying in bulk would be slightly cheaper for a good-sized backyard. 10. Buying top soil by the bag would be less expensive because its weight would cost \$335 and in bulk would cost at least \$345.93.

Practice 11-5
1. 34,992 \( cm^3 \) 2. 400 in.\(^3 \) 3. 10,240 in.\(^3 \) 4. 4800 \( yd^3 \) 5. 314.2 \( in.\(^3 \) 6. 4955.3 \( m^3 \) 7. 1415.8 \( in.\(^3 \) 8. 1005.3 \( m^3 \) 9. 20 10. 2

Guided Problem Solving 11-5
1. a description of a plumb bob, with dimensions 2. the plumb bob’s volume 3. equilateral; 2 cm 4. \( a = \sqrt{3} \text{ cm}; b = 6\sqrt{3} \text{ cm}^2 \) 5. Prism: \( V_1 = 36\sqrt{3} \text{ cm}^3 \); pyramid: \( V_2 = 6\sqrt{3} \text{ cm}^3 \); plum bob: \( V \approx 73 \text{ cm}^3 \) 6. \( \frac{1}{3} \); this seems reasonable 7. 42 \( cm^2 \)

Guided Problem Solving 11-6
1. the diameter of a hailstone, along with its density compared to normal ice 2. the hailstone’s weight 3. \( V = \frac{4}{3}\pi r^3 \) 4. 2.8 in. 5. 91.95 \( in.\(^3 \) 6. 1.7 lb 7. The density of normal ice. This was given for comparison purposes only. 8. 0.01 oz

Practice 11-7
1. 7 : 9 2. 5 : 8 3. yes; 7 : 4 4. not similar 5. 125 \( cm^3 \) 6. 256 \( in.\(^3 \) 7. 25 \( ft^2 \) 8. 600 \( cm^2 \)

Guided Problem Solving 11-7
1. two different sizes for an image on a balloon, and the volume of air in the balloon for one of those sizes 2. the volume of air for the other size 3. The two sizes of the clown face have a ratio of 4 : 8, or 1 : 2. 4. 1 : 8 5. 108 \( in.\(^3 \times 8 = 864 \text{ in.}^3 \) 6. Yes, the two clown face heights are lengths, measured in inches (not in.\(^2 \) or in.\(^3 \)). 7. 351 \( cm^3 \)

11A: Graphic Organizer
1. Surface Area and Volume 2. Answers may vary. Sample: space figures and nets; space figures and drawing; surface areas; and volumes. 3. Check students’ work.

11B: Reading Comprehension
1. Area = \( \frac{1}{2} \) base times height; area of a triangle 2. slope = the difference of the y-coordinates divided by the difference of the x-coordinates; slope of the line between 2 points 3. \( a \) squared plus \( b \) squared equals \( c \) squared, the Pythagorean Theorem; the lengths of the sides of a right triangle 4. area = \( \pi \) times radius squared; area of a circle 5. volume = \( \pi \) times radius squared times height; volume of a cylinder 6. the sum of the \( x \)-coordinates of two points divided by 2, and the sum of the \( y \)-coordinates of two points divided by 2; midpoint of a line segment with endpoints \((x_1, y_1)\) and \((x_2, y_2)\) 7. circumference = two times \( \pi \) times the radius; circumference of a circle 8. volume = one-third \( \pi \) times radius squared times height; volume of a cone 9. distance equals the square root of the quantity of the sum of the square of the difference of the \( x \)-coordinates, squared, plus the difference of the \( y \)-coordinates, squared; the distance between two points \((x_1, y_1)\) and \((x_2, y_2)\) 10. area = one-half the height times the sum of base one plus base two; area of a trapezoid 11. a

11C: Reading/Writing Math Symbols
1. 527.8 \( in.\(^2 \) 2. 136 \( ft^2 \)

11D: Visual Vocabulary Practice
11E: Vocabulary Check

Face: A surface of a polyhedron.
Edge: An intersection of two faces of a polyhedron.
Altitude: For a prism or cylinder, a perpendicular segment that joins the planes of the bases.
Surface area: For a prism, cylinder, pyramid, or cone, the sum of the lateral area and the areas of the bases.
Volume: A measure of the space a figure occupies.

11F: Vocabulary Review
1. polyhedron 2. edge 3. vertex 4. cross section 5. prism 6. altitude 7. right prism 8. slant height 9. cone
10. volume 11. sphere 12. hemispheres 13. similar figures 14. hypotenuse

Chapter 12

Practice 12-1
1. 32 2. 72 3. 15 4. 6 5. \( \sqrt{634} \) 6. \( 4\sqrt{3} \)
7. circumscribed 8. inscribed 9. circumscribed

Guided Problem Solving 12-1
1. The area of one is twice the area of the other.
2. inscribed; circumscribed

3. 5. \( 2r \) 6. \( 4r^2 \) 7. \( \sqrt{2}r \) 8. \( 2\sqrt{2} \) 9. \( 4r^2 = 2(2r^2) \): One area is double the other area.
10. yes 11. The same: The area of one circle is twice the area of the other circle.

Practice 12-2
1. \( r = 13; m\overline{AB} \approx 134.8 \) 2. \( r = 3\sqrt{5}; m\overline{AB} \approx 53.1 \)
3. 4 5. 4 6. 8.5 7. \( \angle Q \equiv \angle T; \overline{PR} \equiv \overline{SU} \)
8. \( \triangle A \equiv \triangle J; \overline{BC} \equiv \overline{KL} \) 9. Because congruent arcs have congruent chords, \( \overline{AB} = \overline{BC} = \overline{CA} \). Then, because an equilateral triangle is equiangular, \( m\angle ABC = m\angle BCA = m\angle CAB \).

Guided Problem Solving 12-2
1. perpendicular 2. bisects 3. 4; 3 4. right 5. radius 6. Pythagorean Theorem 7. 5 8. yes 9. 8; 4

Practice 12-3
1. \( \angle A \) and \( \angle D; \angle B \) and \( \angle C \) 2. \( \angle ADB \) and \( \angle CDB \)
3. 55 4. \( x = 45; y = 50; z = 85 \) 5. \( x = 90; y = 70 \)
6. 180 7. 70 8. \( x = 120; y = 100; z = 140 \)
9a. \( m\angle A = 90 \) 9b. \( m\angle B = 80 \) 9c. \( m\angle C = 90 \)
9d. \( m\angle D = 100 \)

Guided Problem Solving 12-3
1. \( C \) 2. 180 3. a diameter 4. Check students’ work.
5. Check students’ work. 6. Check students’ work.
7. Check students’ work. 8. 90 9. Check students’ work.

Practice 12-4
1. 87 2. 35 3. 120 4. 72 5. \( x = 58; y = 59; z = 63 \)
6. \( x = 30; y = 30; z = 120 \) 7. \( x = 16; y = 52 \)
8. \( x = 138; y = 111; z = 111 \) 9. \( x = 30; y = 60 \)
10. 10 11. 4 12. 3.2 13. 6

Guided Problem Solving 12-4
1. tangent 2. 360. 3. \( \frac{1}{2} \) 4. 85 5. 2 6. 180 7. 95
8. 104; 86; 75 9. 360 10. yes; The sum of the measures of the arcs should be 360.
11. Opposite angles are not supplementary.

Practice 12-5
1. \( C(0,0), r = 6 \) 2. \( C(-1,-6), r = 4 \) 3. \( x^2 + y^2 = 49 \)
4. \( (x-5)^2 + (y-3)^2 = 4 \) 5. \( (x+5)^2 + (y-4)^2 = \frac{1}{4} \)
6. \( (x+2)^2 + (y+5)^2 = 2 \) 7. \( x^2 + y^2 = 4 \)
8. \( x^2 + (y-3)^2 = 16 \) 9. \( (x-7)^2 + (y+2)^2 = 4 \)
10. \( x^2 + (y+20)^2 = 100 \)
Guided Problem Solving 12-5

1. (2, 2); 5
2. \( \frac{3}{4} \)
3. They are perpendicular.
4. \( -\frac{3}{4} \); \( \frac{39}{4} \)
5. \( -\frac{3}{4} \), \( \frac{39}{4} \)
6. \( y = \frac{-3}{4}x + \frac{39}{4} \)
7. \( 13 \)
8. \( \frac{3}{4}x + \frac{39}{4} \); yes
9. \( x \)-intercept: \( a = \frac{25}{3} \); \( y \)-intercept: \( b = \frac{25}{4} \)

Practice 12-6

1. \( x \)-intercept: \( a = \frac{25}{3} \); \( y \)-intercept: \( b = \frac{25}{4} \)

Guided Problem Solving 12-6

1. a line 2. Check students’ work. 3. They are perpendicular. 4. \(-\frac{1}{2}\) 5. 2 6. the midpoint of \( PQ \) 7. (3, 2)
8. \(-4\) 9. \( y = 2x - 4 \) 10. (2, 0); yes 11. \( y = \frac{3}{2}x + 2 \)

12A: Graphic Organizer

1. Circles 2. Answers may vary. Sample: tangent lines; chords and arcs; inscribed angles; and angle measures and segment lengths 3. Check students’ work.

12B: Reading Comprehension

Answers may vary. Sample answers:
1. \( \overrightarrow{AD}, \overrightarrow{CE} \) 2. \( \overrightarrow{BF}, \overrightarrow{CD}, \overrightarrow{AD} \) 3. \( \overrightarrow{HD} \) 4. \( \overrightarrow{AH} \)
5. \( \overrightarrow{AF}, \overrightarrow{AB}, \overrightarrow{BC} \) 6. \( \overrightarrow{CDF}, \overrightarrow{DAB}, \overrightarrow{DAC} \) 7. \( \overrightarrow{AC}, \overrightarrow{ED} \)
8. \( \angle AOC \) 9. \( \angle DCE \) 10. \( \overrightarrow{DEA} \) 11. \( \angle DAH \)
12. \( \angle AG \) 13. \( \angle AHC \) 14. \( \angle DCE \) 15. \( \overrightarrow{AH} \) 16. a

12C: Reading/Writing Math Symbols

1. \( BC \perp DE \) 2. \( EF \) is tangent to \( O \). 3. \( WX \equiv ZY \)
4. \( AB \perp XY \) 5. \( HM = 4 \)

12D: Visual Vocabulary Practice

1. tangent to a circle 2. standard form of an equation of a circle 3. secant 4. chord 5. circumscribed about 6. intercepted arc 7. inscribed in 8. inscribed angle 9. locus

12E: Vocabulary Check

Inscribed in: A circle inside a polygon with the sides of the polygon tangent to the circle.
Chord: A segment whose endpoints are on a circle.
Point of tangency: The single point of intersection of a tangent line and a circle.
Tangent to a circle: A line, segment, or ray in the plane of a circle that intersects the circle in exactly one point.
Circumscribed about: circle outside a polygon with the vertices of the polygon on the circle.

12F: Vocabulary Review

Lesson 1-1 Patterns and Inductive Reasoning

Vocabulary

- Inductive reasoning is reasoning based on patterns you observe.
- A conjecture is a conclusion you reach using inductive reasoning.
- An example is an example for which the conjecture is incorrect.

Example 1
Finding and Using a Pattern: Find the next two terms in each sequence.

a. 1, 2, 4, 7, 11, 16, 22, ...
   Each term is the preceding term plus 1.
   The next two terms are 28 and 34.

b. Monday, Tuesday, Wednesday, ..., ...
   Answers may vary. Sample: Thursday, Friday

Quick Check
1. Find the next two terms in each sequence:
   a. 1, 2, 4, 7, 11, 16, 22, ...
   b. Monday, Tuesday, Wednesday, ..., ...

c. A net is a net for three-dimensional figures.

Lesson Objective

- Use inductive reasoning to make conjectures.

Lesson Objectives

- Inductive reasoning
- Dimension and Shape

Local Standards:

- NAEP 2005 Strand: Geometry
- Mathematical Reasoning

Example 2
Using Inductive Reasoning: Make a conjecture about the sum of the cubes of the first 25 counting numbers.

Think of the sides of the square base as hinges, and "unfold" the figure at these edges to form a net. The base of each of the four isometric triangle faces is a side of the square. Write in the known dimensions.

\[
\begin{align*}
1^3 + 2^3 + 3^3 + \ldots + 25^3 &= \left(1 + 2 + 3 + \ldots + 25\right)^2 \\
&= \left( \frac{25 \times 26}{2} \right)^2 \\
&= \left( \frac{325}{2} \right)^2 \\
&= 325^2 \\
&= 105625
\end{align*}
\]

The sum of the first 25 counting numbers equals the square of the sum of the first 25 counting numbers.

Quick Check
2. Make a conjecture about the sum of the first 35 even numbers. Use your calculator to verify your conjecture.

384, 192, 96, 48, ...

The sum of the first two cubes equals the square of the sum of the first two counting numbers. This pattern continues for the fourth and fifth rows. So a conjecture might be that the sum of the cubes of the first 25 counting numbers equals the square of the sum of the first 25 counting numbers, or (1 + 2 + 3 + \ldots + 25)^2.

Example 3
Drawing a Net: Draw a net for the figure with a square base and four isosceles triangle faces. Label the net with its dimensions.

Think of the sides of the square base as hinges and "unfold" the figure at these edges to form a net. The base of each of the four isosceles triangle faces is a side of the square. Write in the known dimensions.

Quick Check
1. The diagram shows one possible net for the Graham Crackers box.

2. The diagram shows a different net for the box. Show the dimensions in your diagram.

Answers may vary. Example:

- Front: 14 cm
- Top: 14 cm
- Right: 26 cm
Lesson 1-3
Points, Lines, and Planes

**Lesson Objectives**
- Understand basic terms of geometry
- Understand basic postulates of geometry

**Vocabulary and Key Concepts**

**Postulate 1-1**
Through any two points there is exactly one line.

**Postulate 1-2**

A line is a series of points that extends in two opposite directions without end.

A point is a location in the set of all points.

**Postulate 1-3**
Through any three noncollinear points there is exactly one plane.

**Postulate 1-4**
Through any three noncollinear points there is exactly one plane.

**Examples**

1. Identify segments and rays in the figure.

**Quick Check**

1. Use the figure at right. Name three points that are collinear and three points that are not collinear.

**Examples**

1. Identify parallel planes in your classroom.

**Quick Check**

1. Identify parallel lines in the diagram at right.

**Examples**

1. Identify plane geometry in the figure.

**Quick Check**

1. Identify collinear points in the figure.

**Examples**

1. Identify segments and rays in the figure.

**Quick Check**

1. Identify parallel planes in your classroom.

**Examples**

1. Identify points, lines, and planes in the figure.

**Quick Check**

1. Identify segments and rays in the figure.

**Examples**

1. Identify parallel lines in the diagram at right.
Lesson 1-5

Measuring Segments

Objectives:
- Identify special angle pairs

Key Concepts:
- A straight angle has a measurement of exactly 180°.

Postulate 1-5: Ruler Postulate
- The points on a line can be put into one-to-one correspondence with the real numbers so that the distance between any two points is the absolute value of the difference of the corresponding numbers.

Postulate 1-6: Segment Addition Postulate
- If three points $A$, $B$, and $C$ are collinear and $B$ is between $A$ and $C$, then $AB + BC = AC$.

Postulate 1-8: Angle Addition Postulate
- If two angles with a common side are adjacent, then the measure of the angle formed by these two angles is the sum of the measures of the two angles.

Quick Check:
1. a. $A$, $B$, and $C$ are points on a line.
2. Find the value of $x$. Then find $AB$ and $NB$.
3. Using the Segment Addition Postulate, find the value of $x$. Then find $AB$ and $NB$.
4. Find lengths $AB$ and $BC$.

Example 1:
- Using the Segment Addition Postulate

Example 2:
- Using the Angle Addition Postulate

Quick Check:
1. Find the value of $x$.
2. $AB = 25$, $CD = 20$, $EF = 30$.

Lesson 1-6

Measuring Angles

Objectives:
- Find the measures of angles
- Identify special angle pairs

Key Concepts:
- An angle is formed by two rays with the same endpoint. The rays are the sides of the angle and the endpoint is the vertex of the angle.

Postulate 1-7: Protractor Postulate
- The absolute value of the difference of the coordinates of the endpoints of a segment is the length of the segment.

Postulate 1-8: Angle Addition Postulate
- If two angles with a common side are adjacent, then the measure of the angle formed by these two angles is the sum of the measures of the two angles.

Quick Check:
1. a. $m\angle 1 + m\angle 2 = 180°$.
2. $x = 40$.

Example 1:
- Given: $\angle 1$ and $\angle 2$ are supplementary.

Example 2:
- Given: $m\angle 1 = 42$ and $m\angle 2 = 38$.

Quick Check:
1. a. $m\angle 1 = 42$, $m\angle 2 = 38$.
2. $m\angle 1 = 140°$.

Example 3:
- Critical Thinking: Find the value of $x$ in the given figure.

Example 4:
- Find the values of $x$ and $y$.

Quick Check:
1. $x = 120°$.
2. $y = 60°$.

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Lesson 1-7  
Basic Constructions

Lesson Objectives
- Use a compass and a straightedge to construct congruent segments and congruent angles
- Use a compass and a straightedge to bisect segments and angles

Vocabulary
- Construction: a procedure for using a compass and straightedge to draw geometric figures

Examples

1. Constructing Congruent Segments
   - Given: AB
   - Construct: AB congruent to CD
   - Step 1: Draw a ray with endpoint E.
   - Step 2: Open the compass the length of AB.
   - Step 3: With the same compass setting, put the compass point on point F. Draw an arc that intersects the ray. Label the point of intersection W.

Quick Check
1. Use a straightedge to draw ST. Then construct TR so that TR = ST.
2. Draw TR. Construct its perpendicular bisector.

Lesson 1-8  
The Coordinate Plane

Lesson Objectives
- Find the distance between two points in the coordinate plane
- Find the coordinates of the midpoint of a segment in the coordinate plane

Key Concepts

- Formula: The Distance Formula
  \[d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\]

- Formula: The Midpoint Formula
  \[M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\]

Examples

1. Finding the Midpoint
   - Given: \(M(2, 3)\) and \(N(-1, 5)\)
   - Use the Midpoint Formula.
   - Midpoint Formula
   - The coordinates of the midpoint \(M\) are \(\left(\frac{2 + (-1)}{2}, \frac{3 + 5}{2}\right)\)

Quick Check
1. Find the coordinates of the midpoint of \(M(-1, 3)\) with endpoints \(N(2, -4)\) and \(P(5, 1)\).
2. The midpoint of \(M(2, 3)\) has coordinates \((x, y)\). X has coordinate \((2, x)\) and \((y, 4)\). Find the coordinates of \(P\).
Lesson 1-9 Perimeter, Circumference, and Area

Lesson Objectives

- Find perimeters of rectangles and squares, and circumferences of circles
- Find areas of rectangles, squares, and circles

Key Concepts

Vocabulary and Key Concepts

Finding Circumference

C = πd
Finding Area of a Circle

A = πr^2

Examples

1. Finding Circumference

- If a circle has a radius of 5 cm, find the circumference of the circle in terms of π.
- Use the formula: C = 2πr
- C = 2π(5) = 10π cm

2. Finding Area of a Circle

- If a circle has a radius of 3 cm, find the area of the circle in terms of π.
- Use the formula: A = πr^2
- A = π(3)^2 = 9π cm^2

Quick Check

1. Identify the hypothesis and the conclusion of this conditional statement:
   If two lines are parallel, then they are coplanar.

   Hypothesis: Two lines are parallel.
   Conclusion: The lines are coplanar.

2. Write the converse of the following conditional:
   If the measure of an angle is 90°, then it is a right angle.

   Hypothesis: If the measure of an angle is 90°.
   Conclusion: Then it is a right angle.

Local Standards: Mathematical Reasoning

NAEP 2005 Strand: Measurement

Topic: Measuring Physical Attributes

Postulate 5-9

If two figures are congruent, then their areas are equal.

Postulate 5-10

The area of a region is the sum of the areas of non-overlapping parts.

Local Standards: Mathematical Reasoning

NAEP 2005 Strand: Measurement

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Lesson 2-2
Biconditionals and Definitions

Vocabulary and Key Concepts

- **Biconditional Statements**
  - A biconditional contains the words "if and only if.
  - *Examples:*
    - A quadrilateral is a square if and only if it contains four right angles.
    - An integer is even if and only if it is divisible by 2.

- **Biconditional:**
  - A biconditional contains the words "if and only if.
  - *Example:*
    - A quadrilateral is a square if and only if it contains four right angles.

**Examples: Using the Law of Detachment**

1. **Conditional:** If a number ends in 6, then it is divisible by 2.
   - *Converse:* If a number is divisible by 2, then it ends in 6.
   - *Conclusion:* The original statement is always true because the converse is always true.

2. **Conditional:** If she rains, then the garden will be watered.
   - *Converse:* If the garden is watered, then it rains.
   - *Conclusion:* The converse is a true statement.

3. **Conditional:** If a number ends in 4, then it is divisible by 2.
   - *Converse:* If a number is divisible by 2, then it ends in 4.
   - *Conclusion:* The original statement is always true because the converse is always true.

4. **Conditional:** If a number is divisible by 10, then it ends in 0.
   - *Converse:* If a number ends in 0, then it is divisible by 10.
   - *Conclusion:* The original statement is a good definition because it is true for all numbers that are divisible by 10.

Quick Check

1. Identify the true conditional statement. Write its converse. If the converse is also true, combine the statements as a biconditional.
   - **Conditional:** If we have a dead battery, then the car will not start.
   - **Converse:** If the car will not start, then we have a dead battery.
   - **Conclusion:** The original statement is a good definition because the converse is always true.

2. Is the following statement a good definition? Explain.
   - **Statement:** A rectangle has four right angles.
   - **Conclusion:** The statement is not a good definition because a rectangle has four right angles and is not necessarily a square.

3. Suppose that a mechanic begins work on a car and finds that the car will not start. Can the mechanic conclude that the car has a dead battery? Explain.
   - **Conclusion:** Yes, because the hypothesis of the second conditional is the same as the conclusion of the first conditional.

4. Using the Law of Syllogism, use the Law of Detachment to draw a conclusion from the following true statements.
   - **Statement:** If a number is divisible by 10, then it ends in 0.
   - **Conclusion:** The number 15 is not divisible by 10 because it does not end in 0.

5. Using the Law of Syllogism, use the Law of Detachment to draw a conclusion from the following true statements.
   - **Statement:** If a number ends in 6, then it is divisible by 2.
   - **Conclusion:** The number 12 is divisible by 2 because it ends in 2.

6. Using the Law of Syllogism, use the Law of Detachment to draw a conclusion from the following true statements.
   - **Statement:** If a number is divisible by 10, then it ends in 0.
   - **Conclusion:** The number 50 is divisible by 10 because it ends in 0.

7. Using the Law of Syllogism, use the Law of Detachment to draw a conclusion from the following true statements.
   - **Statement:** If a number is divisible by 10, then it ends in 0.
   - **Conclusion:** The number 20 is divisible by 10 because it ends in 0.

8. Using the Law of Syllogism, use the Law of Detachment to draw a conclusion from the following true statements.
   - **Statement:** If a number is divisible by 10, then it ends in 0.
   - **Conclusion:** The number 30 is divisible by 10 because it ends in 0.

9. Using the Law of Syllogism, use the Law of Detachment to draw a conclusion from the following true statements.
   - **Statement:** If a number is divisible by 10, then it ends in 0.
   - **Conclusion:** The number 40 is divisible by 10 because it ends in 0.

10. Using the Law of Syllogism, use the Law of Detachment to draw a conclusion from the following true statements.
    - **Statement:** If a number is divisible by 10, then it ends in 0.
    - **Conclusion:** The number 50 is divisible by 10 because it ends in 0.
Lesson 2-5 Proving Angles Congruent

Lesson Objectives
Prove and apply theorems about congruent angles.

Vocabulary and Key Concepts

Theorem 2-1: Vertical Angles Theorem
Vertical angles are congruent.

Theorem 2-2: Congruent Supplements Theorem
If two angles are supplements of the same angle (or of congruent angles), then the two angles are congruent.

Theorem 2-3: Congruent Complements Theorem
If two angles are complements of the same angle (or of congruent angles), then the two angles are congruent.

Theorem 2-4
All right angles are congruent.

Theorem 2-5
If two angles are congruent and supplementary, then each is a right angle.

Examples

1. Proving Vertical Angles: Find the value of $x$.

Vertical angles are congruent.

2. Proving Congruent Complements: If two angles are complements of the same angle, then they are congruent.

Adjacent angles form a linear pair and supplementary.

Quick Check

1. Fill in each missing reason.

2. Name the property of equality or congruence illustrated.

Local Standards: Geometry

NAEP 2005 Strand: Number and Operations

Specific Standard: 4.N.O.1.4

Geometry Lesson 2-5

Daily Notetaking Guide
Lesson 3-1

Properties of Parallel Lines

Vocabulary and Key Concepts

Postulate 3-1: Corresponding Angles Postulate
If a transversal intersects two parallel lines, then corresponding angles are congruent.

Theorem 3-1: Alternate Interior Angles Theorem
If a transversal intersects two parallel lines, then alternate interior angles are congruent.

Theorem 3-2: Same-Side Interior Angles Theorem
If a transversal intersects two parallel lines, then same-side interior angles are supplementary.

Lesson Objectives

- Identify angles formed by two lines and a transversal
- Use and write properties of parallel lines

Lesson 3-2

Proving Lines Parallel

Vocabulary and Key Concepts

Postulate 3-2: Converse of the Corresponding Angles Postulate
If a transversal intersects two lines, then the lines are parallel if corresponding angles are congruent.

Theorem 3-5: Converse of the Alternate Interior Angles Theorem
If a transversal intersects two lines, then the lines are parallel if alternate interior angles are congruent.

Theorem 3-6: Converse of the Same-Side Interior Angles Theorem
If a transversal intersects two lines, then the lines are parallel if same-side interior angles are supplementary.

Lesson Objectives

- Use a transversal in proving lines parallel
- Identify angles formed by two lines and a transversal

Lesson 3-3

Converse of the Corresponding Angles Postulate

If two lines are parallel, then the corresponding angles are congruent.

Theorem 3-4: Converse of the Alternate Interior Angles Theorem
If two lines are parallel, then the alternate interior angles are congruent.

Theorem 3-7: Converse of the Same-Side Interior Angles Theorem
If two lines are parallel, then the same-side interior angles are supplementary.

Theorem 3-8: Converse of the Same-Side Exterior Angles Theorem
If two lines and a transversal form alternate exterior angles, then the lines are parallel.

Lesson Objectives

- Identify angles formed by two lines and a transversal
- Use and write properties of parallel lines
Lesson 3-3
Parallel and Perpendicular Lines

Key Concepts

Theorem 3-0
If two lines are parallel to the same line, then they are parallel to each other.

Theorem 3-10
In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

Theorem 3-11
In a plane, if a line is perpendicular to one of two parallel lines, then it is also perpendicular to the other.

Examples

1. Critical Thinking
   In a plane, if two lines form congruent angles with a third line, must the lines be parallel? Draw a diagram to support your answer.

   No; answers may vary. Sample:
   \[ \angle 1 = 118°, \quad \angle 2 = 62° \]

   The exterior angle and the angle formed by the back of the chair and the armrest are \( \angle 2 \), which together form a straight angle, \( 180° \). Since these measures are congruent, the other measures are equal. The angle formed by the back of the chair and the armrest increases as one measure increases. Hence, you make a lounge chair recline more.

2. Find the value of \( a \) for which \( \angle 1 \) is a right angle.

   \[ m\angle 1 = 90° \]
   \[ 118° - a = 90° \]
   \[ a = 118° - 90° \]
   \[ a = 28° \]

Quick Check

1. Find the value of \( a \) for which \( \angle 2 \) is a right angle. Explain how you can check your answer.

   \[ m\angle 2 = 90° \]
   \[ 62° + a = 90° \]
   \[ a = 90° - 62° \]
   \[ a = 28° \]

   Draw a diagram to support your answer.

Lesson 3-4
Parallel Lines and the Triangle Angle-Sum Theorem

Examples

1. Applying the Triangle Angle-Sum Theorem
   Given: \( \triangle PQR \)
   Find: \( m\angle Z \)
   \[ m\angle P + m\angle Q + m\angle R = 180° \]
   \[ 45° + 35° + Z = 180° \]
   \[ Z = 180° - 45° - 35° \]
   \[ Z = 100° \]

2. Applying the Triangle Exterior Angle Theorem
   Explain what happens to the angle formed by the back of the chair and the armrest as you make a lounge chair recline more.

   The exterior angle and the angle formed by the back of the chair and the armrest are \( \angle 2 \), which together form a straight angle, \( 180° \). Since these measures are congruent, the other measures are equal. The angle formed by the back of the chair and the armrest increases as one measure increases. Hence, you make a lounge chair recline more.

Quick Check

1. \( \triangle ABC \) is a right triangle. \( \angle B \) is a right angle and \( m\angle A = 35° \). Find \( m\angle C \).

   \[ m\angle A + m\angle B + m\angle C = 180° \]
   \[ 35° + 90° + C = 180° \]
   \[ C = 180° - 35° - 90° \]
   \[ C = 55° \]

2. \( \triangle DEF \) is isosceles. \( \angle D \) is the base angle.

   \[ \angle D \] is the base angle.

   \[ \angle D = \angle F \]

   \[ m\angle D + m\angle F + m\angle E = 180° \]
   \[ m\angle D + m\angle F + 35° = 180° \]
   \[ m\angle D + m\angle F = 145° \]

   \[ m\angle D = m\angle F \]
   \[ 2m\angle D = 145° \]
   \[ m\angle D = 72.5° \]
Lesson 3-5
The Polygon Angle-Sum Theorems

Vocabulary and Key Concepts

Theorem 3-14: Polygon Angle-Sum Theorem
The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360°.

Theorem 3-15: Polygon Exterior Angle-Sum Theorem
The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360°.

Examples

Classifying Polygons
Classify the polygon at the right by its sides. Identify it as convex or concave.

Finding a Polygon Angle Sum
Find the sum of the measures of the angles of a decagon.

Quick Check
1. Find the sum of the measures of the angles of a 13-gon.

Name ___________________________________ Class __________________________ Date __________________

Lesson 3-6
Lines in the Coordinate Plane

Vocabulary

The slope-intercept form of a linear equation is \( y = mx + b \).

The standard form of a linear equation is \( Ax + By = C \).

The point-slope form for a nonvertical line is \( y - y_1 = m(x - x_1) \).

Examples

Graphing Lines Using Intercepts
Use the x-intercept and y-intercept to graph \( 5x - 6y = 30 \).

Quick Check
1. Graph each equation.

Name ___________________________________ Class __________________________ Date __________________
Lesson 3-7
Slopes of Parallel and Perpendicular Lines

Lesson Objectives
\( \checkmark \) Use the method learned for constructing congruent angles.
\( \checkmark \) Construct parallel lines
\( \checkmark \) Construct perpendicular lines

Key Concepts

Slopes of Parallel Lines
If two nonvertical lines are parallel, their slopes are equal.
Any two vertical lines are parallel.

Slopes of Perpendicular Lines
If two nonvertical lines are perpendicular, the product of their slopes is -1.
If the slopes of two lines have a product of -1, the lines are perpendicular.
Any horizontal line and vertical line are perpendicular.

Example

1. Determine whether lines are parallel. Are the lines \( y = 5x + 4 \) and \( x = -y + 4 \) parallel? Explain.

   The equation \( y = 5x + 4 \) is in slope-intercept form. 
   \[ y = 5x + 4 \]
   Subtract \( x \) from each side.
   \[ y - x = 4 \]
   Divide each side by \( -5 \).
   \[ m = \frac{4}{-5} \]
   The line \( x = -y + 4 \) is parallel because their slopes are equal.

Example

2. Find the slope of a line perpendicular to \( 5x + 2y = 1 \). Find the slope of the given line, rewrite the equation in slope-intercept form.

   The line \( 5x + 2y = 1 \) has slope \( m = -\frac{5}{2} \).
   Subtract \( 5x \) from both sides.
   \[ 2y = -5x + 1 \]
   Divide each side by 2.
   \[ y = -\frac{5}{2}x + \frac{1}{2} \]
   Simplify.
   \[ m = -\frac{5}{2} \]

Quick Check

1. Are the lines \( y = \frac{3}{4}x + 5 \) and \( 2x + 4y = 9 \) parallel? Explain.

   Yes; Each line has slope \( \frac{3}{4} \) and the y-intercepts are different.

2. Find the slope of a line perpendicular to \( 5x - x = 10 \).

   \[ m = -5 \]

Example

3. Perpendicular From a Point to a Line

   Find the perpendicular from point \( P \) to line \( AB \).

   Use the method learned for constructing congruent angles.
   Step 1: With the compass point on point \( P \), draw an arc that intersects the sides of \( AB \).
   Step 2: With the same compass setting, put the compass point on point \( A \) and draw an arc.
   Step 3: Put the compass point below point \( N \) where the arc intersects line \( AB \). Keeping the same compass setting, put the compass point above point \( N \) where the arc intersects line \( AB \) and draw an arc to locate point \( E \).
   Step 4: Use a straightedge to draw line \( PE \) through the point you located and point \( N \).

Quick Check

1. Use Example 1. Explain why lines \( c \) and \( d \) must be parallel.

   If corresponding angles are congruent, the lines are parallel by the
   Congruent Angles Postulate.

Lesson 3-8
Constructing Parallel and Perpendicular Lines

Lesson Objectives
\( \checkmark \) Construct parallel lines
\( \checkmark \) Construct perpendicular lines

Key Concepts

Constructing a Parallel Line
Examine the diagram at right. Explain how to construct \( AB \) \( \parallel \) \( \ell \). Construct the angle.

Example

1. Constructing a Parallel Line

   Use the method learned for constructing congruent angles.
   Step 1: With the compass point on point \( E \), draw an arc.
   Step 2: With the same compass setting, put the compass point on point \( F \) and draw an arc.
   Step 3: Put the compass point below point \( N \) where the arc intersects line \( EF \). Keeping the same compass setting, put the compass point above point \( N \) where the arc intersects line \( EF \) and draw an arc to locate point \( G \).
   Step 4: Use a straightedge to draw line \( GJ \) through the point you located and point \( N \).

Quick Check

1. Use Example 1. Explain why lines \( c \) and \( d \) must be parallel.

   If corresponding angles are congruent, the lines are parallel by the
   Congruent Angles Postulate.
Lesson 4-1

Vocabulary and Key Concepts

1. **Congruent Figures**
   - Polygons that have corresponding sides and corresponding angles congruent.

2. **Corresponding Parts**
   - Angles:
     - Corresponding angles of congruent triangles are congruent.
   - Sides:
     - Corresponding sides of congruent triangles are congruent.

3. **Congruent Polygons**
   - Polygons that have corresponding sides and corresponding angles congruent.

**Example**

**Naming Congruent Parts**

- **Prove:** \( \triangle ABC \cong \triangle DEF \)

**Solution:**

- \( \angle A \cong \angle D \)
- \( \angle B \cong \angle E \)
- \( \angle C \cong \angle F \)

4. **Using Correspondence**

- **Prove:** \( \triangle ABC \cong \triangle DEF \)

**Solution:**

- \( \angle A \cong \angle D \)
- \( \angle B \cong \angle E \)
- \( AB = DE \)

**Quick Check**

1. It is given that \( \angle A \cong \angle B \). What other information do you need to prove \( \triangle ABC \cong \triangle DEF \)?

   - Angle: \( \angle A \cong \angle D \)

2. It is given that \( \angle A \cong \angle B \). What is \( m \angle C \)? Explain.

   - \( m \angle C = 65^\circ \)

3. You now have two pairs of corresponding congruent sides. What other information do you need to prove \( \triangle ABC \cong \triangle DEF \)?

   - Angle: \( \angle C \cong \angle F \)

4. You now have two pairs of corresponding congruent sides. What other information do you need to prove \( \triangle ABC \cong \triangle DEF \)?

   - Angle: \( \angle B \cong \angle E \)

**Postulate 4-1: Side-Side-Side (SSS) Postulate**

If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent.

**Example**

**Prove:** \( \triangle ABC \cong \triangle DEF \)

**Given:** \( AB = DE \), \( BC = EF \), \( AC = DF \)

**Steps:**

1. \( AB = DE \)
2. \( BC = EF \)
3. \( AC = DF \)

**Quick Check**

1. Given: \( \angle A \cong \angle B \), \( \angle C \cong \angle D \), \( \angle B \cong \angle E \)

   - **Prove:** \( \triangle ABC \cong \triangle DEF \)

   - **Reason:** SAS

2. Given: \( \angle A \cong \angle B \), \( \angle C \cong \angle D \)

   - **Prove:** \( \triangle ABC \cong \triangle DEF \)

   - **Reason:** SAA

3. Given: \( \angle A \cong \angle B \), \( \angle C \cong \angle D \), \( \angle B \cong \angle E \)

   - **Prove:** \( \triangle ABC \cong \triangle DEF \)

   - **Reason:** SSA (Not Congruent)

4. Given: \( \angle A \cong \angle B \), \( \angle C \cong \angle D \)

   - **Prove:** \( \triangle ABC \cong \triangle DEF \)

   - **Reason:** SSA (Not Congruent)
Lesson 4-3

Triangle Congruence

Using only the information in the diagram, can you conclude that the triangles are congruent? Use triangle congruence and CPCTC to prove that parts of two triangles are corresponding parts of congruent triangles are congruent.

Example 1: Using ASA

Suppose that \( \triangle ABC \) and \( \triangle DEF \) are congruent by the ASA Postulate.

Given: \( \angle A \cong \angle D \) and \( \angle B \cong \angle E \) and \( \angle C \cong \angle F \).

Prove: \( \triangle ABC \cong \triangle DEF \).

Statements

1. \( \angle A \cong \angle D \)
2. \( \angle B \cong \angle E \)
3. \( \angle C \cong \angle F \)
4. \( \triangle ABC \cong \triangle DEF \).

Reasons

1. Given
2. \( \angle B \cong \angle E \) by ASA.
3. \( \angle C \cong \angle F \) by ASA.
4. \( \triangle ABC \cong \triangle DEF \) by ASA Postulate.

Quick Check

1. Using only the information in the diagram, can you conclude that \( \triangle DEF \) is congruent to either of the other two triangles? Explain.

Example 2: Using AAS

Given: \( \triangle ABC \) and \( \triangle DEF \) are right triangles with \( \angle A \cong \angle D \) and \( \angle C \cong \angle F \) and \( \angle B \cong \angle E \).

Prove: \( \triangle ABC \cong \triangle DEF \).

Statements

1. \( \angle A \cong \angle D \)
2. \( \angle B \cong \angle E \)
3. \( \angle C \cong \angle F \)
4. \( \triangle ABC \cong \triangle DEF \).

Reasons

1. Given
2. \( \angle B \cong \angle E \) by AAS.
3. \( \angle C \cong \angle F \) by AAS.
4. \( \triangle ABC \cong \triangle DEF \) by AAS Postulate.

Quick Check

1. In Example 2, explain how you could prove \( \triangle ABC \cong \triangle DEF \) using AAS.

Example 3: Using SAS

Given: \( \triangle ABC \) and \( \triangle DEF \) are congruent by the SAS Postulate.

Prove: \( \angle A \cong \angle D \) and \( \angle B \cong \angle E \) and \( \angle C \cong \angle F \).

Statements

1. \( \angle A \cong \angle D \)
2. \( \angle B \cong \angle E \)
3. \( \angle C \cong \angle F \)
4. \( \triangle ABC \cong \triangle DEF \).

Reasons

1. Given
2. \( \angle B \cong \angle E \) by SAS.
3. \( \angle C \cong \angle F \) by SAS.
4. \( \triangle ABC \cong \triangle DEF \) by SAS Postulate.

Quick Check

1. In Example 3, what can you say about \( \angle A \) and \( \angle D \)? Explain.

Example 4: Using CPCTC

Given: \( \triangle ABC \) and \( \triangle DEF \) are congruent.

Prove: \( \angle A \cong \angle D \) and \( \angle B \cong \angle E \) and \( \angle C \cong \angle F \).

Statements

1. \( \triangle ABC \cong \triangle DEF \)
2. \( \angle A \cong \angle D \) and \( \angle B \cong \angle E \) and \( \angle C \cong \angle F \)
3. \( \angle A \cong \angle D \) and \( \angle B \cong \angle E \) and \( \angle C \cong \angle F \) by CPCTC.

Reasons

1. Given
2. \( \angle A \cong \angle D \) and \( \angle B \cong \angle E \) and \( \angle C \cong \angle F \) by CPCTC.
3. \( \angle A \cong \angle D \) and \( \angle B \cong \angle E \) and \( \angle C \cong \angle F \) by CPCTC.

Quick Check

1. Recall Example 4. About how wide was the river if the officer paced off 20 paces and each pace was about 2 feet long? Explain.

Example 5: Using Right Triangles

According to legend, one of Napoleon’s followers used right triangles to estimate the width of a river. If the man on the bank paced off distances from the tip of his visor to corresponding points on the opposite bank, he could easily determine the width of the river. Explain the method used.

Given: \( \triangle ABC \) is a right triangle with \( \angle A \) a right angle.

Prove: \( \angle A \cong \angle D \) and \( \angle B \cong \angle E \) and \( \angle C \cong \angle F \).

Statements

1. \( \angle A \cong \angle D \)
2. \( \angle B \cong \angle E \)
3. \( \angle C \cong \angle F \)
4. \( \triangle ABC \cong \triangle DEF \).

Reasons

1. \( \angle A \cong \angle D \) by Right Angles.
2. \( \angle B \cong \angle E \) by Right Angles.
3. \( \angle C \cong \angle F \) by Right Angles.
4. \( \triangle ABC \cong \triangle DEF \) by Right Angles.
Lesson 4-5: Isosceles and Equilateral Triangles

Vocabulary and Key Concepts:

- **Isosceles Triangle**: A triangle with at least two congruent sides.
- **Equilateral Triangle**: A triangle with all sides congruent.
- **Base**: The side opposite the vertex angle.
- **Base Angles**: The angles opposite the congruent sides.
- **Vertex Angle**: The angle opposite the base.

**Theorem 4-3: Isosceles Triangle Theorem**
If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

**Theorem 4-4: Converse of Isosceles Triangle Theorem**
If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

**Theorem 4-5**
The diagram shows the following congruent parts. By the HL Theorem, conclude that \( \triangle ABC \) is isosceles.

**Quick Check**

a. In the figure, suppose \( \angle ACB = 55 \). Find \( \angle ABC \) and \( \angle BAC \).

b. In the figure, can you deduce that \( \triangle ABC \) is isosceles? If not, \( \angle ACB \) or \( \angle BAC \) can be shown to be congruent to \( \angle A \).

**Example**

Using the Isosceles Triangle Theorems: Explain why \( \triangle ABC \) is isosceles.

\( \angle ABC \) and \( \angle BAC \) are \( \angle XAB \) angles formed by \( \overline{XD, BC} \) and the transversal \( \overline{XY} \). Because \( \overline{XD} \parallel \overline{BC} \),

\( \angle ABC = \angle BAC \).

The diagram shows that \( \angle XAB = \angle ACB \). By the triangle sum theorem, \( \angle XAB = \angle ACB \).

**Quick Check**

1. In the figure, suppose \( \angle ACB = 55 \). Find \( \angle ABC \) and \( \angle BAC \).

2. In the figure, can you deduce that \( \triangle ABC \) is isosceles? If not, \( \angle ACB \) or \( \angle BAC \) can be shown to be congruent to \( \angle A \).

3. Two-Column Proof—Using the HL Theorem

**Given**: \( \triangle ABC \) and \( \triangle DCB \) are right triangles. \( \overline{AB} \parallel \overline{DC} \)

**Prove**: \( \triangle ABC \cong \triangle DCB \)

**Statements**

1. \( \angle ABC \) and \( \angle DCB \) are right angles.
2. Definition of Right Triangle
3. \( \overline{AB} \parallel \overline{DC} \)
4. Reflexive Property of Congruence
5. \( \triangle ABC \cong \triangle DCB \)

**Quick Check**

1. \( \triangle ABC \) and \( \triangle DCB \) are right triangles.

2. You know that two legs of one right triangle are congruent to two legs of another right triangle. Explain how to prove the triangles are congruent.

3. Which two triangles are congruent by the HL Theorem? Write a correct congruence statement.

4. \( \triangle ABC \cong \triangle DCB \)

5. You know that two legs of one right triangle are congruent to two legs of another right triangle. Explain how to prove the triangles are congruent.

6. Since all right angles are congruent, the triangles are congruent by HL.
Lesson 5-1
Midsegments of Triangles

Lesson Objectives
➢ Use properties of midsegments to solve problems

Vocabulary and Key Concepts
Theorem 5-1: Triangle Midsegment Theorem
If a segment joins the midpoints of two sides of a triangle, then the segment is parallel to the third side, and is half its length.

Examples
1. Finding Lengths In \( \triangle XYZ \), \( M \) and \( N \) are midpoints.
   - The perimeter of \( \triangle MNP \) is 60. Find \( NP \) and \( YZ \).
   - Because the perimeter of \( \triangle MNP \) is 60, you can find \( NP \):
     \[ NP = MN + MP = 60 \quad \text{(definition of perimeter)} \]
   - \( NP = 60 \)
   - \( NP = \frac{1}{2} \cdot 60 \) (by the Distance Formula)
   - Use the Triangle Midsegment Theorem to find \( YZ \):
     \[ MP = YZ \quad \text{(Triangle Midsegment Theorem)} \]
     \[ MP = \frac{1}{2} \cdot 60 \]
     \[ MP = 30 \]
     \[ YZ = 30 \times 2 = 60 \]

Quick Check
1. The diagram shows triangles in the scaffoldings that workers used when they repaired and cleaned the Statue of Liberty.
   a. Name the common side in \( \triangle ABC \) and \( \triangle BCD \).
   b. Name another pair of triangles that share a common side. Name the common side.

2. Write a two-column proof.
   Given: \( \overline{MP} = \overline{PQ} = \overline{QR} \)
   Prove: \( \triangle MNP \) is equilateral

Statements Reasons
1. \( \overline{MP} = \overline{PQ} = \overline{QR} \) 1. Given
2. \( \angle MPQ = \angle MPQ \) 2. Reflexive Property of Congruence
3. \( \triangle MPQ \) is equilateral 3. Definition of an equilateral triangle
4. \( \triangle MNP \) is equilateral 4. CPCTC
5. \( \angle NMP = \angle NPM \) 5. Reflexive Property of Congruence
6. \( \angle NMZ = \angle NMP \) 6. SAS

Local Standards: ____________________________________
Relationships Among Geometric Figures

Critical Thinking
Find \( m\angle XYZ \). Justify your answer.

1. The diagram shows triangles \( \triangle UVZ \) and \( \triangle XYZ \).
   - \( \triangle UVZ \) is similar to \( \triangle XYZ \).
   - Find \( m\angle XYZ \). Justify your answer.

2. \( \angle XYZ \) is 60°; so \( \triangle UVZ \) is 60°.
   - \( \angle UVZ \) is 60°.
   - \( \angle UVZ \) is 60°.
   - \( \angle UVZ \) is 60°.

3. \( \angle UVZ \) is 60°; so \( \triangle UVZ \) is 60°.
   - \( \angle UVZ \) is 60°.
   - \( \angle UVZ \) is 60°.
   - \( \angle UVZ \) is 60°.

4. \( \angle UVZ \) is 60°; so \( \triangle UVZ \) is 60°.
   - \( \angle UVZ \) is 60°.
   - \( \angle UVZ \) is 60°.
   - \( \angle UVZ \) is 60°.

5. \( \angle UVZ \) is 60°; so \( \triangle UVZ \) is 60°.
   - \( \angle UVZ \) is 60°.
   - \( \angle UVZ \) is 60°.
   - \( \angle UVZ \) is 60°.

6. \( \angle UVZ \) is 60°; so \( \triangle UVZ \) is 60°.
   - \( \angle UVZ \) is 60°.
   - \( \angle UVZ \) is 60°.
   - \( \angle UVZ \) is 60°.

7. \( \angle UVZ \) is 60°; so \( \triangle UVZ \) is 60°.
   - \( \angle UVZ \) is 60°.
   - \( \angle UVZ \) is 60°.
   - \( \angle UVZ \) is 60°.

8. \( \angle UVZ \) is 60°; so \( \triangle UVZ \) is 60°.
   - \( \angle UVZ \) is 60°.
   - \( \angle UVZ \) is 60°.
   - \( \angle UVZ \) is 60°.

9. \( \angle UVZ \) is 60°; so \( \triangle UVZ \) is 60°.
   - \( \angle UVZ \) is 60°.
   - \( \angle UVZ \) is 60°.
   - \( \angle UVZ \) is 60°.

10. \( \angle UVZ \) is 60°; so \( \triangle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.

11. \( \angle UVZ \) is 60°; so \( \triangle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.

12. \( \angle UVZ \) is 60°; so \( \triangle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.

13. \( \angle UVZ \) is 60°; so \( \triangle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.

14. \( \angle UVZ \) is 60°; so \( \triangle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.

15. \( \angle UVZ \) is 60°; so \( \triangle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.

16. \( \angle UVZ \) is 60°; so \( \triangle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.

17. \( \angle UVZ \) is 60°; so \( \triangle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.

18. \( \angle UVZ \) is 60°; so \( \triangle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.

19. \( \angle UVZ \) is 60°; so \( \triangle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.

20. \( \angle UVZ \) is 60°; so \( \triangle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.
    - \( \angle UVZ \) is 60°.
Lesson 5-2
Bisectors in Triangles

Lesson Objective
Identify properties of perpendicular bisectors and angle bisectors.

Vocabulary and Key Concepts

Theorem 5-2: Perpendicular Bisector Theorem
If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

Theorem 5-3: Converse of the Perpendicular Bisector Theorem
If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

Theorem 5-4: Angle Bisector Theorem
If a point is equidistant from the sides of an angle, then it is on the angle bisector.

Theorem 5-5: Converse of the Angle Bisector Theorem
If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.

Quick Check
a. According to the diagram, how far is K from \( \overline{FD} \)?

b. What can you conclude about \( \overline{FD} \)?

Example
Using the Angle Bisector Theorem
Find \( \overline{FB} \) and \( \overline{FD} \) in the diagram at right.

\[ \overline{FD} = \frac{1}{2}(7 + 37) = 22 \]

Quick Check
a. Find the value of \( x \).

b. Critical Thinking In Example 1, explain why it is not necessary to find the third perpendicular bisector.

Critical Thinking
To find the third perpendicular bisector, you need to find the equation of the bisector of the angle formed by the two existing perpendicular bisectors. However, in the given problem, you only need to find the bisector of the angle formed by \( \overline{FD} \) and \( \overline{FB} \), which is sufficient to determine its location. Therefore, the third bisector is not necessary.

Lesson 5-3
Concurrent Lines, Medians, and Altitudes

Lesson Objective
Identify properties of perpendicular bisectors and angle bisectors.

Vocabulary
Concurrent lines are three or more lines that meet in one point.

Finding the Circumcenter
Find the center of the circle that circumscribes \( \triangle XYZ \).

Example
Finding the Circumcenter
Find the center of the circle that circumscribes \( \triangle XYZ \).

The perpendicular bisector of \( \overline{YZ} \) is the horizontal line that passes through \( (4, 0) \), and the equation of the perpendicular bisector is \( y = 0 \).

Because \( Y \) has coordinates \((0, 6)\) and \( Z \) has coordinates \((5, 0)\), the perpendicular bisector of \( \overline{YZ} \) is the vertical line that passes through \( (4, 0) \), and the equation of the perpendicular bisector is \( x = 4 \).

The lines \( y = 0 \) and \( x = 4 \) intersect at the point \((4, 0)\), which is the center of the circle that circumscribes \( \triangle XYZ \).
Lesson 5-4

Inverses, Contrapositives, and Indirect Reasoning

<table>
<thead>
<tr>
<th>Lesson Objective</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use inverses, contrapositives, and indirect reasoning</td>
<td></td>
</tr>
</tbody>
</table>

Vocabulary and Key Concepts

Negation of a Conditional Statement

- The opposite of a conditional statement has the same truth value.

Examples

1. Writing the Negation of a Statement
   - Write the negation of "It is not true that all triangles are isosceles.

Quick Check

1. Write the negation of each statement:
   - a. If you stand for something, you won't fall for anything.
   - b. Today is not Tuesday.

2. Write (a) the inverse and (b) the contrapositive of the statement "If a polygon is not a convex polygon, then it is not a straight angle.

Lesson 5-5

Inequalities in Triangles

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Key Concepts

- Corollary to the Triangle Exterior Angle Theorem
- The measure of an angle of a triangle is greater than the measure of each of its remote angles.

Examples

1. Applying Theorem 5-12
   - In \( \triangle ABC \), \( \angle B = 14 \), \( \angle C = 12 \), and \( \angle A = 20 \).
   - Find the angle opposite each side.

Quick Check

1. List the sides of \( \triangle ABC \) in order from smallest to largest.

2. Can a triangle have sides with the given lengths? Explain:
   - a. 6, 8, 10
   - b. 7, 10, 13

Local Standards: ____________________________________

Mathematical Reasoning

Topics:

- NAEP 2005 Strand: Geometry
- Topic: Relationships Among Geometric Figures

Local Standards: ____________________________________

Mathematical Reasoning

Topics:

- NAEP 2005 Strand: Geometry
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Local Standards: ____________________________________

NAEP 2005 Strand: Geometry

Topic: Relationships Among Geometric Figures

Local Standards: ____________________________________
Lesson 6-1
Classifying Quadrilaterals

Lesson Objective
✓ Use relationships among sides and angles of quadrilaterals
✓ Use relationships involving diagonals of parallelograms and transversals

Vocabulary and Key Concepts

Theorem 6-1
Opposite sides of a parallelogram are congruent.

Theorem 6-2
Opposite angles of a parallelogram are congruent.

Theorem 6-3
The diagonals of a parallelogram bisect each other.

Examples

Using Algebra

Find the values of x in \( \text{ABCD} \). Then find \( m\angle A \).

\[ x + 15 = 135 - x \]

Add \( x \) to both sides.

\[ 135 - 45 \]

Substitute \( x = 15 \).

Quick Check

1. Find the value of \( y \) in \( \text{ABCD} \).

2. Find the value of \( x \) in \( \text{ABCD} \).

Quick Check

1. Find the slope of each side.

2. Find the slope of each side.

Lesson 6-2
Properties of Parallelograms

Lesson Objective
✓ Use relationships among sides and angles of parallelograms
✓ Use relationships involving diagonals of parallelograms and transversals

Vocabulary and Key Concepts

Theorem 6-1
Opposite sides of a parallelogram are congruent.

Theorem 6-2
Opposite angles of a parallelogram are congruent.

Theorem 6-3
The diagonals of a parallelogram bisect each other.

Examples

Using Algebra

Find the values of \( x \) and \( y \) in \( \text{KLNM} \).

\[ 2x + 5 = 3y \]

Subtract 5 from each side.

\[ 2y - 18 \]

Subtract 5 from each side.

Quick Check

1. Find the values of \( x \) and \( y \) in \( \text{ABCD} \).

2. Find the values of \( x \) and \( y \) in \( \text{ABCD} \).

Quick Check

1. Find the values of \( x \) and \( y \) in \( \text{ABCD} \).

2. Find the values of \( x \) and \( y \) in \( \text{ABCD} \).
Lesson 6-3

**Key Concepts**

- **Theorem 6-3**: If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
- **Theorem 6-6**: If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
- **Theorem 6-7**: If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
- **Theorem 6-8**: If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.

**Examples**

1. **Finding Values for Parallelograms**: Find values for which \(ABCD\) is a parallelogram.
   
   - **Theorem 6-9**: \(ABCD\) for which \(x\) and \(y\) must be parallel.
   - **Theorem 6-10**: \(ABCD\) bisect each other.
   - **Theorem 6-11**: \(ABCD\) perpendicular.

2. **Finding Angle Measures**: Find the measure of the numbered angles in the rhombus.
   - **Theorem 6-13**: \(ABCD\) bisects two angles of the rhombus, so \(m\angle 1 = 78\).
   - **Theorem 6-14**: \(ABCD\) perpendicular.

3. **Finding Diagonal Length**: Find the length of each diagonal.
   - **Theorem 6-15**: \(ABCD\) parallel and congruent (Theorem 6-4).
   - **Theorem 6-16**: \(ABCD\) congruent.
   - **Theorem 6-17**: \(ABCD\) and \(BCDE\) from each side.
   - **Theorem 6-18**: \(ABCD\) each side of each.

4. **Quick Check**: Determine whether a parallelogram is a parallelogram from what is given? Explain.
   - **Theorem 6-19**: \(ABCD\) congruent.

**Lesson Objectives**

- Use properties of diagonals of a rhombus.
- Use properties of diagonals of a rectangle.

**Local Standards**

- **NAEP 2005 Strand**: Geometry
- **Local Standards**: ____________________________
Lesson 6-5

Trapezoids and Kites

Lesson Objective
- Prove properties of trapezoids and kites

Vocabulary and Key Concepts

- Trapezoids
- Theorem 6-15
- Theorem 6-16
- Kites
- Theorem 6-17

Examples

1. Finding Angle Measures in Trapezoids: WXYZ is an isosceles trapezoid, and \( m\angle W = 120 \). Find \( m\angle Y \), \( m\angle X \), \( m\angle Z \), and \( m\angle W \).

   - \( m\angle W + m\angle Z = 180 \) (base angles of an isosceles trapezoid are congruent).
   - \( m\angle W = 120 \), so \( m\angle Z = 180 - 120 = 60 \).
   - \( m\angle X + m\angle Y = 180 \) (base angles of an isosceles trapezoid are congruent).
   - \( m\angle Y = 180 - 120 = 60 \).

Quick Check

1. In the isosceles trapezoid, \( m\angle X = 70 \). Find \( m\angle P \), \( m\angle Q \), and \( m\angle R \).

Lesson 6-6

Placing Figures in the Coordinate Plane

Lesson Objective
- Use coordinates of special figures to determine their positions and directions.

Vocabulary and Key Concepts

- Coordinates of Special Figures
- Position and Direction

Examples

1. Proving Congruency: Show that \( \triangle ABC \) is a parallelogram by proving pairs of opposite sides congruent.

   - \( A(0, 0) \), \( B(x, 0) \), \( C(x, y) \), and \( D(0, y) \) are the coordinates of the vertices of \( \triangle ABC \).
   - \( A'B'C'D' \) is the image of \( \triangle ABC \) after a reflection over the y-axis.
   - \( A'B' \) is parallel to \( BC \) and \( AB \) is parallel to \( CD \).

Quick Check

1. Use the diagram above. Use a different method: Show that \( \triangle ABC \) is a parallelogram by finding the midpoints of the diagonals.

   - Midpoint of \( AC \) is \( (\frac{x}{2}, \frac{y}{2}) \).
   - Midpoint of \( BD \) is \( (\frac{x}{2}, \frac{y}{2}) \).

   Thus, the diagonals bisect each other, and \( \triangle ABC \) is a parallelogram.
Lesson 6-7

Proofs Using Coordinate Geometry

Vocabulary and Key Concepts

Theorem 6-18: Trapezoid Midsegment Theorem

The midsegment of a trapezoid is the segment that joins the midpoints of the nonparallel sides of the trapezoid. The length of this segment is equal to one half the sum of the lengths of the bases.

A scale model of a car is 4 in. long. The actual car is 15 ft long. What is the ratio of the length of the model to the length of the car?

Finding Ratios

If the diagonals of a parallelogram are equal, then the parallelogram is a rectangle.

Quick Check

1. A photo that is 8 in. wide and 12 in. high is enlarged to a poster that is 2 ft wide and 3 ft high. What is the ratio of the height of the photo to the height of the poster?

2. Solve each proportion.
   a. \( \frac{\text{Cross}}{\text{Product Property}} \)
   b. \( \frac{\text{Cross}}{\text{Product Property}} \)
   c. \( \frac{\text{Cross}}{\text{Product Property}} \)

Lesson 6-6

From Lesson 6-6, you know that XYZW is a rectangle.

Lesson 6-7

A parallelogram is a from Theorem 6–14. If the diagonals of a parallelogram are equal, then the parallelogram is a rectangle.

Quick Check

Use the diagonals of a rectangle to find the lengths of its diagonals, and then compare them to show that they are equal.

If the diagonals of a parallelogram are equal, then the parallelogram is a rectangle.
Lesson 7-2

**Lesson Objectives:**
- Identify similar polygons
- Apply similar polygons
- Identify corresponding parts of similar triangles

**Vocabulary:**
- Similarity ratio
- Golden proportion

**Examples:**

1. **Understanding Similarity:** \( \triangle ABC \sim \triangle DEF \)
   - Complete:
   - **A**
   - **B**
   - **C**
   - **D**
   - **E**
   - **F**

2. **Quick Check:**
   - Complete:
   - **m**
   - **s**
   - **x**
   - **y**

**Using Similarity Theorems:**
- Explain why the triangles must be similar. Write a similarity statement.
- **Example:**
  - Given: \( \angle ABC \sim \angle DEF \)
  - **Solution:** \( \triangle ABC \sim \triangle DEF \) because corresponding angles are congruent.

**Quick Check:**
- 1. Refer to the diagram for Example 1. Complete:
  - **m**
  - **s**
  - **x**
  - **y**

**Lesson 7-3**

**Lesson Objectives:**
- Use AA, SAS, and SSS similarity statements
- Apply AA, SAS, and SSS similarity statements

**Vocabulary and Key Concepts:**
- Postulate 7-1: Angle-Angle Similarity (AA~) Postulate
- Postulate 7-2: Side-Angle-Side Similarity (SAS~) Theorem
- Theorem 7-1: Side-Angle-Side Similarity (SAS~) Theorem
- Theorem 7-2: Side-Side-Side Similarity (SSS~) Theorem

**Examples:**

1. **Using AA~ Postulate:** \( \angle P \sim \angle Q \)
   - Explain why the triangles are similar. Write a similarity statement.
   - **Solution:** \( \triangle ABC \sim \triangle DEF \) because \( \angle A \sim \angle D \) and \( \angle B \sim \angle E \).

2. **Quick Check:**
   - 1. In Example 1, you have enough information to write a similarity statement. Do you have enough information to find the similarity ratio? Explain.

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**Local Standards:**
- NAEP 2005 Strand: Geometry and Measurement

**Topics:**
- Transformation of Shapes and Preservation of Scale

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**Geometric Mean:**
- The geometric mean of two positive numbers \( a \) and \( b \) is \( \sqrt{ab} \).
- The geometric mean can be used to find an unknown length in a right triangle by the geometric mean theorem.

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**Golden Ratio:**
- The golden ratio is approximately 1.618 and is denoted by \( \phi \).
- The golden ratio is the ratio of the length to the width of any golden rectangle, about 1:1.618.

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**Indirect Measurement:**
- A way of measuring things that are difficult to measure directly.

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**Golden Rectangle:**
- A rectangle with side lengths in the golden ratio, about 1:1.618.
Lesson 7-4

Lesson Objective
Find and use relationships in similar right triangles.

Vocabulary and Key Concepts

1. **Examples**

   **Theorem 7-3**
   The altitude to the hypotenuse of a right triangle divides the triangle into two triangles that are similar to each other.

   **Corollary 1 to Theorem 7-3**
   The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments of the hypotenuse.

   **Theorem 7-4**
   If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the other two sides proportionally.

   **Corollary 2 to Theorem 7-4**
   The geometric mean of two positive numbers $a$ and $b$ is the positive number $x$ such that $x^2 = ab$.

   **Finding the Geometric Mean**
   The geometric mean of two positive numbers is the positive square root of the product of the numbers.

   **Finding Distance**
   Maria drove her ball 192 yd straight toward the cup. Her brother Gabriel drove his ball straight 240 yd, but not toward the cup. The diagram shows the results. Find $x$ and $y$, their remaining distances from the cup.

   **Quick Check**
   1. Find the geometric mean of 5 and 20.
   2. Recall Example 2: Find the distance between Maria’s ball and Gabriel’s ball.

   **Lesson Objectives**
   - Use the Triangle-Angle-Bisector Theorem to find the value of $x$.
   - Use Corollary 2 of Theorem 7-3 to solve for $y$.

   **Local Standards**
   - NAEP 2005 Strand: Geometry
   - Topic: Transformation of Shapes and Preservation of Properties

   **Using the Triangle-Angle-Bisector Theorem**
   If a ray bisects an angle of a triangle, then it divides the opposite side into two segments that are proportional to the other two sides of the triangle.

   **Using the Triangle-Angle-Bisector Theorem**
   Find and use relationships in similar right triangles.
Lesson 8-1: The Pythagorean Theorem and Its Converse

**Vocabulary and Key Concepts**

- **Pythagorean Theorem**
- In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

\[ a^2 + b^2 = c^2 \]

**Examples**

- **Pythagorean Triples**
  - Find the length of the hypotenuse of \( \triangle ABC \). Do the lengths of the sides of \( \triangle ABC \) form a Pythagorean triple?
  - \( a = 6, b = 8, c = 10 \)
  - Hypotenuse: \( c = \sqrt{a^2 + b^2} = \sqrt{36 + 64} = \sqrt{100} = 10 \)
  - Yes, it is a Pythagorean triple.

**Quick Check**

1. **Is it a Right Triangle?** Is this triangle a right triangle?
   - \( a = 3, b = 4, c = 5 \)
   - Yes, it is a right triangle.

2. A triangle has sides of lengths 4, 5, and 7. Is it a right triangle?
   - No, it is not a right triangle.

**Lesson 8-2: Special Right Triangles**

**Lesson Objectives**
- Use the properties of 45°-45°-90° triangles.
- Use the properties of 30°-60°-90° triangles.

**Key Concepts**

- **45°-45°-90° Triangle Theorem**
  - In a 45°-45°-90° triangle, both legs are congruent, and the length of the hypotenuse is \( \sqrt{2} \) times the length of a leg.

- **30°-60°-90° Triangle Theorem**
  - In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the shorter leg.

**Examples**

1. **Finding the Length of the Hypotenuse**
   - Find the value of each variable.
   - Use the 45°-45°-90° Theorem to find the hypotenuse.
   - \( a = \frac{x}{2} \) hypotenuse \( = \sqrt{2} \) leg
   - \( a = \frac{x}{2} \)
   - Simplify
   - \( a = \frac{x}{2} \)
   - The length of the hypotenuse is \( \sqrt{2} \).

**Quick Check**

1. **Using the Length of One Side**
   - Find the value of each variable.
   - Use the 30°-60°-90° Theorem to find the lengths of the legs.
   - \( y = \frac{x}{2} \) shorter leg
   - \( y = \frac{x}{2} \)
   - Simplify
   - \( y = \frac{x}{2} \)

   - The length of the shorter leg is \( \frac{x}{2} \) and the length of the longer leg is \( x \).

2. **Finding the Length of the Hypotenuse of a 45°-45°-90° Triangle with Legs of Length 3 and 3**
   - \( a = 3 \) and \( b = 3 \)
   - \( c = \sqrt{a^2 + b^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2} \)

   - The length of the hypotenuse is \( 3\sqrt{2} \).
Lesson 8-3

The Tangent Ratio

Vocabulary
The tangent of a is the ratio of the length of the leg opposite \( \angle A \) to the length of the leg adjacent to \( \angle A \).

Writing Tangent Ratios
Write the tangent ratios for \( \angle A \) and \( \angle B \).

\[
\tan A = \frac{\text{opposite}}{\text{adjacent}}
\]

\[
\tan B = \frac{\text{opposite}}{\text{adjacent}}
\]

Examples
1. Writing Tangent Ratios: Write the tangent ratios for \( \angle A \) and \( \angle B \).

Lesson 8-4

Sine and Cosine Ratios

Vocabulary
The sine of \( \angle A \) is the ratio of the length of the leg opposite \( \angle A \) to the hypotenuse.

The cosine of \( \angle A \) is the ratio of the length of the leg adjacent to \( \angle A \) to the hypotenuse.

Writing Sine and Cosine Ratios
Use the triangle to find \( \sin T \), \( \cos T \), \( \sin G \), and \( \cos G \). Write your answers in simplest form.

\[
\sin T = \frac{\text{opposite}}{\text{hypotenuse}}
\]

\[
\cos T = \frac{\text{adjacent}}{\text{hypotenuse}}
\]

\[
\sin G = \frac{\text{opposite}}{\text{hypotenuse}}
\]

\[
\cos G = \frac{\text{adjacent}}{\text{hypotenuse}}
\]
Lesson 8-5
Angles of Elevation and Depression

Lesson Objective
Identifying Angles of Elevation and Depression

Vocabulary
- An angle of elevation is the angle formed by a horizontal line and the line of sight to an object above the horizontal line.
- An angle of depression is the angle formed by a horizontal line and the line of sight to an object below the horizontal line.

Quick Check
1. Describe each angle as it relates to the situation shown.
   - a. ______
   - b. ______

2. An airplane pilot sees a life raft at a 26° angle of depression.
   a. Calculate the airplane's altitude is 3 km. What is the airplane's surface distance from the raft?
   b. ______

Lesson 8-6
Vectors

Lesson Objective
Adding Vectors

Vocabulary and Key Concepts
- A vector is any quantity with magnitude (size) and direction.
- A vector can be represented by an arrow.
- The magnitude of a vector is its size, or length.
- The initial point of a vector is the point at which it begins.
- The terminal point of a vector is the point at which it ends.
- A resultant vector is the sum of other vectors.

Quick Check
1. Describe the vector as an ordered pair. Give the coordinates to the nearest tenth.
   - ______

2. Write the sum of the two vectors (-2, 3) and (4, -5) as an ordered pair.
   ______

Examples
1. Describing a Vector
   Describe \( \vec{C} \) as an ordered pair.
   \( \vec{C} = (3, 4) \)
   - The coordinates are (3, 4).
   - The magnitude of the vector is \( \sqrt{3^2 + 4^2} = 5 \).
Lesson 9-1
Translations

**Lesson Objectives**
- Identify isometries
- Find translation images of figures

**Vocabulary**
A transformation of a geometric figure is a change in its position, shape, or size. In a transformation, the **preimage** is the original image before changes are made. In a transformation, the image is the resulting figure after changes are made. An **isometry** is a transformation in which the preimage and the image are congruent. A translation (slide) is a transformation that maps all points the same distance and in the same direction. A composition of transformations is a combination of two or more transformations. Includes reflection, rotation, and translation.

**Examples**
1. **Identifying Isometries**
   - Does the transformation appear to be an isometry? Explain.
   - The image appears to be the same as the preimage, but the transformation appears to be an isometry.

Quick Check
1. What are the coordinates of the image of point A if the reflection line is y = -3?
   - (-3, -6)

Lesson 9-2
Reflections

**Lesson Objectives**
- Find reflection images of figures

**Vocabulary**
A reflection in line \( r \) is a transformation such that if a point \( A \) is on line \( r \), then its image \( B \) is the point such that \( r \) is the perpendicular bisector of \( AB \). The line of reflection is the perpendicular bisector of \( AB \) if \( B \) is in \( r \).

**Examples**
1. **Finding Reflection Images**
   - If point \( Q(-1, 2) \) is reflected across line \( r \), what are the coordinates of its reflection image?
   - \( Q' \) is the point such that \( r \) is the perpendicular bisector of \( QQ' \).

Quick Check
1. What is the coordinate of the image of point \( Q \) if the reflection line is \( y = -3 \)?
   - \((-3, -6)\)

Lesson 9-3
Reflections

**Lesson Objectives**
- Find reflection images of figures

**Vocabulary**
A reflection in line \( r \) is a transformation such that if a point \( A \) is on line \( r \), then its image \( B \) is the point such that \( r \) is the perpendicular bisector of \( AB \). The line of reflection is the perpendicular bisector of \( AB \) if \( B \) is in \( r \).

**Examples**
1. **Finding Reflection Images**
   - If point \( Q(-1, 2) \) is reflected across line \( r \), what are the coordinates of its reflection image?

Quick Check
1. What is the coordinate of the image of point \( Q \) if the reflection line is \( y = -3 \)?
   - \((3, -6)\)
Lesson 9-4  Symmetry

Lesson Objective
Identify the type of symmetry in a figure

Vocabulary
A figure has **symmetry** if there is an isometry that maps the figure onto itself.

- **Reflectional Symmetry**: If there is symmetry that maps the figure onto itself.
- **Line Symmetry**: Same as reflectional symmetry.
- **Rotational Symmetry**: If the figure is its own image for some rotation of 180° or less.

Examples
1. Draw a rectangle and all of its lines of symmetry.
2. A regular pentagon has 5 congruent triangles. Name the image of ABCDEF for a 180° rotation about the center of the pentagon.

Quick Check
1. Draw the image of LLOM for a 90° rotation about point O. Label the vertices of the image.
2. A regular pentagon PENTA is divided into 5 congruent triangles. Name the image of T for a 144° rotation about point A.

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Lesson 9-5

**Lesson Objectives**
- Locate dilation images of figures

**Vocabulary**
A dilation is a transformation with center C and scale factor k for which the following are true:
- The image of C is C
- For any point R, R is on CR and CR
- The dilation is a reduction with center (0, 0) and scale factor k.

**Finding a Scale Factor**
A is a dilation with a scale factor less than 1.
An enlargement is a dilation with a scale factor greater than 1.

**Theorem 9-5: Isometry Classification Theorem**
- The image of R is on and CR
- The dilation is a reduction with center C and scale factor k.

**Example**
1. Find the image of R through another reflection across line x and then in line y.
2. The height of a tractor-trailer truck is 4.2 m. The scale factor for a model truck is 1/100. Find the height of the model to the nearest centimeter.

**Quick Check**
- 1. Quadrilateral JKLM is a dilation image of quadrilateral JKLM. Describe the dilation.
- 2. The composition of two reflections across intersecting lines is a rotation. The composition of two reflections across parallel lines is a reflection. Describe the resulting rotation.

Lesson 9-6

**Lesson Objectives**
- Use a composition of reflections
- Identify glide reflections

**Vocabulary and Key Concepts**
- Theorem 9-1: A translation or rotation is a composition of two reflections.
- Theorem 9-2: A composition of reflections across two parallel lines is a translation.
- Theorem 9-3: A composition of reflections across two intersecting lines is a rotation.
- **Theorem 9-4: Fundamental Theorem of Isometries**
- There are only four isometries. They are the following:
  - Reflection
  - Translation
  - Rotation
  - Glide reflection

A glide reflection is the composition of a glide (translation) and a reflection across a line parallel to the direction of translation.

**Quick Check**
- Describe the composition of reflections in intersecting lines.
- Describe the composition of reflections in parallel lines.
- Describe the composition of reflections in intersecting and parallel lines.

Lesson 9-7

**Lesson Objectives**
- Use a composition of translations
- Identify glide reflections

**Vocabulary and Key Concepts**
- Theorem 9-1: A translation or rotation is a composition of two reflections.
- Theorem 9-2: A composition of translations across two parallel lines is a translation.
- Theorem 9-3: A composition of translations across two intersecting lines is a rotation.
- **Theorem 9-4: Fundamental Theorem of Isometries**
- There are only four isometries. They are the following:
  - Reflection
  - Translation
  - Rotation
  - Glide reflection

A glide reflection is the composition of a glide (translation) and a reflection across a line parallel to the direction of translation.

**Quick Check**
- Describe the composition of translations in intersecting lines.
- Describe the composition of translations in parallel lines.
- Describe the composition of translations in intersecting and parallel lines.

Lesson 9-8

**Lesson Objectives**
- Use a composition of rotations
- Identify glide reflections

**Vocabulary and Key Concepts**
- Theorem 9-1: A translation or rotation is a composition of two reflections.
- Theorem 9-2: A composition of rotations across two parallel lines is a translation.
- Theorem 9-3: A composition of rotations across two intersecting lines is a rotation.
- **Theorem 9-4: Fundamental Theorem of Isometries**
- There are only four isometries. They are the following:
  - Reflection
  - Translation
  - Rotation
  - Glide reflection

A glide reflection is the composition of a glide (translation) and a reflection across a line parallel to the direction of translation.

**Quick Check**
- Describe the composition of rotations in intersecting lines.
- Describe the composition of rotations in parallel lines.
- Describe the composition of rotations in intersecting and parallel lines.

Lesson 9-9

**Lesson Objectives**
- Use a composition of glide reflections
- Identify glide reflections

**Vocabulary and Key Concepts**
- Theorem 9-1: A translation or rotation is a composition of two reflections.
- Theorem 9-2: A composition of glide reflections across two parallel lines is a translation.
- Theorem 9-3: A composition of glide reflections across two intersecting lines is a rotation.
- **Theorem 9-4: Fundamental Theorem of Isometries**
- There are only four isometries. They are the following:
  - Reflection
  - Translation
  - Rotation
  - Glide reflection

A glide reflection is the composition of a glide (translation) and a reflection across a line parallel to the direction of translation.

**Quick Check**
- Describe the composition of glide reflections in intersecting lines.
- Describe the composition of glide reflections in parallel lines.
- Describe the composition of glide reflections in intersecting and parallel lines.

Lesson 9-10

**Lesson Objectives**
- Use a composition of reflections and rotations
- Identify glide reflections

**Vocabulary and Key Concepts**
- Theorem 9-1: A translation or rotation is a composition of two reflections.
- Theorem 9-2: A composition of reflections and rotations across two parallel lines is a translation.
- Theorem 9-3: A composition of reflections and rotations across two intersecting lines is a rotation.
- **Theorem 9-4: Fundamental Theorem of Isometries**
- There are only four isometries. They are the following:
  - Reflection
  - Translation
  - Rotation
  - Glide reflection

A glide reflection is the composition of a glide (translation) and a reflection across a line parallel to the direction of translation.

**Quick Check**
- Describe the composition of reflections and rotations in intersecting lines.
- Describe the composition of reflections and rotations in parallel lines.
- Describe the composition of reflections and rotations in intersecting and parallel lines.
Lesson 9-7

**Tessellations**

**Vocabulary and Key Concepts**

- **Theorem 9-6:** Every triangle tessellates.
- **Theorem 9-7:** Every quadrilateral tessellates.

- **Example:** A tessellation, or tiling, is a repeated combination of a shape and one adjoining shape that will completely cover the plane without gaps or overlaps. Use arrows to show a translation.

- **Example:** A repeated pattern of figures that completely covers a plane, with no gaps or overlaps.

- **Theorem:** A tessellation has translational symmetry, as can be seen by sliding any triangle onto a copy of itself along any of the lines.

- **Quick Check 1:** Identify the symmetries in the tessellation.
  - Translational symmetry: 
  - Glide reflectional symmetry: 
  - Reflectional symmetry: 
  - Rotational symmetry: 

- **Quick Check 2:** List the symmetries in the tessellation.
  - Translational symmetry: 
  - Glide reflectional symmetry: 
  - Reflectional symmetry: 
  - Rotational symmetry: 

Lesson 10-1

**Areas of Parallelograms and Triangles**

**Vocabulary and Key Concepts**

- **Theorem 10-1:** Area of a Rectangle
  - The area of a rectangle is the product of its base and height.
  - \( A = bh \)

- **Theorem 10-2:** Area of a Parallelogram
  - The area of a parallelogram is the product of its base and the corresponding height.
  - \( A = bh \)

- **Theorem 10-3:** Area of a Triangle
  - The area of a triangle is one half the product of its base and the corresponding height.
  - \( A = \frac{1}{2}bh \)

- **Area of a Parallelogram**
  - A base of a parallelogram is any of its sides.
  - The altitude of a parallelogram corresponding to a base is the segment perpendicular to the line containing that base drawn from the opposite side.
  - The height of a parallelogram is the length of its altitude.
  - \( A = bh \)

- **Area of a Triangle**
  - A base of a triangle is any of its sides.
  - The height of a triangle is the length of the altitude to the line containing that base.
  - \( A = \frac{1}{2}bh \)
Lesson 10-2
Areas of Trapezoids, Rhombuses, and Kites

Lesson Objectives

- Find the area of a trapezoid
- Find the area of a rhombus or a kite
- Find the area of a regular polygon

Vocabulary and Key Concepts

Theorem 10-6: Area of a Trapezoid

The area of a trapezoid is half the product of the height and the sum of the bases.

\[ A = \frac{1}{2} \times (b_1 + b_2) \times h \]

Theorem 10-7: Area of a Rhombus or a Kite

The area of a rhombus is half the product of the lengths of its diagonals.

\[ A = \frac{1}{2} d_1 \times d_2 \]

Finding the Area of a Regular Polygon

The area of a regular polygon is half the product of the perimeter and the apothem.

\[ A = \frac{1}{2} \times p \times a \]

Examples

1. Applying the Area of a Trapezoid
   A car window is shaped like the trapezoid shown. Find the area of the window.

   \[ A = \frac{1}{2} \times (20 + 36) \times 16 = 192 \text{ square inches} \]

2. Applying the Area of a Rhombus
   Find the area of a rhombus with diagonals measuring 24 m and 10 m.

   \[ A = \frac{1}{2} \times 24 \times 10 = 120 \text{ square meters} \]

3. Applying the Area of a Kite
   Find the area of a kite with diagonals measuring 12 m and 8 m.

   \[ A = \frac{1}{2} \times 12 \times 8 = 48 \text{ square meters} \]

4. Applying the Area of a Regular Polygon
   A regular hexagon has an apothem and finding angle measures.

   - The center of a regular polygon is the center of the circumscribed circle.
   - The apothem bisects the vertex angle of the regular polygon.
   - The radii of a regular polygon connect the center to a vertex.
   - The diagonals of a rhombus bisect each other.
   - The diagonals of a kite are perpendicular.

Finding Angle Measures

The measure of each numbered angle.

1. \( m \angle 1 = 60 \)  
2. \( m \angle 2 = m \angle 1 \)  
3. \( m \angle 2 = m \angle 1 \)  
4. \( m \angle 3 = m \angle 1 \)  
5. \( m \angle 3 = m \angle 2 \)  
6. The sum of the measures of the angles of a triangle is 180.

Quick Check

1. Find the area of a regular polygon with 11.6-cm sides and an 8-cm apothem.

2. Find the area of a trapezoid with height 7 cm and bases 12 cm and 15 cm.
Lesson 10-4
Perimeters and Areas of Similar Figures

Lesson Objective
Find the perimeters and areas of similar figures.

Key Concepts

- Finding Ratios in Similar Figures: The triangles at the right are similar. Find the ratio (larger to smaller) of their perimeters and of their areas.
  - The shortest side of the left-hand triangle has length $\frac{1}{2}$ and the shortest side of the right-hand triangle has length $\frac{1}{3}$.
  - By the Perimeters and Areas of Similar Figures Theorem, the ratio of the perimeters is $\frac{1}{2} : \frac{1}{3}$.
  - The ratio of the areas is $(\frac{1}{2})^2 : (\frac{1}{3})^2 = \frac{1}{4} : \frac{1}{9}$.

- Finding Areas Using Similar Figures: The ratio of the lengths of the corresponding sides of the regular octagon is $3 : 1$. The area of the larger octagon is $320$ ft$^2$. Find the area of the smaller octagon.
  - Because the ratio of the lengths of the corresponding sides of the regular octagon is $3 : 1$, the ratio of their areas is $(3 : 1)^2 = 9 : 1$.
  - The area of the smaller octagon is $\frac{1}{9}$ of the area of the larger octagon.
  - So, the area of the smaller octagon is $\frac{320}{9}$ ft$^2$.

- Quick Check
  1. Two similar polygons have corresponding sides in the ratio $5 : 7$. Find the ratio of their perimeters.
  2. The corresponding sides of two similar parallelograms are in the ratio $\frac{1}{2}$. The area of the smaller parallelogram is $54$ in.$^2$. Find the area of the larger parallelogram.
  3. The similarity ratio of the dimensions of two similar pieces of window glass is $3 : 5$. The smaller piece costs $2.50. What should be the cost of the larger piece?

Lesson 10-5
Trigonometry and Area

Lesson Objective
Find the area of a regular polygon using trigonometry. Find the area of a triangle using trigonometry.

Key Concepts

- The area of a triangle is one half the product of the lengths of two sides and the sine of the included angle.
  - The formula is $A = \frac{1}{2}ab \sin C$.

- Finding the area of a regular polygon using trigonometry:
  - Find the area of a regular pentagon with a perimeter of $50$ ft. Give the area to the nearest tenth.
    - The area of a regular pentagon is $\frac{1}{4} \times \left( \frac{n-2}{2} \right) \times s^2 \times \tan \left( \frac{180}{n} \right)$, where $n$ is the number of sides and $s$ is the length of a side.
    - Substituting $n = 5$ and $s = 10$, we get $A = \frac{1}{4} \times \left( \frac{5-2}{2} \right) \times 10^2 \times \tan \left( \frac{180}{5} \right)
    - $A = 100 \times \tan(36^\circ)$.
    - Using a calculator, we find $A = 100 \times 0.7265366897 = 72.654$ ft$^2$.

- Quick Check
  1. Find the area of a regular octagon with a perimeter of $80$ in. Give the area to the nearest tenth.
  2. Find the area of a triangle using trigonometry.

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### Lesson 10-6  Circles and Arcs

**Vocabulary and Key Concepts**

- **Postulate 10-1: Arc Addition Postulate**
  - The measure of the arc formed by two adjacent arcs is the sum of the measures of the two arcs.

- **Theorem 10-9: Circumference of a Circle**
  - The circumference of a circle is \(2\pi r\) or \(\pi d\), where \(r\) is the radius and \(d\) is the diameter.

- **Arc Addition Postulate**
  - The measure of the arc formed by two adjacent arcs is the sum of the measures of the two arcs.

**Examples**

1. **Finding the Length of \(\mathbf{AB}\)**
   - \(\mathbf{AB}\) intercepts arc \(\mathbf{XY}\) with measure \(100^\circ\) and arc \(\mathbf{M}\) with measure \(56^\circ\).
   - **Solution:**
     - \(\mathbf{AB}\) intercepts arc \(\mathbf{XY}\) with measure 100 degrees and arc \(\mathbf{M}\) with measure 56 degrees. The measure of arc \(\mathbf{AB}\) is \(100^\circ + 56^\circ = 156^\circ\).

2. **Finding the Area of a Sector**
   - **Example:**
     - **Problem:** The radius of a semicircle is 4 cm. Find the length of the arc.\(\text{(in terms of } \pi\text{)}\)
     - **Solution:**
       - The length of the arc is \(\frac{1}{2} \cdot 2\pi r = \pi r\), where \(r\) is the radius.
       - Substitute \(r = 4\) cm.
       - \(\text{Length} = \pi \cdot 4 = 4\pi\) cm.

3. **Finding the Area of a Segment**
   - **Example:**
     - **Problem:** Find the area of the sector of a circle with radius 10 cm and central angle \(60^\circ\).
     - **Solution:**
       - **Step 1:** Find the area of the sector.
       - **Step 2:** Subtract the area of the triangle formed by the central angle.

**Quick Check**

1. **Finding the Length of \(\mathbf{AB}\)**
   - **Example:**
     - **Problem:** Find the length of \(\mathbf{AB}\) in terms of \(x\).
     - **Solution:**
       - The length of \(\mathbf{AB}\) is \(\frac{x}{3}\) in terms of \(x\).

2. **Finding the Area of a Sector**
   - **Example:**
     - **Problem:** Find the area of the sector of a circle with radius 10 cm and central angle \(120^\circ\).
     - **Solution:**
       - **Step 1:** Find the area of the sector.
       - **Step 2:** Subtract the area of the triangle formed by the central angle.

---

**Lesson 10-7  Areas of Circles and Sectors**

**Vocabulary and Key Concepts**

- **Theorem 10-11: Area of a Circle**
  - The area of a circle is \(\pi r^2\), where \(r\) is the radius.

- **Theorem 10-12: Area of a Sector of a Circle**
  - The area of a sector of a circle is \(\frac{m}{360} \cdot \pi r^2\), where \(m\) is the measure of the arc and \(r\) is the radius.

**Examples**

1. **Finding the Area of a Sector**
   - **Example:**
     - **Problem:** Find the area of sector \(\mathbf{ACB}\) in circle \(C\).
     - **Solution:**
       - **Step 1:** Find the area of the sector.
       - **Step 2:** Subtract the area of the triangle formed by the central angle.

2. **Finding the Area of a Segment**
   - **Example:**
     - **Problem:** Find the area of the segment of a circle with radius 10 cm and central angle \(90^\circ\).
     - **Solution:**
       - **Step 1:** Find the area of the sector.
       - **Step 2:** Subtract the area of the triangle formed by the central angle.

---

**Quick Check**

1. **Finding the Area of a Sector**
   - **Example:**
     - **Problem:** Find the area of sector \(\mathbf{ACD}\) in circle \(D\).
     - **Solution:**
       - **Step 1:** Find the area of the sector.
       - **Step 2:** Subtract the area of the triangle formed by the central angle.

2. **Finding the Area of a Segment**
   - **Example:**
     - **Problem:** Find the area of segment \(\mathbf{EFG}\) in circle \(E\).
     - **Solution:**
       - **Step 1:** Find the area of the sector.
       - **Step 2:** Subtract the area of the triangle formed by the central angle.
Lesson 11-1  Space Figures and Cross Sections

**Lesson Objective**
- Identify vertices, edges, and faces of space figures.

**Vocabulary**
- **Face**: A flat surface of a polyhedron.
- **Vertex**: A point where three or more edges intersect.
- **Edge**: A segment that lies in the intersection of two faces.

**Identifying Vertices, Edges, and Faces**
- There are 4 vertices: A, B, C, and D.
- There are 6 edges: AB, AD, BC, BD, CD, and CB.
- There are 4 faces: \( \triangle ABC, \triangle ABD, \triangle BCD, \) and \( \triangle ACD \).

**Examples**
1. **Identifying Vertices, Edges, and Faces**
   - How many vertices, edges, and faces are there in the polyhedron shown? Give a list of each.
   - There are 4 vertices: A, B, C, and D.
   - There are 6 edges: AB, AD, BC, BD, CD, and CB.
   - There are 4 faces: \( \triangle ABC, \triangle ABD, \triangle BCD, \) and \( \triangle ACD \).

**Finding Probability Using Segments**
- A gnat lands at random on the edge of the ruler below. Find the probability that the gnat lands on a point between 2 and 10.

2. **Finding Probability Using Segments**
   - The length of the segment between 2 and 10 is 8.
   - The length of the ruler is 10.
   - **Quick Check**
     - A point on \( \overline{CD} \) is selected at random. What is the probability that it is a point on \( \overline{AB} \)?
     - A point on \( \overline{CD} \) is selected at random. What is the probability that it is a point on \( \overline{AB} \)?

**Using Euler’s Formula**
- Use Euler’s Formula to find the number of edges on a polyhedron with 6 faces and 8 vertices.

3. **Using Euler’s Formula**
   - A solid with 6 faces and 8 vertices has ______ edges.

**Quick Check**
1. **Finding Probability Using Segments**
   - Use segments and area models to find the probabilities of events.
   - Find the area of the square: \( A = \pi r^2 \).
   - Find the area of the circle: \( A = \pi r^2 \).
   - Find the area of the region between the square and the circle: \( A = (400 - \frac{100}{\pi}) \text{ cm}^2 \).

2. **Using Euler’s Formula**
   - A solid with 6 faces and 8 vertices has ______ edges.

3. **Quick Check**
   - Find the area of the region between the square and the circle: \( A = (400 - \frac{100}{\pi}) \text{ cm}^2 \).

**Finding Probability Using Area**
- Find the area of the region between the square and the circle.

4. **Finding Probability Using Area**
   - Find the area of the region between the square and the circle: \( A = (400 - \frac{100}{\pi}) \text{ cm}^2 \).

5. **Quick Check**
   - Use areas to calculate the probability that a dart landing randomly in the square does not land within the circle. Use a calculator. Round to the nearest thousandth.
   - The probability that a dart landing randomly in the square does not land within the circle is about ______.

**Finding Probability Using Segments**
- A gnat lands randomly on the edge of the ruler below. Find the probability that the gnat lands on a point between 2 and 10.

6. **Finding Probability Using Segments**
   - The length of the segment between 2 and 10 is 8.
   - The length of the ruler is 10.

7. **Quick Check**
   - A point on \( \overline{CD} \) is selected at random. What is the probability that it is a point on \( \overline{AB} \)?

8. **Using Euler’s Formula**
   - Use Euler’s Formula to find the number of edges on a polyhedron with 6 faces and 8 vertices.

9. **Quick Check**
   - Find the area of the region between the square and the circle: \( A = (400 - \frac{100}{\pi}) \text{ cm}^2 \).

10. **Finding Probability Using Area**
    - Find the area of the region between the square and the circle: \( A = (400 - \frac{100}{\pi}) \text{ cm}^2 \).

11. **Quick Check**
    - Use areas to calculate the probability that a dart landing randomly in the square does not land within the circle. Use a calculator. Round to the nearest thousandth.

12. **Finding Probability Using Segments**
    - Find the probability that a dart landing randomly in the square does not land within the circle. Use a calculator. Round to the nearest thousandth.

13. **Quick Check**
    - The probability that a dart landing randomly in the square does not land within the circle is about ______.

14. **Finding Probability Using Segments**
    - Use segments and area models to find the probabilities of events.

15. **Quick Check**
    - Find the area of the region between the square and the circle: \( A = (400 - \frac{100}{\pi}) \text{ cm}^2 \).

16. **Finding Probability Using Area**
    - Find the area of the region between the square and the circle: \( A = (400 - \frac{100}{\pi}) \text{ cm}^2 \).

17. **Quick Check**
    - Use areas to calculate the probability that a dart landing randomly in the square does not land within the circle. Use a calculator. Round to the nearest thousandth.

18. **Finding Probability Using Segments**
    - Find the probability that a dart landing randomly in the square does not land within the circle. Use a calculator. Round to the nearest thousandth.

19. **Quick Check**
    - The probability that a dart landing randomly in the square does not land within the circle is about ______.
Lesson 11-2
Surface Areas of Prisms and Cylinders

Vocabulary and Key Concepts

Theorem 11-1: Lateral and Surface Area of a Cylinder

- The lateral area of a cylinder is the sum of the areas of the two bases and the lateral faces.
- The surface area of a cylinder is the sum of the lateral area and the areas of the two bases.

A prism is a polyhedron with exactly two congruent, parallel faces called bases. The lateral faces are the faces on a prism that are not bases.

A cylinder is a three-dimensional figure with two congruent circular bases that lie in parallel planes. In a cylinder, the bases are circular disks.

Lesson 11-3
Surface Areas of Pyramids and Cones

Vocabulary and Key Concepts

Theorem 11-2: Lateral and Surface Area of a Regular Pyramid

- The lateral area of a regular pyramid is the sum of the areas of the lateral faces.
- The surface area of a regular pyramid is the sum of the lateral area and the area of the base.

A regular pyramid is a pyramid whose base is a regular polygon and whose lateral faces are congruent isosceles triangles.

Example: Finding Surface Area of a Pyramid

- Find the surface area of a square pyramid with base edges 7.5 ft and slant height 12 ft.

- Use the formula for lateral area of a pyramid.
  \[ L.A. = \frac{1}{2} \times p \times l \]
- Find the area of the square base.
  \[ B = 7.5^2 = 56.25 \text{ ft}^2 \]
- Use the formula for surface area of a pyramid.
  \[ S.A. = B + L.A. \]

Quick Check

a. Find the surface area of a cylinder with height 10 cm and radius 10 cm in terms of \( \pi \).

b. The company in the Example wants to make a box to cover the cylindrical container. The label will cover the container all the way around, but will not cover any part of the top or bottom. What is the area of the label to the nearest tenth of a square inch?

- Surface Area of a Cylinder
  \[ S.A. = 2\pi rh + 2\pi r^2 \]
  \[ S.A. = 2\pi (10)(10) + 2\pi (10)^2 \]
  \[ S.A. = 200\pi + 200\pi \]
  \[ S.A. = 400\pi \text{ cm}^2 \]

- Surface Area of a Box
  \[ S.A. = 2(lw + lh + wh) \]
  \[ S.A. = 2(10)(10) + 2(10)(10) + 2(10)(10) \]
  \[ S.A. = 200 + 200 + 200 \]
  \[ S.A. = 600 \text{ in}^2 \]

- Label Area
  \[ \text{Label Area} = \pi (2r)h = \pi (20)(10) \]
  \[ \text{Label Area} = 200\pi \text{ in}^2 \]

Quick Check Continued

Find the surface area of a square pyramid with base edges 7.5 ft and slant height 12 ft.

- Use the formula for lateral area of a square pyramid.
  \[ L.A. = \frac{1}{2} \times p \times l \]
- Use the formula for surface area of a square pyramid.
  \[ S.A. = B + L.A. \]

- The surface area of a square pyramid is also equal to \( 2B + 2L.A. \).
Lesson 11-4
Volumes of Prisms and Cylinders

**Lesson Objectives**
- Find the volume of a prism
- Find the volume of a cylinder

**Vocabulary and Key Concepts**

**Theorem 11-8: Volume of a Pyramid**
The volume of a pyramid is one third the product of the area of the base and the height of the pyramid.

**Theorem 11-7: Volume of a Cylinder**
The volume of a cylinder is the product of the area of the base and the height of the cylinder.

### Examples

**Finding Volume of a Cylinder**
Find the volume of the cylinder.

- **Problem:** Find the volume of a cylinder with a radius of 4 cm and a height of 9 cm.
  - **Solution:**
    - **Formula:** \( V = \pi r^2 h \)
    - **Substitution:** \( r = 4 \) cm, \( h = 9 \) cm
    - **Calculation:** \( V = \pi (4)^2 (9) = 144\pi \) cm³

**Finding Volume of a Pyramid**
Find the volume of a pyramid.

- **Problem:** Find the volume of a square pyramid with a base area of 24 m² and a height of 10 m.
  - **Solution:**
    - **Formula:** \( V = \frac{1}{3}Bh \)
    - **Substitution:** \( B = 24 \) m², \( h = 10 \) m
    - **Calculation:** \( V = \frac{1}{3}(24)(10) = 80 \) m³

**Finding Volume of an Oblique Cone**
Find the volume of an oblique cone.

- **Problem:** Find the volume of an oblique cone with a base radius of 5 in and a height of 12 in.
  - **Solution:**
    - **Formula:** \( V = \frac{1}{3}\pi r^2 h \)
    - **Substitution:** \( r = 5 \) in, \( h = 12 \) in
    - **Calculation:** \( V = \frac{1}{3}\pi (5)^2 (12) = 100\pi \) in³

**Quick Check**

1. Find the volume of a right triangular prism with a height of 10 m and a base area of 20 m².
2. A small child's teepee is in the shape of a cone 6 ft tall and 7 ft in diameter. Find the volume of the teepee to the nearest cubic foot.
Lesson 11-6  Surface Areas and Volumes of Spheres

**Lesson Objectives**
- Find the surface area and volume of a sphere

**Vocabulary and Key Concepts**

**Theorem 11-10: Surface Area of a Sphere**
The surface area of a sphere is four times the product of the square of the radius and 
π. 

\[ S.A. = 4\pi r^2 \]

**Theorem 11-11: Volume of a Sphere**
The volume of a sphere is four thirds the product of the cube of the radius and π. 

\[ V = \frac{4}{3}\pi r^3 \]

A great circle is the intersection of a sphere and a plane containing the diameter of the sphere.

The diameter of a sphere is a segment that has one endpoint at the center and the other endpoint on the sphere.

The radius of a sphere is the distance from the center to the other endpoint on the sphere.

The volume of the sphere is \( \frac{4}{3}\pi r^3 \) cubic units.

The surface area of a sphere is \( 4\pi r^2 \) square units.

The circumference of a sphere is \( 2\pi r \) units.

The great circle is a great circle of the sphere.

**Quick Check**

1. Find the surface area of a sphere with \( r = 14 \) in. Give your answer in terms of \( \pi \) and rounded to the nearest square inch.

\[
S.A. = 4\pi r^2 = 4\pi (14)^2 = 784\pi \approx 2465.8 
\]

2. Find the volume of a sphere with diameter 6 ft.

\[
V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (3)^3 = 36\pi \approx 113.1 
\]

**Examples**

1. **Finding Surface Area**: The circumference of a rubber ball is 13 cm. Calculate the surface area to the nearest whole number.

   **Step 1**
   - Find the radius:
   \[
   r = \frac{13}{2\pi} 
   \]
   - Calculate the surface area:
   \[
   S.A. = 4\pi r^2 = 4\pi \left(\frac{13}{2\pi}\right)^2 = \frac{169}{2} \approx 84.5 
   \]

2. **Finding Volume**: Find the volume of a sphere.

   **Step 2**
   - Use the formula for volume:
   \[
   V = \frac{4}{3}\pi r^3 
   \]
   - Substitute the given radius:
   \[
   V = \frac{4}{3}\pi (3)^3 = 36\pi \approx 113.1 
   \]

**Local Standards**
- Systems of Measurement

- Measurement
Lesson 12-2

Vocabulary and Key Concepts

Theorem 12-1
If a line is tangent to a circle, then the line is perpendicular to the radius drawn to the point of tangency.

Theorem 12-2
If a line in the plane of a circle is perpendicular to a radius at its endpoint on the circle, then the line is tangent to the circle.

Theorem 12-3
The two segments tangent to a circle from one point outside the circle are congruent.

A triangle inscribed in a circle is circumscribed about a circle.

A triangle is inscribed in a circle if each side of the triangle is tangent to the circle.

An arc whose endpoints are on a circle is called a chord.

The distance between the centers of the pulleys is about 16 in.

Find OD.

Because opposite sides of a rectangle have the same measure, OW = DW and OD = DP.

Because ODP is the supplement of right angle AOE, ODP is also a right angle and ODP is a right angle.

Because the radius of is 7 cm, OD = OP = 7 cm.

OD^2 + PD^2 = OP^2

Use the Pythagorean Theorem to solve the equation.

OD = 2.4 in.

Use a calculator to find the square root.

The distance between the centers of the pulleys is about 16 in.

Quick Check
1. A belt fits tightly around two circular pulleys, as shown. Find the distance between the centers of the pulleys. Round your answer to the nearest tenth.

2. Find the value of x in the circle at the right.

Name_____________________________________ Class____________________________ Date ________________
Lesson 12-3
Inscribed Angles

Example:

Using the Inscribed Angle Theorem

Find the values of $x$ and $y$.

$x = \frac{1}{2} \angle EDF$

$y = \frac{1}{2} \angle EFG$

Corollaries to the Inscribed Angle Theorem

1. Two inscribed angles that intercept the same arc are

   # Inscribed Angle

2. An angle inscribed in a semicircle is a right angle.

   # Inscribed Angle

3. The opposite angles of a quadrilateral inscribed in a circle are supplementary.

   # Inscribed Angle

Lesson 12-4
Angle Measures and Segment Lengths

Example:

Finding Arc Measures: An advertising agency wants a frontal photo of a "flying saucer" ride at an amusement park. The photographer stands at the vertex of the angle formed by tangents to the "flying saucer." What is the angle measure of the arc that will be in the photograph? In the diagram, the photographer stands at point $A$, and $\angle XAY$ and $\angle XZT$ intercept minor arc $AXY$ and major arc $XZF$, respectively.

Quick Check

In the Example, find $m\angle DEF$.

$\angle DEF = \frac{1}{2} \angle EFG$

Local Standards: ____________________________________

NAEP 2005 Strand: __________

Topic: ____________________

Geometry

Geometry: All-In-One Answers Version B (continued)
Lesson 12-6
Locus: A Set of Points

Examples:
1. Write the equation of a circle with center (5, 8) that passes through the point (15, 15).

   a. Write the standard equation of a circle with center (5, 8) and radius 1.5 cm.
   
   \[ (x - 5)^2 + (y - 8)^2 = 1.5^2 \]

   b. Use the Distance Formula to find the center of the circle.
   
   \[ \sqrt{(15 - 5)^2 + (15 - 8)^2} = \sqrt{10^2 + 7^2} = \sqrt{100 + 49} = \sqrt{149} \]

2. Write the standard equation of the circle with center (2, 3) that passes through the point (4, 2).

   a. Write the standard equation of a circle with center (2, 3) and radius .
   
   \[ (x - 2)^2 + (y - 3)^2 = r^2 \]

   b. Use the Distance Formula to find the radius.
   
   \[ r = \sqrt{(4 - 2)^2 + (2 - 3)^2} = \sqrt{4 + 1} = \sqrt{5} \]

3. Find the center and radius of the circle with equation \((x - 2)^2 + (y + 3)^2 = 100\). Then graph the circle.

   a. Write the standard equation of the circle with center (2, 3) and radius 10.
   
   \[ (x - 2)^2 + (y - (-3))^2 = 10^2 \]

   b. Find the distance between the points (2, 3) and (13, 5).
   
   \[ \sqrt{(13 - 2)^2 + (5 - 3)^2} = \sqrt{11^2 + 2^2} = \sqrt{121 + 4} = \sqrt{125} \]

   c. Find the center and radius of the circle with equation \((x + 2)^2 + (y - 3)^2 = 25\). Then graph the circle.
   
   \[ (x + 2)^2 + (y - 3)^2 = 5^2 \]

   d. Write an equation of a circle that is parallel to and equidistant from each.
   
   \[ (x - 1)^2 + (y - 2)^2 = 4 \]

   e. Graph a circle given its equation. Find the center and radius of the circle with equation \((x - 1)^2 + (y - 2)^2 = 25\). Then graph the circle.
   
   \[ (x - 1)^2 + (y - 2)^2 = 5^2 \]

   f. Write the equation of the circle with center (1, 2) and the radius .

   \[ (x - 1)^2 + (y - 2)^2 = r^2 \]
Data Analysis and Probability Workbook Answers

Section 1 Graphs

1. Frequency Tables, Line Plots, and Histograms

<table>
<thead>
<tr>
<th>Number of Rose Bushes</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tally</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>6</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

2. Rose Bushes Wanted

<table>
<thead>
<tr>
<th>Number of Rose Bushes</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tally</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>6</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

3. Answers may vary. Sample:

<table>
<thead>
<tr>
<th>Rose Bushes Wanted</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
</tr>
<tr>
<td>x</td>
</tr>
<tr>
<td>x</td>
</tr>
<tr>
<td>x</td>
</tr>
<tr>
<td>x</td>
</tr>
<tr>
<td>x</td>
</tr>
<tr>
<td>x</td>
</tr>
<tr>
<td>x</td>
</tr>
</tbody>
</table>

Page 2 Practice: Frequency Tables, Line Plots, and Histograms

1. Boxes of Juice Sold

<table>
<thead>
<tr>
<th>21</th>
<th>22</th>
<th>23</th>
<th>24</th>
<th>25</th>
<th>26</th>
<th>27</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

2. a student  3. 3 students  4. 13 students  5. 80 and 85, 75 and 90  6. No; the interval includes 2–2.75 h.  7. 10 students

Page 3 Reteaching 1: Frequency Tables and Line Plots

1. Inches | 3 | 4 | 5 | 6 | 7 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

2. Inches | 0 | 1 | 2 | 3 | 4 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>5</td>
<td>2</td>
<td>0</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

3. Inches | 3 | 4 | 5 | 6 | 7 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Page 4 Reteaching 2: Frequency Distributions

<table>
<thead>
<tr>
<th>Interval</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>95–100</td>
<td>II</td>
<td>2</td>
</tr>
<tr>
<td>90–94</td>
<td>III</td>
<td>4</td>
</tr>
<tr>
<td>85–89</td>
<td>II</td>
<td>2</td>
</tr>
<tr>
<td>80–84</td>
<td>III</td>
<td>7</td>
</tr>
<tr>
<td>75–79</td>
<td>III</td>
<td>4</td>
</tr>
<tr>
<td>70–74</td>
<td>III</td>
<td>4</td>
</tr>
<tr>
<td>under 70</td>
<td>II</td>
<td>2</td>
</tr>
</tbody>
</table>

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Data Analysis and Probability Workbook Answers

2. | Interval   | Tally | Frequency |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>700–749</td>
<td>III</td>
<td>3</td>
</tr>
<tr>
<td>750–799</td>
<td>III</td>
<td>4</td>
</tr>
<tr>
<td>800–849</td>
<td>III</td>
<td>3</td>
</tr>
<tr>
<td>850–899</td>
<td></td>
<td></td>
</tr>
<tr>
<td>900–949</td>
<td></td>
<td></td>
</tr>
<tr>
<td>950–999</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000 and over</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. | Interval   | Tally | Frequency |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>60–79</td>
<td></td>
<td></td>
</tr>
<tr>
<td>80–99</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100–119</td>
<td></td>
<td></td>
</tr>
<tr>
<td>120–139</td>
<td></td>
<td></td>
</tr>
<tr>
<td>140–159</td>
<td></td>
<td></td>
</tr>
<tr>
<td>160–179</td>
<td></td>
<td></td>
</tr>
<tr>
<td>180 and over</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

page 5 Reteaching 3: Histograms
1. Answers may vary. Sample:

<table>
<thead>
<tr>
<th>Number of Raisins</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>25–29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30–34</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35–39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40–44</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. 

3. 

4. Daily Use of Petroleum in the U.S.

5. Answers may vary. Sample: Daily petroleum use has been declining since 1970.

page 8 Stacked Bar and Multiple Line Graphs
1. grade 7  2. grade 7; grade 8  3. Sample: Find the total, subtract the bottom part.  4. 1993  5. Find the number of each item sold and add the two.

page 9 Practice: Stacked Bar and Multiple Line Graphs
1. Average Viewing Time
8:00 P.M.–11:00 P.M. (Mon.–Sun.)

<table>
<thead>
<tr>
<th>Time (Hours)</th>
<th>Males</th>
<th>Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>14 12 10 8 6 4 2 0 2 4 6 8 10 12 14</td>
<td>55+</td>
<td>25–54</td>
</tr>
<tr>
<td></td>
<td></td>
<td>18–24</td>
</tr>
</tbody>
</table>

page 6 Bar and Line Graphs
1. tens  2. fives  3. thousands  4. hundreds  5. fives  6. tens  7. Bar graph; data shows amounts, but not changes over time.  8. Line graph; data shows change over time.

page 7 Practice: Bar and Line Graphs
1. names of the athletes  2. Answers may vary. Sample: 5
Data Analysis and Probability Workbook Answers

2. Average Viewing Time

<table>
<thead>
<tr>
<th>Time (Hours)</th>
<th>Ages</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–3</td>
<td>18–24</td>
</tr>
<tr>
<td>3–6</td>
<td>25–54</td>
</tr>
<tr>
<td>6–9</td>
<td>55+</td>
</tr>
</tbody>
</table>

3. double bar graph
4. sliding bar graph

5. Population

<table>
<thead>
<tr>
<th>Year</th>
<th>Population (100,000s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>1.0</td>
</tr>
<tr>
<td>1970</td>
<td>1.5</td>
</tr>
<tr>
<td>1980</td>
<td>2.0</td>
</tr>
<tr>
<td>1990</td>
<td>2.5</td>
</tr>
<tr>
<td>2000</td>
<td>3.0</td>
</tr>
</tbody>
</table>

6. Population

<table>
<thead>
<tr>
<th>Year</th>
<th>Population (100,000s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>1.0</td>
</tr>
<tr>
<td>1970</td>
<td>1.5</td>
</tr>
<tr>
<td>1980</td>
<td>2.0</td>
</tr>
<tr>
<td>1990</td>
<td>2.5</td>
</tr>
<tr>
<td>2000</td>
<td>3.0</td>
</tr>
</tbody>
</table>

7. multiple line graph

9. Budget

<table>
<thead>
<tr>
<th>Category</th>
<th>Amount Budgeted</th>
<th>Percent of Total</th>
<th>Degrees in Central Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas</td>
<td>$200</td>
<td>10%</td>
<td>36°</td>
</tr>
<tr>
<td>Meals</td>
<td>$400</td>
<td>20%</td>
<td>72°</td>
</tr>
<tr>
<td>Motels</td>
<td>$500</td>
<td>30%</td>
<td>108°</td>
</tr>
<tr>
<td>Other</td>
<td>$800</td>
<td>40%</td>
<td>144°</td>
</tr>
</tbody>
</table>

10. Children per Family

<table>
<thead>
<tr>
<th>Ages</th>
<th>Time (Hours)</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>18–24</td>
<td>8:00 P.M.–11:00 P.M. (Mon.–Sun.)</td>
<td></td>
</tr>
</tbody>
</table>

page 11 Practice: Circle Graphs
1–4. Sample graph shown.

Rahman Family Budget

<table>
<thead>
<tr>
<th>Category</th>
<th>Amount Budgeted</th>
<th>Percent of Total</th>
<th>Degrees in Central Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas</td>
<td>$200</td>
<td>10%</td>
<td>36°</td>
</tr>
<tr>
<td>Meals</td>
<td>$400</td>
<td>20%</td>
<td>72°</td>
</tr>
<tr>
<td>Motels</td>
<td>$500</td>
<td>30%</td>
<td>108°</td>
</tr>
<tr>
<td>Other</td>
<td>$800</td>
<td>40%</td>
<td>144°</td>
</tr>
</tbody>
</table>
Data Analysis and Probability Workbook Answers

5–8. Sample graph shown.

<table>
<thead>
<tr>
<th>Category</th>
<th>Amount Budgeted</th>
<th>Percent of Total</th>
<th>Degrees in Central Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>5. Clothing</td>
<td>$50</td>
<td>40%</td>
<td>36°</td>
</tr>
<tr>
<td>6. Entertainment</td>
<td>$40</td>
<td>32%</td>
<td>72°</td>
</tr>
<tr>
<td>7. Savings</td>
<td>$25</td>
<td>20%</td>
<td>108°</td>
</tr>
<tr>
<td>8. Transportation</td>
<td>$10</td>
<td>8%</td>
<td>144°</td>
</tr>
</tbody>
</table>

Lucy’s Budget

<table>
<thead>
<tr>
<th>Category</th>
<th>Amount Budgeted</th>
<th>Percent of Total</th>
<th>Degrees in Central Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>5. Clothing</td>
<td>$50</td>
<td>40%</td>
<td>36°</td>
</tr>
<tr>
<td>6. Entertainment</td>
<td>$40</td>
<td>32%</td>
<td>72°</td>
</tr>
<tr>
<td>7. Savings</td>
<td>$25</td>
<td>20%</td>
<td>108°</td>
</tr>
<tr>
<td>8. Transportation</td>
<td>$10</td>
<td>8%</td>
<td>144°</td>
</tr>
</tbody>
</table>

Milk Drunk

<table>
<thead>
<tr>
<th>Category</th>
<th>Amount Budgeted</th>
<th>Percent of Total</th>
<th>Degrees in Central Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>5. Clothing</td>
<td>$50</td>
<td>40%</td>
<td>36°</td>
</tr>
<tr>
<td>6. Entertainment</td>
<td>$40</td>
<td>32%</td>
<td>72°</td>
</tr>
<tr>
<td>7. Savings</td>
<td>$25</td>
<td>20%</td>
<td>108°</td>
</tr>
<tr>
<td>8. Transportation</td>
<td>$10</td>
<td>8%</td>
<td>144°</td>
</tr>
</tbody>
</table>

page 14  Practice: Stem-and-Leaf Plots

1. 10, 11, 12, 13  2. 4, 5, 9, 9  3. 115; 104 and 115; 38

<table>
<thead>
<tr>
<th>Category</th>
<th>Amount Budgeted</th>
<th>Percent of Total</th>
<th>Degrees in Central Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>5. Clothing</td>
<td>$50</td>
<td>40%</td>
<td>36°</td>
</tr>
<tr>
<td>6. Entertainment</td>
<td>$40</td>
<td>32%</td>
<td>72°</td>
</tr>
<tr>
<td>7. Savings</td>
<td>$25</td>
<td>20%</td>
<td>108°</td>
</tr>
<tr>
<td>8. Transportation</td>
<td>$10</td>
<td>8%</td>
<td>144°</td>
</tr>
</tbody>
</table>

page 12  Reteaching: Circle Graphs

1. 97°  2. 86°  3. 47°  4. 65°  5. 65°

page 13  Stem-and-Leaf Plots

1. Times for the 200-M Dash
   38 | 5 6
   37 | 3
   36 | 0 4 7 9
   35 | 2 3
   34 | 4 7 7 7
   33 | 4
   32 | 1 2 2 5
   31 | 4 7 9
   30 | 2
   29 | 3 5
   38 | 8 means 38.8

2. 34.7 s  3. 9.3 s  4. 34.7 s  5. 16 students

38 | 8 means 38.8

4. 10 6 8
   1 5 6 9
   2 0 3 4 4 7
   3 5 6 8
   3 | 8 means 38
   23; 24; 32
5. 6 3 3
   7 0 1 4 5 9
   8 1 2 2 6
   9 0 1 6 9
   9 | 9 means 9.9
   8.1; 8.2 and 6.3; 3.6
6. 4 17 36 70
   5 21 26 86
   6 34 75 92
   7 19
   8 17
   4 | 17 means 17.4
   586; no mode; 400
7. 17 4 6 9
   18 5 5 6
   19 4 5
   20 4
   17 | 4 means 17.4
   18.5; 18.5; 3
8. 15°  9. 64°  10. 69°  11. 76; 75; 79; 80

page 15  Reteaching: Stem-and-Leaf Plots

1. 1 5 6
   2 4 7
   3 6 6 9
   4 2 5
   5 1 4 9
   6 1 3 4
   1 | 5 means 15

3. 14 7 8 9
   4 45 0 1 3
   8 2 3 4 4 5 6 6 7 9
   5 5 6 7 8
   9 0 1 2
   6 2 3 5 6 6 6
   7 1 4
   8 1 2 3 5
   8 1 1 means 81
page 16  Activity: Relating Stem-and-Leaf Plots to Histograms

16 a. Stem | Leaf
--- | ---
3 | 2 7
4 | 4 4 6 7
5 | 7 8
6 | 1 3 4 8
7 | 6 9

b. Frequency Table

| Grouping Intervals | Frequency |
--- | ---|
30–39 | 2 |
40–49 | 4 |
50–59 | 2 |
60–69 | 4 |
70–79 | 2 |

c. Number of Teams

| Score |
--- |
30–39 |
40–49 |
50–59 |
60–69 |
70–79 |

page 17  Puzzle: Keeping Score

| Stem | Leaf |
--- | ---|
5 | .75,.90 |
6 | .35,.45,.75 |
7 | .20,.50,.50,.40,.75 |
8 | .75,.75,.75 |
7 | .20 means 7.20 |

page 18  Box-and-Whisker Plots

1. 98, 80, 5, 118

2. 70 90 110 130 150 170

2. 13, 4, 21

0 10 20 30 40

page 19  Practice: Box-and-Whisker Plots

1. 55 miles, 15 miles
2. 35 miles
3. 75%
4. 6 runners

5. 10 15 20 25 30

page 20  Reteaching: Box-and-Whisker Plots

1. 1 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 4 4 4 4 4 4 4 5 5 6 6 6 7 7 7 7 8 8 8 8 8 8 8 9 9 9 10 11 13
2. median = 4 3. lower median = 3; upper median = 8

4. 1 2 3 4 5 6 7 8 9 10 11 12 13

page 21  Reading and Understanding Graphs

1. 20 books
2. January
3. 10 books
4. October and November
5. English and Spanish
6. 12 people
7. 12 people
8. Polish

page 22  Practice: Reading and Understanding Graphs

1. oxygen
2. oxygen, carbon, hydrogen
3. There are other elements in quantities too small to be labeled individually
4. Western Europe
5. South and Central America, Middle East, Africa
6. North America
7. A greater percentage of men are reaching ages 25–29 without having married.
8. 1960 to 1970
9. C

page 23  Reading Graphs Critically

1. the first graph
2. about 3 times
3. The second graph; since the scale is smaller, the bars can be read more accurately.
4. By using the break, most of the bar for the United States has been left out.

page 24  Practice: Reading Graphs Critically

1. 45–64 year olds
2. 45–64 year olds
3. Answers may vary.
Check students’ graphs.
4. Answers may vary.
5. Answers may vary.
Check students’ graphs.
6. Answers may vary.
Check students’ graphs.
7. Answers may vary.
8. Answers may vary. Sample: to convince people that one program is more popular than it is

page 25  Activity: Organizing and Analyzing Data

1. no; too wide a range of data
2. D.C., N.J., and R.I.
3. possible answer: 100 people per square mile
4. People might assume that the two places have similar densities
5. You could list the three places individually
6. nothing
Data Analysis and Probability Workbook Answers

page 26 Scatter Plots and Trends
1. Test Scores and TV

<table>
<thead>
<tr>
<th>Hours of TV</th>
<th>Test Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>90</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
</tr>
<tr>
<td>3</td>
<td>70</td>
</tr>
<tr>
<td>4</td>
<td>60</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
</tr>
</tbody>
</table>

2. Negative; as one value goes up, the other goes down.
3. The more TV students watch, the lower their test scores.

page 27 Practice: Scatter Plots and Trends
1. positive trend
2. negative trend
3. no trend
4. Students' graphs should show a negative trend. Sample:

page 28 Activity 1: Making a Scatter Plot
1–2. Check student's work. 3–4. Check students' work. There should be a positive correlation.

page 29 Activity 2: Making a Scatter Plot
1. yes, no
2. no, yes

page 30 Analyzing Scatter Plots
1. negatively correlated
2. unrelated
3. positively correlated
4. positively correlated
5. negatively correlated
6. unrelated
7a. b. negatively correlated

page 31 Reteaching: Scatter Plots and Trends
1. Weight and Height Survey

<table>
<thead>
<tr>
<th>Weight in Pounds</th>
<th>Height in Inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

2. about 61 in. 3. about 63 lb 4. yes 5. Sample: As height increases, weight increases.
Data Analysis and Probability Workbook Answers

**Section 2 Measures of Central Tendency**

**page 33** Measures of Central Tendency

**1.** mean: 3; median: 3; mode: 4  
**2.** mean: 8.1; median: 8; mode: 5  
**3.** mean: 5.25; median: 5; mode: 0, 1  
**4.** mean: $3.25; median: $4  
**5.** Median; there is no mode and the median is greater than the mean.  
**6.** Mode; both the median and the mean are less than the mode.

**page 34** Practice 1: Measures of Central Tendency

**1.** 1.5; 2; 2  
**2.** 3; 3; 1; 0  
**3.** 29.1; 29.5; 25; 30, and 35  
**4.** 8.1; 9; 9; 5; 73.6; 72; no mode  
**5.** 153.2; 146; no mode  
**6.** Mean; data are nonnumerical.  
**7.** Mean; data are nonnumerical.  
**8.** Mean; there should be no outliers.  
**9.** Mean or median; use median if there are outliers.  
**10.** At least 87  
**11.** 92  
**12.** At least 87

**page 35** Practice 2: Mean, Median, Mode, and Outliers

**1.** 23 students  
**2.** At least 12 students; up to 11 students  
**3.** Mean: 287.5, median: 300, mode: 200  
**4.** 50  
**5.** Lower  
**6.** The median, because there is an outlier.  
**7.** 8 points  
**8.** 23 points  
**9.** Answers will vary.

**page 36** Activity 1: Choosing an Appropriate Measure

**1.** Check students' work.  
**2.** a, b, c, d, g  
**3.** All of them, where the mode exists  
**4-6.** Check students' work.

**page 37** Activity 2: Average Temperature

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>0 2 3 3</td>
</tr>
<tr>
<td>7</td>
<td>0 4 4 5 5 5 6 6 7 7 8 8 8 8 9 9 9 9 9</td>
</tr>
<tr>
<td>8</td>
<td>0 0 0 1 1 2 3 4 5 5 6 6 7 7 8 8 8 8 9 9 9 9 9</td>
</tr>
<tr>
<td>9</td>
<td>0 1 6</td>
</tr>
<tr>
<td>10</td>
<td>2 7</td>
</tr>
</tbody>
</table>

2. Answers will vary.  
3. Mean = 81.05; median = 80.5; mode = 78 and 89 (bimodal)  
No city has the mean, although Columbus and Seattle are close. No city has the median, although Albany, Pittsburgh, St. Louis, and Salt Lake City lie just below; and Columbus and Seattle lie just above. The data is bimodal: Denver, Detroit, Spokane, Topeka, and Tulsa all have 78; Albuquerque, Portland, Raleigh, Richmond, and San Antonio all have 89.  
4-6. Answers depend upon class data.

**page 38** Activity 3: Wink Count

1-6. Answers depend upon class data.  
**7.** They must score at least 12 runs during the 3 games. One way to find the different ways in which this can be done is to make a chart.

<table>
<thead>
<tr>
<th>Game 1</th>
<th>Game 2</th>
<th>Game 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>0</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>11</td>
<td>0</td>
</tr>
</tbody>
</table>

etc.

There are 91 ways in all in which the additional 12 runs may be scored.

**page 39** Puzzle: Mean, Median, and Mode

**1.** [2, 4, 5, 7, 7]  
**2.** All should have found the same set.  
**3.** One  
**4.** a, 0  
**5.** a, 0  
**6.** Several solutions are possible, including [5, 5, 5, 5, 5], [4, 5, 5, 5, 5], [3, 5, 5, 5, 5], etc.  
**7.** Conditions: 0 ≤ a ≤ b ≤ c ≤ d ≤ e ≤ 20, and 2a + d + e ≤ 4c. Several solutions are possible, including [0, 0, 0, 0, 20].  
**8.** Conditions: a + b + c + d + e < 5c; and a + b + c + d + e < mode. One possible solution is [0, 0, 5, 5, 5].  
**9.** No solution  
**10.** Answers may vary. Samples: [0, 0, 0, 20, 20] and [0, 0, 20, 20, 20]

**page 40** Reteaching: Mean, Median, and Mode

**1.** 12, 11.5, none  
**2.** 87c, 86.5c, none  
**3.** 231, 231, none  
**4.** 50, 49, 43  
**5.** $130, $129, none

**page 41** Assessment 2

**1.** 3:03  
**2.** 2:04; 1:55  
**3.** 1:58  
**4.** 2:03  
**5.** Answers may vary. Sample: The median, because it eliminates the effect of the outlier and gives a representative time.  
**6.** Answers may vary. Sample: The mode, because it is arbitrary that two skiers happened to have the exact time.  
**7.** 1:18; the difference between the fastest and the slowest time.  
**8.** Below; 23 degrees  
**9.** 41 degrees  
**10.** 63 degrees

**Section 3 Use and Misuse of Data Displays**

**page 42** Choosing an Appropriate Graph

**1.** Multiple line graph: shows changes in two sets of data over time  
**2.** Circle graph: shows how the club's budget is divided into parts  
**3.** Double bar graph: compares two sets of data  
**4.** Circle graph: shows how 100% is divided into parts  
**5.** Line graph: shows change over time
Answers

1. 1. line graph; shows change over time
   2. bar graph, compares quantities
   3. line graph
   4. bar graph
   5. scatter plot, shows a relationship between sets of data
   6. circle graph, compares parts of a whole

2. bar graph: See students' graphs. Sample:

3. JFM

4. Answers may vary. Sample:

5. Answers may vary. Sample:

6. The number of people who prefer Yummy Cereal is increasing sharply.

7. The vertical axis changes units; the graph continues through a break.

8. You may wish to have students compare their graphs.

9. No

10. 2 cars

11. Probably not; car sales vary greatly and information from the first three months is not necessarily indicative of the whole year.

12. Vertical scale is unbroken on one, and broken on the other; difference in the bars seems greater on graph D.

13. The years on the horizontal axis are spaced differently; graph A seems to rise more steeply than graph B.

14. Answers may vary. Sample:

15. Lois's graph has intervals of 5 years and Harold's has intervals of varying numbers of years. Lois is more useful.

16. Lois's graph is more useful.

17. Numbers of Teachers

18. Numbers of Teachers

19. Numbers of Clear Days

20. Numbers of Clear Days

21. Miles

22. Mammals

23. Reptiles

24. Amphibians

25. Fish

26. Snails

27. Crustaceans

28. U.S. Endangered Animals

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Using Graphs to Persuade 2

1. Gasoline Retail Prices

The first graph implies prices decreased rapidly from 1997 to 1998, then increased rapidly from 1999 to 2000. The second graph implies slower changes.

2. Gasoline Retail Prices

3. The first graph implies prices decreased rapidly from 1997 to 1998, then increased rapidly from 1999 to 2000. The second graph implies slower changes.

Practice: Using Graphs to Persuade

1. birds
2. no
3. the break in the vertical axis

4. U.S. Endangered Species

5. The differences seem much less.

50. Practice: Using Graphs to Persuade

1. birds
2. no
3. the break in the vertical axis

4. U.S. Endangered Species

5. The differences seem much less.

U.S. Union Membership

6. The horizontal scales are different.

Page 51 Analyzing Games and Making Predictions

1. No; it suggests that Player A wins 3 and Player B wins 5.
2. HTHHT, HHTHT, HTHTH, HTHTT, HHTTH, THTHT, THTHT, TTHTH, Player A wins 4 and Player B wins 6.
4. Yes; in 16 rounds, both players are likely to win 8 points.

Page 52 Practice: Analyzing Games and Making Predictions

1. not fair
2. fair
3. fair
4. not fair
5. Possible answers: 0–1 for correct, 2–9 for incorrect
6. Check students’ work.
7. possible answer: 0–4 for correct, 5–9 for incorrect
8. Check students’ work. \( \frac{29}{50} = 58\% \) if answer shown here for Exercise 5 is used.
9. Possible answer: 0–7 for correct, 8–9 for incorrect
10. Check students’ work. \( \frac{41}{50} = 82\% \) if answer shown here for Exercise 9 is used.
Data Analysis and Probability Workbook Answers

Page 53 Assessment 3
1. Answers will vary. Sample:

2. Answers will vary. Sample:

3. Answers will vary. Graph A is good for giving the impression that something is decreasing rapidly. Sample: showing the change in a bank account over time to explain why a bigger allowance is needed. Graph B is good for giving the impression that something is decreasing more slowly. Sample: showing the change in a bank account over time to convince one’s self that a job is needed.

Section 4 Counting Principles
Page 54 Counting Outcomes
1. 12
2. 15
3. 16
4. 20
5. 12
6. 30

Page 55 Practice: Counting Outcomes
1. 1st choice 2nd choice 3rd choice
- pizza
- milk
- pudding
- apple
- juice

Page 56 Permutations
1. 120
2. 240
3. 720
4. 5,040
5. 720

Page 57 Practice: Permutations
1. 720
2. 24
3. 336
4. 336
5. 1,200
6. 3,628,800

Page 58 Combinations
1. 10
2. 45
3. 210
4. 2,016
5. 30

Page 59 Practice 1: Combinations
1. 10
2. 720
3. 4,032
4. 1,008
5. 120
6. 5,040
7. 1,008

Page 60 Practice 2: Permutations and Combinations
1. 120
2. 4,032
3. 1,008
4. 1,008
5. 120
6. 5,040

Page 61 Assessment 4
1. 1,008
2. There would be 8 times as many (or 40,320).
3. 15
4. \( \frac{1}{2} \) or 20%
5. 60 ways
6. Answers will vary. Sample: \( \binom{3}{2} \) could represent how many ways you could make a 3-digit combination out of the numbers 1 to 6, \( \binom{5}{3} \) could represent how many ways you select three students from a group of six. \( \binom{6}{2} \), \( \frac{1}{2} \binom{6}{3} \), and 24!
Section 5 Theoretical Probability

page 62 Theoretical Probability

1. 1; 0.2; 20%  
2. 5; 0.4; 40%  
3. 7; 0.5; 50%  
4. 3; 0.3; 30%  
5. 8; 0.9; 90%  

page 63 Practice: Theoretical Probability

1. 1; 0.1; 10%  
2. 2; 0.5; 50%  
3. 3; 1.0; 100%  
4. 4; 0.2; 20%  
5. 5; 0.348%  
6. 9; 34.8%  
7. Add a marble that is not blue.  
8. 8; 1; 10%  
10. 9; 3; 30%  

page 64 Reteaching: Theoretical Probability

1. 1; 0.2; 20%  
2. 2; 0.3; 30%  
3. 3; 0.5; 50%  
4. 4; 0.7; 70%  

page 65 Activity: Finding Errors in Statistical Analysis

1. Incorrect probability for each event  
\[ P(\text{even}) = P(2) + P(4) + P(6) = \frac{2}{6} = \frac{1}{3} \]  
2. Cannot count the same event twice  
\[ P(\text{not a multiple of } 3) = P(1) + P(2) + P(4) + P(5) + P(7) + P(8) - P(1\cap 2\cap 4\cap 5\cap 7\cap 8) \]  
3. Labels on the horizontal and vertical axes are transposed.

page 66 Sample Spaces

1. 12  
2. 10

page 67 Practice: Sample Spaces

1. 1  
2. 2  
3. 3  
4. 4  
5. 5  
6. 6  
7. 7  
8. 8  
9. 9  
10. 10

page 68 Counting Outcomes and Probability

1. 34 possible outcomes  
2. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10  

page 69 Practice: Counting Outcomes and Probability

1. 61 options: AM, AN, BM, BN, CM, CN  
2. 4 choices: AM, AN, BM, BN  
3. 6 combinations: P1C1, P1C2, P2C1, P2C2, P3C1, P3C2  

page 70 Independent and Dependent Events

1. Independent  
2. Independent  
3. Independent  
4. Dependent  

page 71 Practice 1: Independent and Dependent Events

1. 1; 0.2; 20%  
2. 2; 0.2; 20%  
3. 3; 0.2; 20%  
4. 4; 0.2; 20%  
5. 5; 0.2; 20%  
6. 6; 0.2; 20%  
7. Dependent; the second guest's choice is limited by the first guest's choice.  
8. Independent; the second flip is not affected by the first.

page 72 Practice 2: Independent and Dependent Events

1. 1; 0.2; 20%  
2. 2; 0.2; 20%  
3. 3; 0.2; 20%  
4. 4; 0.2; 20%  
5. 5; 0.2; 20%  
6. 6; 0.2; 20%  
7. 7; 0.2; 20%  
8. 8; 0.2; 20%  
9. 9; 0.2; 20%
Data Analysis and Probability Workbook Answers

11a. trash
   laundry
   2/7 brown 3/13
   5/7 black 4/13

11b.

page 73 Reteaching: Independent and Dependent Events
1. 1/5 2. 2/5 3. 3/5 4. 4/5 5. 6/10 6. 7/10 7. 5/10 8. 1/10

page 74 Analyzing Events Not Equally Likely
1. 1/3 2. 1/12 3. 1/5 4. 1/7 5. 1/12 6. 1/7 7. 8/12 8. 1/8
9. 1/4 10. 3/13 11. 1/2 12. 2/3 13. 5/13 14. 9/16 15. 8/16 16. 7/16

page 76 Activity: Analyzing Events Not Equally Likely
1. 4/7 2. 0.77 3. 0.9999 4. 8/19 5. 2/13 6. 12/13

page 77 Puzzle: Analyzing Theoretical Probability
1. 1/2 2. 2/3 3. 3/4 4. 1/3 5. 2/5 6. 3/5

page 82 Activity: 2: Theoretical and Experimental Probability
1–2. Check students’ work. 3. no 4. 1/6 5. no 6. For a large number of rolls, you will likely roll a 3 one-sixth of the time. 7. Since P (any number from 1 through 6) = 1, students should be able to predict that this result will occur 12 times. Since P(7) = 0, students should be able to predict that this result will occur 0 times. 8. Yes, because through repeated trials the experimental results should approach the theoretical probability. 9–10. Check students’ work.

page 83 Activity: 3: Increasing the Number of Trials
Check students’ tables. 1. 40 2. Answer should be close to 24 3. answer should be about 3.16 4. Check students’ calculations.

page 84 Predictions Based on Experimental Probabilities
1. P(broken) = 1/20 = 0.05 2. Yes; P(broken) = 1/50 = 0.02 3. 0.04 × 18,000 = 720 crayons 4. 0.016 × 18,000 = 288 crayons 5. 98.4%

page 85 Practice: Predictions Based on Experimental Probabilities
1. 1/20 2. yes; P(defective) now is 3/10, which is less than 1/20 3. 175 g 4. 250 g 100 g 5. 150 g 6. The mass must be within 75 g of 175 g 7. 87 cars 8. 8,000 bearings

page 86 Reteaching: Predictions Based on Experimental Probabilities
1. 160 shirts 2. yes; P(defective) now is 3/10, which is less than 1/20 3. 480 games, 400 = 12,000 4. 450 games, 300 = 12,000 5. 456 games, 190 = 12,000 6. 459 games, 180 = 12,000

page 87 Reteaching: Predictions Based on Experimental Probabilities
1. 1/20 2. yes; P(defective) now is 3/10, which is less than 1/20 3. 480 games, 400 = 12,000 4. 450 games, 300 = 12,000 5. 456 games, 190 = 12,000 6. 459 games, 180 = 12,000

Section 6 Experimental Probability
page 79 Theoretical and Experimental Probability

page 80 Practice: Theoretical and Experimental Probability
1. 1/6 2. 5/12 3. 1/3 4. 5/12 5. 5/12 6. 5/12 7. 1/4 8. 2/12 9. 2/12 10. 1/3 11. 5/12 12. 2/12 13. 4 to 11 14. 8 to 37 15. 3/2 16. 2/3 17a. 1/1200 = 1 to 1199 17b. 1/1200 = 1 to 1199 18. 0.55; 90 slices 19. 1/12000 = 1 to 9,999

page 81 Activity 1: Theoretical and Experimental Probability
1a-b. Check students’ work. 1c. Answers may vary; graphs should approach 50% as number of tosses increases.

Section 7 Statistical Investigations and Simulations
page 88 Random Samples and Biased Questions
1. C 2. B 3. No; you are more likely to interview homeowners. 4. No; you are more likely to interview renters. 5. Yes; you can’t tell if people own or rent.

page 89 Practice: Random Samples and Biased Questions
1–10. Answers may vary. Samples are given. 1. random sample 2. Not a random sample; students who use the vending machine may not represent all types of students.

Answers Data Analysis and Probability Teacher’s Guide
Data Analysis and Probability Workbook Answers

page 90  Planning A Survey
1. No; the sample includes only students interested in art, not the whole population.  2. Yes; every student has an equal chance of being selected.  3. Closed-option; Which sport do you think is the most exciting?  4. Open-option; Did you enjoy the book we just read in English?  5. Questioner is suggesting a preferred answer.  6. Question assumes that the movie was boring.

page 91  Practice 1: Planning a Survey
1. shoppers in a mall who are at least 16 years old  2. 96 never, 644 occasionally, 536 regularly  3. 2,146 people  4. no  5. open  6. The question makes gourmet meals seem better than plain ones.  7. The question makes shortened school days seem inferior.  8. No. People who work full-time are more likely to shop after work than midday.  9. Answers may vary. Some things to consider are using good questions, getting a large enough random sample, and avoiding biased questions.

page 92  Practice 2: Random Samples and Surveys
1. 320 students  2. 352 students  3. 200 students  4. 192 students  5. The views of people coming out of a computer store may not represent the views of other voters. This is not a good sample because it is not random.  6. The city telephone book may cover more than one school district. It would also include people who do not vote. This is not a good sample because it does not represent the population.  7. This is a good sample. It is selected at random from the population you want to study.

page 93  Conducting a Statistical Investigation
1. Check students' work.

page 94  Investigation 1: Happy Birthday
1–6. Check students’ work. EXTRA: James Polk (1796) and Warren Harding (1865) were born on November 2. John Adams (1826), Thomas Jefferson (1826), and James Monroe (1831) all died on July 4.

page 95  Investigation 2: Hits and Misses (Geometric Probability)
1.  2.  3.  4.  Answers may vary.

page 96  Investigation 3: Sticky Dot Number Cubes
1–5. Answers may vary.  6.  7.  Answers may vary.

page 97  Planning a Simulation
1. the probability that out of 3 children, one will be a girl  2. a coin toss  3. three tosses of a coin  4. the probability that out of 6 tees, there will be 2 red, 2 white, and 2 blue  5. a number cube with 2 numbers assigned to each color  6. 6 rolls of the number cube  7. a number cube with 2 numbers assigned to each color  8. 12 rolls of the number cube

page 98  Simulation 1: A Multiple Choice Test
1.  2. Answers may vary. Theoretically, 8.4 rolls  3. Answers may vary. The theoretical probability is .  4. Answers may vary. The theoretical probability is .  5. Answers may vary. The theoretical value is 20.  6. Answers may vary. The theoretical probability is .  7. Answers may vary. The theoretical value is 7.5.  8. Answers may vary. The theoretical probability is .  9. Probability of getting 4 or more answers correct.

page 99  Simulation 2: Dyeing T-Shirts
1.  2.  3.  4.  5. Answers may vary. Theoretically, 11.25 shirts.  6. Answers may vary. The theoretical probability is .  7. Answers may vary. The theoretical value is 18.75.  8. Answers may vary. The theoretical probability is .  9. Answers may vary. The theoretical value is 5.  10. Answers may vary. The theoretical probability is .

page 100  Simulation 3: A Fair Game?
In game 1, there is 1 chance out of 3 to match. In game 2, chances are 1 out of 2 that there will be a match.

page 101  Analyzing Simulations
1.  2.  3.  4.  5.  6.  7.  8.  9.  10.  11. Answers may vary. The probability of each person winning the lottery is much less than .

page 102  Reteaching 1: Simulations
1. Sample: Use a set of number cards 1–10 and draw one card at a time from a bag. Then replace the card.  2. Sample: Roll a number cube letting each of the numbers represent a different symbol. (Ignore 6.)  3. Sample: Use a spinner with 8 equal parts.  4. Sample: Roll a 12-sided number cube, letting each of the numbers represent a different symbol.

page 103  Reteaching 2: Simulations
1. Sample: Use a spinner divided into four parts, one for each symbol.  2. Sample: Roll two number cubes, letting each of the 36 combinations represent a different saying.  3. Sample: Toss a number cube.  4. Sample: Use two sets of number cards 1–10 and draw them from a bag.

page 104  Practice 1: Simulations
1a.  1b.  2a.  2b.  3. Check students’ work. Answer should be about 12.5%.  4. Check students’ work. Answer should be about 37.5%.  5. Check students’ work. Answer should be about 37.5%.  6. Check students’ work. Answer should be about 12.5%.  7. Possible answer: Let 1 = basket made, 2–6 = basket missed.  8. Check students’ work.

page 105  Practice 2: Simulations
1. Sample: Use a number cube with one number for W, I, E, and R, and two numbers for N.

page 106  Assessment 7
1. Answers may vary. Sample: Registered drivers selected randomly is best because that represents a fair sample of the
relevant population, while people at the restaurant, waiting to buy tickets, or visiting the dealership probably do not.

2. Open and fair. The question lets people answer generally rather than having them pick from limited choices, so it is open. It asks their opinion neutrally without suggesting an answer so it is fair.

3. 260 people

4. $20,000

5. The simulation shows that even with a 50/50 chance, the outcomes will not be exactly equal. In the simulation, the 75 coin flips come out 41 heads and 34 tails. The number of students is 10 times the number of coin flips, so this is equivalent to 410 students preferring one beverage and 340 wanting the other drink (rather than exactly 375 of each). Based on this result, the beverage committee should buy 35 extras of each drink (410 instead of 375) if they want a good chance of being able to satisfy everyone.

pages 107–108 Cumulative Assessment

1. 

2. 86.8, 88, 88

3. The median, because there is an outlier and there is no mode.

4. Any number greater than or equal to 27.

5. 44

6. One or both of the axes can be stretched or compressed so that the rate of change looks bigger or smaller than it really is. Or the y-axis can be broken so that the differences in magnitude are exaggerated.

7. 35, 210

8. 13, 440

9. 9,000 more

10. $\frac{1}{256}$

11. $\frac{8}{8}$

12. $\frac{1}{17}$

13. $\frac{1}{27}$

14. $\frac{1}{100}$

15. 8

16. 160

17. $\frac{4}{25}$

18. Answers will vary. Sample answer: Do you prefer the greasy pizza from Joe’s or the fancier pizza from Pistachios? Re-write: Which pizza do you prefer, Joe’s or Pistachios?

19. 40,000 people
Answers

Activity 1
1. Check students' work. 2. 1 3. 2
4.

<table>
<thead>
<tr>
<th>Number of twists</th>
<th>Number of strips when cut along the center line</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

5. Sample: A Möbius strip with 5 twists may produce 1 new strip, and one with 6 twists may produce 2 new strips.
6. Sample: By examining the information from the experiments, it is likely that Möbius strips with an odd number of twists result in 1 new strip when cut down the middle. However, Möbius strips with an even number of twists result in 2 new strips when cut down the middle.

Activity 2
1a. no 1b. There is an infinite number of ways.
2a. no 2b. There is an infinite number of ways. 3. 1 way
4a. Sample: The cardboard is not stable. 4b. There is an infinite number of ways. 5a. Sample: Yes, it is possible. However, if all four erasers are not the same length, they cannot all touch the cardboard. 5b. The cardboard is stable. 5c. The cardboard is not stable. 6. Sample: Three distinct points are needed to determine a unique plane. However, if one line and another point are present, a unique plane can be determined.

Activity 3
1. Check students' work. 2. Check students' work.
3. ABCD, ABFE, EFGH, CDHG, ADHE, BCFG, ABGH, CDEF, BCHE, ADGF, ACGE, BDHF 4. 12

Activity 4
1.–2.

<table>
<thead>
<tr>
<th>Hypothesis (p)</th>
<th>T or F</th>
<th>Negation (~p)</th>
<th>T or F</th>
</tr>
</thead>
<tbody>
<tr>
<td>X &gt; Y (6 &gt; 5)</td>
<td>T</td>
<td>X not greater than Y (6 &gt; 5)</td>
<td>F</td>
</tr>
<tr>
<td>X &lt; Y (6 &lt; 5)</td>
<td>F</td>
<td>X not less than Y (6 &lt; 5)</td>
<td>T</td>
</tr>
<tr>
<td>X = Y (6 = 5)</td>
<td>F</td>
<td>X ≠ Y (6 ≠ 5)</td>
<td>T</td>
</tr>
</tbody>
</table>

3. IF X > Y THEN PRINT Y, X; IF X < Y THEN PRINT X, Y; IF X = Y THEN PRINT X = Y 4. IF 3 < 10 THEN PRINT 3 = 10 5. False; the hypothesis is true, and the conclusion is false.

Activity 5

<table>
<thead>
<tr>
<th>Category numbers</th>
<th>Possible stores for best buy</th>
<th>Store with best buy so that each buy occurs exactly twice</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 and 2</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>1 and 3</td>
<td>C, D</td>
<td>C</td>
</tr>
<tr>
<td>1 and 4</td>
<td>A, C</td>
<td>A</td>
</tr>
<tr>
<td>1 and 5</td>
<td>D, E</td>
<td>E</td>
</tr>
<tr>
<td>2 and 3</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>2 and 4</td>
<td>A, B</td>
<td>B</td>
</tr>
<tr>
<td>2 and 5</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>3 and 4</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>3 and 5</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>4 and 5</td>
<td>E</td>
<td>E</td>
</tr>
</tbody>
</table>

Activity 6

Activity 7
1a. The 50-yd line is a transversal across the players' paths. The measures of the same-side interior ∠s on the west side of the 50-yd line are 65° for player #21 and 90° for player #13.
1b. Because the sum of the measures of the same-side interior ∠s is less than 180°, the two pathways meet beyond the 50-yd line. Since player #13 has less distance to travel, he should be able to overtake player #21. 2. Measurements may vary, however, the alternate interior ∠s will not be ≅. 3. No; the alternate interior ∠s formed are not ≅. 4. It was not, because BX is not || to the 40-yd line.

Activity 8

<table>
<thead>
<tr>
<th>Triangle</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMQ</td>
<td>acute, scalene</td>
</tr>
<tr>
<td>AHD</td>
<td>obtuse, scalene</td>
</tr>
<tr>
<td>BEJ</td>
<td>acute, equilateral</td>
</tr>
<tr>
<td>FLK</td>
<td>acute, isosceles</td>
</tr>
<tr>
<td>GRC</td>
<td>right, scalene</td>
</tr>
<tr>
<td>STP</td>
<td>obtuse, scalene</td>
</tr>
</tbody>
</table>
Activity 9
1a. 1b.
2. 3.
4a. 4b.

Activity 10
1.–3. Check students’ work. 4. congruent intersecting circles 5. congruent crescentlike shapes; congruent triangular shapes; congruent oval shapes 6. two pairs of congruent triangles 7. three pairs of congruent triangles 8. Check students’ designs for requirements.

Activity 11
1. Because all pins are equidistant, each side length is the sum of two equal distances. So \( AB \equiv JK \), \( BC \equiv KL \), and \( AC \equiv JL \). Therefore, \( \triangle ABC \equiv \triangle JKL \) by SSS.
2. \( \triangle TSR \equiv \triangle VUQ \). The pairs of corresponding sides are \( TS \), \( VR \), \( SR \), and \( UQ \), and \( TR \) and \( VQ \). By the previous reasoning (each side is the sum of \( \equiv \) line segments), the pairs of corresponding sides are \( \equiv \). So the triangles are \( \equiv \) by SSS.
3a. Sample: \( A \), \( B \), \( C \), \( D \), \( E \), \( F \), \( G \), \( H \), \( I \), \( J \)
3b. Sample: \( \triangle ABE \), \( \triangle BFC \), \( \triangle CDG \), \( \triangle BEF \), \( \triangle CFG \), \( \triangle EFH \), \( \triangle FGI \), \( \triangle HIJ \), \( \triangle FHI \)

Activity 12
1. The number of smaller triangles in an \( n \)-frequency dome is \( n^2 \). 2. As the frequency increases, the dome becomes more spherical in shape.

Activity 13
1. the width of the straightedge 2. They are equal.
3. They are equal. 4. Point \( T \) lies on \( QS \). 5. They are equal. 6. \( QS \) is the bisector of \( \angle PQR \) in \( \triangle PQR \).

Activity 14
1.
2.
3.
4.
5.
6.
7.
Answers (continued)

**Activity 15**

<table>
<thead>
<tr>
<th>Number of toothpicks</th>
<th>Number of different-shaped triangles</th>
<th>Lengths of sides</th>
<th>Number of equilateral triangles</th>
<th>Number of isosceles triangles (non-equilateral)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>1, 1, 1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>Not possible</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>2, 2, 1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>2, 2, 2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>1, 3, 3 and 2, 2, 3</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>2, 3, 3</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>3, 3, 3; 2, 3, 4; 1, 4, 4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>2, 4, 4; 3, 3, 4</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>11</td>
<td>4</td>
<td>1, 5, 5; 2, 4, 5; 3, 4, 4; 3, 3, 5</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>2, 5, 5; 3, 4, 5; 4, 4, 4</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

1. A triangle cannot be formed.  
2. 5, 7, 8, 9, 10, 11, 12; there does not seem to be a pattern.  
3. 3, 6, 9, 12; yes, the pattern seems to be multiples of 3; prediction: 15.  
4. 1, 2, 3, 4; prediction: 5; the prediction is correct.  
5. Prediction: 6; the prediction is not correct.  
6. greater than; cannot

**Activity 16**

1. All triangles, when combined with a congruent triangle, form a parallelogram. This is true because any parallelogram split in half on its diagonal produces two congruent triangles.  
2. The two acute isosceles triangles yielded a rhombus. Because of the two equal sides of each triangle, the two triangles form four congruent sides when a pair of sides are placed together.  
3. The two right scalene triangles yielded another triangle. This triangle is classified as isosceles. This is possible because of the 90° angle in each triangle. When joined, these two 90° angles form a straight angle.  
4. The two obtuse scalene triangles yielded a concave quadrilateral. When these triangles are joined at their shorter sides, an interior angle greater than 180° is formed.  
5. All three pairs of triangles yielded a kite. When the triangles are joined at their longer sides, congruent sides are adjacent to each other, which is the definition of a kite.  
6. parallelogram, rhombus  
7. parallelogram, rhombus

**Activity 17**

<table>
<thead>
<tr>
<th>Number of toothpicks (y)</th>
<th>Sketch of arrangement(s)</th>
<th>Number of squares (x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td><img src="image1" alt="Sketch" /></td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td><img src="image2" alt="Sketch" /></td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td><img src="image3" alt="Sketch" /></td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td><img src="image4" alt="Sketch" /></td>
<td>3; 2</td>
</tr>
<tr>
<td>11</td>
<td><img src="image5" alt="Sketch" /></td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td><img src="image6" alt="Sketch" /></td>
<td>1; 5</td>
</tr>
<tr>
<td>13</td>
<td><img src="image7" alt="Sketch" /></td>
<td>4</td>
</tr>
</tbody>
</table>

1. 1 2 4 3. Answers may vary. Sample: If the number of toothpicks is divisible by 4, there can be 1 square formed.

**Activity 18**

1. square, rectangle, rhombus, or parallelogram  
2. square or rectangle  
3. square or rhombus  
4.
Answers (continued)

Activity 19
1. Check students’ work.  
2. Sample: The area of $JKLM$ is one-fifth the area of $ABCD.$
3–5.

6. $AF: y = \frac{1}{2}x, CH: y = \frac{1}{2}x + 5, BG: y = -2x + 20, ED: y = -2x + 10; AF$ and $CH$ have the same slopes; $BG$ and $ED$ have the same slopes. The slopes of $AF$ and $CH$ are the negative reciprocals of the slopes of $BG$ and $ED.$
7. They are parallel. $FG$ and $ED$ are also parallel. $JKLM$ is a parallelogram.
8. $J(4, 2)$
9. $K(8, 4), L(6, 8), M(2, 6)$
10. area of $JKLM = 20$ square units; area of $ABCD = 100$ square units; ratio $= \frac{1}{5}$
11. Check students’ work.

Activity 20
1a. Check students’ work.  
1b. Check students’ work.
2. rectangle: $b = \sqrt{18}, h = \sqrt{2}$; kite: $d_1 = 2$, $d_2 = 6$; parallelogram: $b = 3, h = 2$; trapezoid: $b_1 = 4, b_2 = 2, h = 2$
3a. Sample:

3b. Sample:

<table>
<thead>
<tr>
<th>Quadrilaterals with area 10 square units</th>
<th>Classification</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>rectangle</td>
<td>$b = 5, h = 2$</td>
<td></td>
</tr>
<tr>
<td>parallelogram</td>
<td>$b = 2, h = 5$</td>
<td></td>
</tr>
<tr>
<td>isosceles trapezoid</td>
<td>$b_1 = 6, b_2 = 4, h = 2$</td>
<td></td>
</tr>
<tr>
<td>kite</td>
<td>$d_1 = 4, d_2 = 5$</td>
<td></td>
</tr>
<tr>
<td>trapezoid</td>
<td>$b_1 = 3, b_2 = 1, h = 5$</td>
<td></td>
</tr>
<tr>
<td>square</td>
<td>$s = \sqrt{10}$</td>
<td></td>
</tr>
</tbody>
</table>

4a. Sample:

4b. Sample:

<table>
<thead>
<tr>
<th>Quadrilaterals with area 12 square units</th>
<th>Classification</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>rectangle</td>
<td>$b = 6, h = 2$</td>
<td></td>
</tr>
<tr>
<td>rectangle</td>
<td>$b = 4, h = 3$</td>
<td></td>
</tr>
<tr>
<td>isosceles trapezoid</td>
<td>$b_1 = 5, b_2 = 3, h = 3$</td>
<td></td>
</tr>
<tr>
<td>kite</td>
<td>$d_1 = 6, d_2 = 4$</td>
<td></td>
</tr>
<tr>
<td>parallelogram</td>
<td>$b = 3, h = 4$</td>
<td></td>
</tr>
<tr>
<td>trapezoid</td>
<td>$b_1 = 3, b_2 = 1, h = 6$</td>
<td></td>
</tr>
</tbody>
</table>

5a. Sample:

5b. Sample:

<table>
<thead>
<tr>
<th>Quadrilaterals with area 15 square units</th>
<th>Classification</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>rectangle</td>
<td>$b = 3, h = 5$</td>
<td></td>
</tr>
<tr>
<td>parallelogram</td>
<td>$b = 5, h = 3$</td>
<td></td>
</tr>
<tr>
<td>isosceles trapezoid</td>
<td>$b_1 = 4, b_2 = 2, h = 5$</td>
<td></td>
</tr>
<tr>
<td>rectangle</td>
<td>$b = 15, h = 1$</td>
<td></td>
</tr>
<tr>
<td>trapezoid</td>
<td>$b_1 = 7, b_2 = 3, h = 3$</td>
<td></td>
</tr>
</tbody>
</table>

6. Sample: Use factors of the given area.  
7. Sample: Work backward from area formulas of parallelograms, trapezoids, and kites.
Answers (continued)

Activity 21
1. 100 square units  
2. \(PS = 6; SG = 8\)  
3. \(\triangle YPS \cong \triangle SGA\) by \(\text{SAS}\), so \(\triangle YS \cong \triangle SY\) by \(\text{CPCTC}\).  
4. \(\angle 1 \equiv \angle 4\) and \(\angle 2 \equiv \angle 3\) by \(\text{CPCTC}\).  
5. They are complementary.  
6. 90  
7. Check students’ work; area: \(c^2 = 100\).  
8..original squares: \(8^2 + 6^2 = 64 + 36 = 100\);  
new squares: \(10 \cdot 10 = 100\)  
9. Check students’ work.  
10. Using the figure above, the area of square \(XYZU\) is \(a^2\), and the area of square \(RZST\) is \(b^2\). So the area of figure \(XYTSRU\) is \(a^2 + b^2\). If you cut out \(\triangle XYV\), \(\triangle VST\), and \(\triangle XVSRU\) and then rearrange them to form a square with sides \(c\), the area of the square formed is \(c^2\). Therefore, \(a^2 + b^2 = c^2\).

Activity 22
1.  
2.  
3.  
4.  
5.  
6.  
7. and 8.  
9.  
10.  

Activity 23
1. Check students’ work.  
2. Check students’ work.  
3. Triangle A: hypotenuse = 5 cm; Triangle B: hypotenuse = 13 cm; Triangle C: hypotenuse = 10 cm; Triangle D: hypotenuse = 15 cm; Triangle E: hypotenuse = 17 cm  
4. 3, 4, 5; 5, 12, 13; 6, 8, 10; 9, 12, 15; 10, 15, 17  
5. 3, 4, 5; 6, 8, 10; 9, 12, 15  
6. Sample: Each side of a bigger triangle is a multiple of its corresponding side of a smaller triangle. For triangles to be similar, they must have a common ratio among their sides.  
7. Sample: 10, 24, 26; 16, 30, 34  
8. Sample: similar to triangle B: 10, 24, 26; 15, 36, 39; similar to triangle E: 16, 30, 34; 24, 45, 51  
9a. \(7^2 + 24^2 = 25^2\); \(49 + 576 = 625\); \(625 = 625\)  
9b. Sample: 14, 48, 50; 21, 72, 75

Activity 24
1. Check students’ work.  
2.  

| \(\triangle ABC\) | \(m \angle A = 90\) | \(m \angle B = 27\) | \(m \angle C = 63\) | \(\triangle MNP\) | \(m \angle M = 90\) | \(m \angle N = 27\) | \(m \angle P = 63\) | \(\triangle RST\) | \(m \angle R = 90\) | \(m \angle S = 27\) | \(m \angle T = 63\) | \(\triangle ABC\) | \(AB = 3\) cm | \(BC = 3.4\) cm | \(CA = 1.5\) cm | \(\triangle MNP\) | \(MN = 4\) cm | \(NP = 4.5\) cm | \(PM = 2\) cm | \(\triangle RST\) | \(RS = 5\) cm | \(ST = 5.6\) cm | \(TR = 2.5\) cm

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Answers (continued)

3a. These sides are approximately proportional.  
3b. Yes
3c. Yes; the sides of $\triangle ABC$ are proportional to the sides of $\triangle RST$, and the sides of $\triangle MNP$ are proportional to the sides of $\triangle RST$.  
4. Yes; by Theorem 8-2, if the ratios are equal, then the triangles are similar.  
5a. They are parallel.
5b. $MW$ is half the length of $MF$. $MX$ is half the length of $MN$.  
6. The length of the part of the shorter leg is half the length of the part of the longer leg.

Activity 25

1. Check students’ work.  
2. Triangle A: hypotenuse $\approx 3.6$ cm; Triangle B: hypotenuse $\approx 7.2$ cm; Triangle C: hypotenuse $\approx 14.4$ cm.  
3. The corresponding sides are proportional.  
4. The three triangles are similar.  
5. The corresponding angles are congruent.

<table>
<thead>
<tr>
<th>Smaller angle</th>
<th>$\frac{2}{3}$</th>
<th>$\frac{1}{1.8}$</th>
<th>$\frac{1}{1.2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triangle B</td>
<td>$\frac{2}{3}$</td>
<td>$\frac{1}{1.8}$</td>
<td>$\frac{1}{1.2}$</td>
</tr>
<tr>
<td>Triangle C</td>
<td>$\frac{2}{3}$</td>
<td>$\frac{1}{1.8}$</td>
<td>$\frac{1}{1.2}$</td>
</tr>
</tbody>
</table>

6. The ratios in each column are equal.

<table>
<thead>
<tr>
<th>Larger angle</th>
<th>$\frac{3}{2}$</th>
<th>$\frac{1}{1.2}$</th>
<th>$\frac{1}{1.8}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triangle B</td>
<td>$\frac{3}{2}$</td>
<td>$\frac{1}{1.2}$</td>
<td>$\frac{1}{1.8}$</td>
</tr>
<tr>
<td>Triangle C</td>
<td>$\frac{3}{2}$</td>
<td>$\frac{1}{1.2}$</td>
<td>$\frac{1}{1.8}$</td>
</tr>
</tbody>
</table>

Activity 26

1–4. Check students’ work.

Activity 27

1. $180$  
2. $270$  
3. $315$  
4. $112.5$
5. The paper moves faster than the speed of the force with which you rolled it. Its actual speed is the sum of the force with which you rolled the paper and the force with which you blew on it.

6. The paper moves slower than the speed of the force with which you rolled it. Its actual speed is the difference between the force with which you rolled the paper and the force with which you blew on it.

Activity 28

1. Sample: $G \rightarrow \overline{AB} \rightarrow D$, $A \rightarrow \overline{CD} \rightarrow E$, $C \rightarrow \overline{EG} \rightarrow A$  
2. triangular prism  
3. $B \rightarrow \overline{CD} \rightarrow E$  
4. $A \rightarrow \overline{EC} \rightarrow G$  
5. $E \rightarrow \overline{CD} \rightarrow A$  
6. $\angle AEC$ and $\angle BGD$
Answers (continued)

Activity 29
1. Check students’ work.
2.

<table>
<thead>
<tr>
<th>Circle</th>
<th>Radius of circle (R)</th>
<th>Take-out angle (x)</th>
<th>Cone angle (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10 cm</td>
<td>72</td>
<td>288</td>
</tr>
<tr>
<td>B</td>
<td>10 cm</td>
<td>90</td>
<td>270</td>
</tr>
<tr>
<td>C</td>
<td>10 cm</td>
<td>180</td>
<td>180</td>
</tr>
<tr>
<td>D</td>
<td>10 cm</td>
<td>216</td>
<td>144</td>
</tr>
</tbody>
</table>

3. The radius of the circle is the slant height of the cone.
4. Slant height of cone
5. The values are equal.
6. \(2\pi r = \frac{360}{\pi} - x (2\pi R)\)
7. \(r = \frac{(360 - x)R}{360}\)
8. \(\frac{r}{R} = \frac{360}{360 - x}\)
9. circle \(A\): \(h = 6\) cm; circle \(B\): \(h \approx 6.6\) cm; circle \(C\): \(h \approx 8.7\) cm; circle \(D\): \(h \approx 9.2\) cm
10. Check students’ work.

Activity 30
1. Check students’ work.
2. They are congruent.
3. The sum will always be greater than 180°.
4. Sample: Any two points on a sphere lie on the same great circle. Therefore, the shortest distance between any two locations on Earth will be along a great-circle route.
5. The center of the nine-point circle lies on the Euler line and is midway between the circumcenter and the orthocenter.
6. The radius of the nine-point circle is equal to half the radius of the circumscribed circle of the triangle.

Activity 31
Steps 1.–5.

Activity 32
1. 3.–6., and 8.

Activity 33
1. 90
2. 180
3. semicircle
4. Sample: \(AG = 42\) mm; \(BG = 42\) mm; \(CG = 42\) mm; \(DG = 42\) mm
5. \(G\) is the center of the circle.
6. Check students’ work.

Activity 34
1. Sample:
2. Sample:
Answers (continued)

3. Sample:

4. Sample:

5. 

6. 

7. 

8. 

9. perpendicular bisector  

10. Locate the perpendicular bisector of the base of each triangle, and then reflect, or draw a mirror image of, the triangle on each side of this line.  

11. Check students’ work.

Activity 35
1. Figures 4 and 6  2. Figures 3 and 1  3. Figure 5  
4. Figure 2  5.–8. Check students’ work.

Activity 36
1. Sample:

2. 60  3. 40  4. 5  5. 2  6. 90; 65; 5; 2  7. Check students’ work.
Answers

Screening Test

Benchmark Test 1
31. B 32. F 33. C

Benchmark Test 2

Benchmark Test 3

Benchmark Test 4

Benchmark Test 5

Quarter 1 Test, Form A
1. 13, 21 2. Sample: a tiger 3. right triangle
4. 5. $\frac{2}{7}$ 6. $x = 3$ 7. 25
8. 
9. Subtraction Property
10. 
11. 70 12. 62 13. 29 14. Suppose that $2x + 4 = 6$. Subtract 4 from both sides of the equations by the Subtraction Property. This gives an equation of $2x = 6 - 4$, which reduces to $2x = 2$. Now divide both sides of the equation by 2 by using the Division Property. This gives an equation of $\frac{2x}{2} = \frac{2}{2}$, which simplifies to $x = 1$. 15. $AC$
16. $12\pi$ m 17. Sample: A student can drive if and only if he or she is over the age of sixteen. If a student can drive, then he or she is over the age of sixteen. If a student is over the age of sixteen, then he or she can drive. 18. 144
19. (1, 2.5) 20. $-\frac{2}{3}$ 21. $\angle 2 \cong \angle 4$ by the Converse of the Corresponding Angles Theorem or $\angle 2 + \angle 3 = 180$ by the Converse of the Same-Side Interior Angles Theorem 22. $2\sqrt{17}$ 23. no intersection or a point 24. Two lines must not intersect and be in the same plane to be parallel. 25. 40 26. 50 27. 90 28. 75 29. hypothesis: if you are not at school; conclusion: you are at home 30. If you are at home, then you are not at school. 31. 20 in. 32. Transitive Property 33. center $(5, -5)$; radius $= \sqrt{5}$
Quarter 1 Test, Form B
1. 8, 5  2. Sample: obtuse triangle  3. isosceles triangle
4. [Diagram of a rectangular prism with dimensions 14 cm by 6 cm by 3 cm]
5. \(-\frac{1}{8}\)  6. \(x = 6\)  7. 7
8. [Diagram of triangle with vertices A, B, and C]
9. Addition Property
10. [Diagram of a cube]
11. 112  12. 60  13. 44  14. Suppose that \(3x + 5 = 11\). Subtract 5 from both sides of the equation by the Subtraction Property. This gives an equation of \(3x = 6\), which reduces to \(x = 2\).  15. point R
16. 26\(\pi\) m  17. Sample: A student can drive if and only if he or she is over the age of sixteen. If a student can drive, then he or she is over the age of sixteen. If a student is over the age of sixteen, then he or she can drive.  18. 135
19. \((-6, 11)\)  20. \(\frac{3}{2}\)  21. Sample: \(\angle 1 \equiv \angle 3\) or \(\angle 2 \equiv \angle 4\) by the Converse of the Corresponding Angles Postulate; \(\angle 2 + \angle 3 = 180\) by the Converse of the Same-Side Interior Angles Theorem  22. 5  23. no intersection or a line
24. A triangle cannot have two sides and no sides equal at the same time.  25. 60  26. 90  27. 70  28. 45
29. hypothesis: if you are not at work; conclusion: you are at school  30. If you are at school, then you are not at work.
31. 96 in.  32. Substitution Property  33. center (2, 1); radius \(= \sqrt{117}\)

Quarter 1 Test, Form D
1. 21, 25  2. scalene triangle  3. 36\(\pi\) m\(^2\)  4. A  5. 5  6. 9
7. [Diagram of a rectangle with dimensions 3 x 1]
8. 138  9. 46  10. C  11. 40 cm  12. \(\overline{BF}\)  13. 10\(\pi\) m
14. 120  15. \((-2, 4)\)  16. \(\frac{1}{2}\)  17. B  18. 5  19. 45  20. 90
21. 60  22. 85  23. hypothesis: two angles have a sum of 180°; conclusion: the angles are supplementary angles
24. G  25. D

Quarter 1 Test, Form E
1. 26, 31  2. equilateral triangle  3. 100\(\pi\) m\(^2\)  4. 5  5. 6
6. [Diagram of a trapezoid with dimensions 2 x 2]
7. 110  8. 70  9. B  10. G  11. 60 in.  12. \(\overline{AB}\)  13. 14\(\pi\) m
14. 135  15. (1, 5)  16. \(\frac{1}{2}\)  17. D  18. 10  19. 72  20. 55
21. 50  22. 75  23. G  24. Hypothesis: two lines form right angles; conclusion: the lines are perpendicular.  25. C

Quarter 2 Test, Form A
1. \(\overline{RS}\)  2. 20  3. trapezoid  4. 50  5. 40; 82  6. SSS Postulate  7. SAS Postulate  8. \((c - a, b)\)
9. Sample: \((0, 4), (4, 0), (-4, 0), (0, -4)\)  10. \(\angle R, \angle P, \angle Q\)  11. AAS, \(\triangle ABC \equiv \triangle ABD\)  12. 133; 47  13. Sample: 4 ft, 3 ft, and 8 ft
14. 4  15. Sample: The diagonals must be perpendicular bisectors of each other, or all sides must be congruent.
16. The orthocenter is the intersection of the three altitudes of a triangle.  17. an altitude  18. 45  19. Hypotenuse-Leg Theorem  20. centroid  21. It is the median and altitude of the triangle.  22. Suppose that a triangle had more than one right angle. If this were true, then two of the angle measures alone would add up to 180, and the third angle would have a measure that would contradict the Triangle Angle-Sum Theorem.  23. \(\overline{AB}, \overline{BC}, \overline{AC}\)  24. 90; 30  25. rhombus
26. 65  27. c, e, b, a, d or e, c, b, a  28. No; the midsegment is not half the length of the third side.  29. a kite
30. 31  31. They must be perpendicular bisectors of each other.  32. \(> AC\)
Quarter 2 Test, Form B

1. $\angle B$ 2. 10 3. parallelogram 4. 58 5. 41; 75 6. ASA
Postulate 7. AAS Theorem 8. $(c + a, b)$ 9. Sample: $(0,4), (8,0), (0, -4)$ 10. $\angle L, \angle N, \angle M$ 11. SAS. 12. $\triangle ACD \cong \triangle CAB$ 13. Sample: 4 ft, 3 ft, and 8 ft 14. 4 15. Sample: The diagonals must be perpendicular bisectors of each other and all sides must be congruent, or all sides and angles must be congruent. 16. The incenter is the intersection of the three angle bisectors of a triangle. 17. a median 18. 41 19. Hypotenuse-Leg Theorem 20. centroid 21. It is the median and altitude of the triangle. 22. Suppose that a triangle had more than one obtuse angle. If this were true, then two of the angle measures would add up to more than 180, and the third angle would have a measure that would contradict the Triangle Angle-Sum Theorem. 23. $AB, BC, AC$ 24. 90; 50 25. rectangle 26. 70 27. a, c, e, b, d or c, a, e, b, d 28. Yes; the midsegment is half the length of the third side. 29. a parallelogram 30. 31. They must be equal in measure. 32. $AC + BC$

Quarter 2 Test, Form D

1. $XZ$ 2. 4 3. D 4. 55 5. $x = 85, y = 60$ 6. J 7. $\angle C, \angle A, \angle B$ 8. $m \angle 1 = 54.5, m \angle 2 = 54.5, m \angle 3 = 71$ 9. D 10. C 11. B 12. E 13. 55 14. 56 15. H 16. $m \angle 1 = 55, m \angle 2 = 45, m \angle 3 = 45$, 17. $m \angle 1 = 25, m \angle 2 = 25, m \angle 3 = 130$ 18. A 19. parallelogram, rectangle, square, rhombus 20. 164 21. $\angle B \equiv \angle D$ 22. altitude 23. 33

Quarter 2 Test, Form E

1. $\angle R$ 2. 5 3. C 4. 50, 65 5. $x = 100, y = 80$ 6. F 7. $\angle C, \angle B, \angle A$ 8. $m \angle 1 = 122, m \angle 2 = 29, m \angle 3 = 29$ 9. E 10. A 11. C 12. B 13. 78 14. 58 15. G 16. $m \angle 1 = 44, m \angle 2 = 84, m \angle 3 = 40$, 17. $m \angle 1 = 65, m \angle 2 = 65, m \angle 3 = 50$ 18. D 19. parallelogram, rectangle, square, rhombus 20. 187 21. $BX \equiv DX$ 22. perpendicular bisector 23. 40

Quarter 3 Test, Form A

1. 14 2. $\triangle ABC \sim \triangle JKL$ by SSS ~ Theorem 3. $\triangle TUV \sim \triangle WXY$ by AA ~ Postulate 4. obtuse 5. 56.3 6. 10.7 7. reflectional symmetry and 180° rotational symmetry (point symmetry) 8. $2\sqrt{26}$ 9. sin $A = \frac{8}{15}$; cos $A = \frac{12}{15}$; tan $A = \frac{5}{12}$ 10. $A'(7, 2), B'(11, 2), C'(10, -1)$, $D'(6, -1)$ 11. $A'(5, -4), B'(-9, 4), C'(-8, 1), D'(-4, 1)$ 12. $A'(-3, -4), B'(-7, -4), C'(-6, -1), D'(-2, -1)$ 13. 150 in.$^2$ 14. 53.9 mi/h 18.1° south of east 15. A 16. (3, 3) 17. $4\sqrt{3}$ 18. 38 19. 6 20. 18.4 21. Check students’ work. 22. No; there will be gaps when the pattern is repeated. 23. $A'(-2, 6), B'(0, 14), C'(8, 4)$ 24. 48 ft

Quarter 3 Test, Form B

1. 14 2. not similar 3. $\triangle ABC \sim \triangle LPR$ by SAS ~ Theorem 4. obtuse 5. 14.9 6. 55.0 7. reflectional symmetry 8. $\sqrt{66}$ 9. sin $A = \frac{3}{5}$; cos $A = \frac{4}{5}$; tan $A = \frac{3}{4}$ 10. $A'(-6, -3), B'(-2, -3), C'(-2, 0), D'(-5, 0)$ 11. $A'(-18, 15), B'(-6, 15), C'(-6, 6), D'(-15, 6)$ 12. $A'(-9, 6), B'(-5, 6), C'(-5, 3), D'(-8, 3)$ 13. 18 cm$^2$ 14. 40.3 mi/h; 29.7° north of west 15. D 16. $(-4, 1)$ 17. $2\sqrt{34}$ 18. 4 19. 63 20. 33.7 21. Check students’ work. 22. yes

Quarter 3 Test, Form C

1. $\triangle BRG \sim \triangle NDK$ by SAS ~ Theorem 2. $\triangle AYM \sim \triangle XQH$ by AA ~ Postulate 3. 12.8 4. 32.2 5. 13.5 6. D 7. $4\sqrt{5}$ 8. sin $A = \frac{15}{17}$; cos $A = \frac{12}{17}$; tan $A = \frac{5}{12}$ 9. (10, 0) 10. G 11. 112 in.$^2$ 12. A 13. 5$\sqrt{7}$ 14. $x = 9, y = 15$ 15. $y = 14$ 16. $X'(4, 6)$, $Y'(5, 10), Z'(8, 7)$ 17. $X'(-2, -1), Y'(-3, -5), Z'(-6, -2)$ 18. $X'(-2, 1), Y'(3, -5), Z'(6, -2)$ 19. $6\sqrt{3}$ 20. 88 ft 21. 12 in. 22. J

Quarter 3 Test, Form D

1. $\triangle WLB \sim \triangle HER$ by AA ~ Postulate 2. $\triangle CMJ \sim \triangle FKP$ by SAS ~ Theorem 3. 12.2 4. 63.0 5. 6. C 7. $6\sqrt{3}$ 8. sin $A = \frac{15}{17}$; cos $A = \frac{8}{17}$; tan $A = \frac{15}{8}$.
Quarter 4 Test, Form A
1. 268.1 cm$^3$
2. 66.3 cm$^2$
3. 25
4. 32
5. 144
6. Sample: Draw $d_1$, a diagonal of the kite that divides it into two congruent triangles. Let $d_1$ represent the base of each triangle. The area of one triangle is $\frac{1}{2}d_1h_1$. The area of the other triangle is $\frac{1}{2}d_1h_2$. Because the triangles are congruent, $h_1 = h_2$. The other diagonal, $d_2$, is the sum of $h_1$ and $h_2$. Therefore, the area of a kite is $\frac{1}{2}d_1d_2$.
7. 14.14 cm$^2$
8. 12.5
9. a circle with center ($-2, -3$) and radius 5 units
10. 113.1 m$^2$
11. 81 m$^2$
12. 6
13. center: (5, 6); radius: 4
14. 73.7
15. 92
16. 400 ft$^3$
17. 15 rolls of wallpaper
18. 30
19. 60
20. 270
21. 8
22. 310.4 in.$^2$
23. b, c, a, d
24. $(x - 2)^2 + (y - 2)^2 = 4$
25. 188.1 in.$^2$
26. 2π in.
27. Sample: $QR$ is tangent to $\odot P$. $QR$ is perpendicular to a radius of $\odot P$.
28. 284.7 cm$^2$

Quarter 4 Test, Form B
1. 1436.8 in.$^3$
2. 63.4 in.$^2$
3. 39.5
4. 25.5
5. 108
6. Sample: Draw $d_1$, a diagonal of the rhombus that divides it into two congruent triangles. Let $d_1$ represent the base of each triangle. The area of one triangle is $\frac{1}{2}d_1h_1$. The area of the other triangle is $\frac{1}{2}d_1h_2$. Because the triangles are congruent, $h_1 = h_2$. The other diagonal, $d_2$, is the sum of $h_1$ and $h_1$. Therefore, the area of a rhombus is $\frac{1}{2}d_1d_2$.
7. 19.63 in.$^2$
8. 6
9. a circle with center ($-4, 6$) and radius 8 units
10. 466.5 ft$^2$
11. 42 in.$^2$
12. 12
13. center: ($-3, 5$); radius: 3
14. 122
15. 91.1
16. 75.9 cm$^3$
17. 2 cans of paint
18. 55
19. 35
20. 270
21. 11
22. 93.6 in.$^2$
23. b, c, a, d
24. $(x - 3)^2 + (y + 2)^2 = 9$
25. 452.3 cm$^2$
26. $\frac{7}{2}π$ ft
27. Sample: $RS$ is an arc of $\odot Q$. The measure of $\angle RQS$ is equal to the measure of $\angle Q$.}

Quarter 4 Test, Form D
1. 904.8 in.$^2$
2. 4 : 7, 16 : 49
3. 90
4. 75
5. B
6. 96 cm$^2$
7. 8
8. 245.0 ft$^2$
9. 40 cm$^2$
10. 5
11. 9
12. center: (0, 0); radius: 5
13. 104
14. 90
15. 128 cm$^2$
16. 184 in.$^2$
17. 180 cm$^2$
18. 201.1 ft$^2$
19. 133
20. 180
21. 192.5 in.$^2$
22. 150.80 m$^2$
23. G
24. 188.5 cm$^2$

Quarter 4 Test, Form E
1. 1436.8 cm$^3$
2. 3 : 8, 9 : 64
3. 65
4. 140
5. B
6. 255 in.$^2$
7. 16
8. 326.7 in.$^2$
9. 95 m$^2$
10. 10
11. 15
12. center: (0, 0); radius: 4
13. 90
14. 87
15. 125 cm$^3$
16. 804.2 m$^2$
17. 483.8 in.$^2$
18. 262 ft$^2$
19. 59
20. 239
21. 173.8 in.$^2$
22. 94.25 cm$^2$
23. H
24. 55.0 in.$^2$

Mid-Course Test, Form A
1. 63; 127
2. 86
3. 12
4. 7
5. rectangle
6. 12
7. Sample: (2, 0), (8, 0), (10, 5), (4, 5)
8. $\angle B$, $\angle A$, $\angle C$
9. 36, 36
10a. If it is summer, then it is sunny.
10b. If it is not sunny, then it is not summer.
11. C
12. 26
13. 68 cm
14. 60
15. $\triangle CAB \cong \triangle BDC$ by SAS.
16. (0, b)
17a. $\angle 3$ and $\angle 5$ or $\angle 4$ and $\angle 6$
17b. $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, or $\angle 4$ and $\angle 8$
18. 90; 20
Answers (continued)

19. 

\[
\begin{array}{c}
\text{3} \\
\text{1} \\
\text{2}
\end{array}
\]

Front

Right

20. 115; 65 21. always 22. never 23. never 24. sometimes 25. \(x = 34; y = 24; z = 88\) 26. Sample: A rectangle always has opposite sides parallel, making it a parallelogram. A parallelogram doesn’t always have four right angles, so it is not always a rectangle.

27. \(\angle C, \angle A, \angle B\) 28. rectangle 29. rhombus 30. square 31. trapezoid 32. rectangle 33. rhombus 34. 17 35. 48; 90; 42; 57; 33 36. \(x = 25; y = 31\) 37. (2.5, 1) 38. J

39. 118; 34 40. 7.2 41. area = 452.4 in.\(^2\); circumference = 75.4 in. 42. 74 43. HL 44. not possible 45. SSS 46. AAS 47. not possible 48. SAS 49. ASA or AAS 50. SSS or SAS

Mid-Course Test, Form B

1. 22, 29 2. 120 3. 17 4. 12 5. parallelogram 6. 7 7. Sample: (0, 4), (–6, 0), (0, –4) and (6, 0) 8. \(\angle B, \angle C, \angle A\) 9. \(x = 120; y = 30\) If the ground is wet, then it is raining. 10a. If it is not raining, then the ground is not wet. 11. D 12. 12 13. 120 m 14. 36 15. \(\triangle WXY \cong \triangle WXZ\) by AAS 16. \((a – b, c)\) 17a. \(\angle 2\) and \(\angle 3\) or \(\angle 6\) and \(\angle 7\) 17b. \(\angle 1\) and \(\angle 3\), \(\angle 2\) and \(\angle 4\), \(\angle 5\) and \(\angle 7\), or \(\angle 6\) and \(\angle 8\) 17c. \(\angle 2\) and \(\angle 6\) or \(\angle 3\) and \(\angle 7\) 18. 34; 68

Mid-Course Test, Form E

1. –19, –23 2. 13 3. 12 4. \(\overline{RT}, \overline{RS}, \overline{TS}\) 5. B 6. 24 7. F 8. 60 9. \(C \approx 94.2\) in., \(A \approx 706.9\) in.\(^2\) 10. 13 11. \(m\angle 1 = 47, m\angle 2 = 43\) 12. AAS 13. ASA 14. SAS 15. not possible 16. HL 17. SSS 18. 50 19. D 20. J 21. \(m\angle 1 = 80, m\angle 2 = 80, m\angle 3 = 100, m\angle 4 = 80\) 22. square, rhombus 23. A 24. Hypothesis: a transversal intersects two parallel lines; conclusion: corresponding angles are congruent.

Mid-Course Test, Form D

1. –9, –11 2. 11 3. 16 4. \(\angle T, \angle R, \angle S\) 5. D 6. 14 7. G 8. 45 9. \(C \approx 100.5\) in., \(A \approx 804.2\) in.\(^2\) 10. 10 11. \(m\angle 1 = 38, m\angle 2 = 52\) 12. SAS 13. HL 14. not possible 15. SSS 16. AAS 17. ASA 18. 19 19. A
Answers (continued)

29.

30. 52  31. (2, 1)  32. 160 cm  33. J  34. 35. 47
36. 80  37. 67  38. MZ \equiv TR or WZ \equiv WR
39. m\angle 1 = 52, m\angle 2 = 30, m\angle 3 = 98
40. m\angle 1 = 59, m\angle 2 = 62

Final Test, Form A

1. 36  2. (2.5, 1)  3. B  4. G  5. (6, -5)  6. 128 cm²
7. 48 cm²  8. rotational and rotational symmetry  9. D
10. 11. 14.0 in.
12. 13. 120 cm²
14. 1296 cm²  15. two parallel, horizontal lines, one 3 units above and one 3 units below y = -2
16. 40 m²  17. 15.0  18. F  19. 6.5  20. 7.5  21. 54 cm²
22. 110 ft  23. 64 ft  24. (9, 1)  25. 7  26. (-2, -2)
27. \angle A, \angle C, \angle B  28. 17.9  29. never  30. sometimes
31. always  32. sometimes  33. never  34. 18  35. 34
36. 92.3 in²  37. 100  38. 25 : 4  39. 199.0 cm²  40. 9.7
41. 42. \frac{5}{7} = 55.6%  43a. 1809.6 cm²  43b. 7238.2 cm³  44a. If two angles have the same measure, then they are congruent.
44b. If two angles are not congruent, then they do not have the same measure.  45a. PQ  45b. \angle F  46. 13.0
47. AAS  48. rhombus  49. 113  50. y = -\frac{1}{3}x + 10

Final Test, Form B

1. 45  2. (9, 2)  3. C  4. J  5. (-1, -1)  6. 216 \sqrt{3} cm²
7. 96 in²  8. rotational and reflectional symmetry  9. B
10. surface area: 84 cm²
11. 13.6 ft
12. 13. 47.0 cm²
14. 1280 m³  15. The locus would consist of a congruent segment 5 cm above and another 5 cm below the given segment. The endpoints of these two segments would be connected with semicircles of radius 5 cm.
16. 144\sqrt{3} in³  17. 8.9  18. H  19. 6.4  20. 10.5
21. 175 cm²  22. 63 m  23. 44 m
24. 53.2 mi/h; 41.2° west of north  25. 11  26. (-1, -6)
27. \angle C, \angle B, \angle A  28. 10.8  29. always  30. sometimes
31. never  32. sometimes  33. sometimes  34. 5.5  35. 32.5
36. 24.6 cm²  37. 80  38. 81 : 25  39. 182.8 cm²  40. 6
41.
Answers (continued)

42. $\frac{1}{3} = 25\%$  
43a. $282.7 \text{ m}^2$  
43b. $314.2 \text{ m}^3$

44a. If two angles are congruent, then they are vertical.  
44b. If two angles are not vertical, then they are not congruent.

45a. $AE$  
45b. $/H11028$

46. 17.3

47. HL

48. square

49. 125

50. $y = -\frac{1}{2}x - 2$

Final Test, Form D

1. $x = 19, y = 135$  
2. D  
3. Hypothesis: if two sides of a triangle are congruent, then the angles opposite those sides are also congruent. Conclusion: the angles opposite those sides are also congruent.

4. H  
5. A  
6. 62  
7. 74  
8. 91  
9. G  
10. B  
11. C  
12. A  
13. D  
14. $A \approx 38.1 \text{ in.}^2, C \approx 69.1 \text{ in.}$  
15. $(-3, -4)$  
16. $y = -x + 1$  
17. ASA Theorem  
18. ASA Postulate  
19. not possible  
20. SAS Postulate  
21. HL Theorem  
22. $x = 8, y = 4\sqrt{3}$  
23. F  
24. $200\pi \text{ cm}^3$  
25. 40  
26. B  
27. 90  
28. 100 m$^2$  
29. 11 ft  
30. 17  
31. H  
32. 28  
33. 71  
34. 180  
35. acute  
36. center: $(-4, 3)$; radius: 7  
37. $AC \approx 41.3, CD \approx 33.4$  
38. 288 square units  
39. 8  
40. 43.30 cm$^2$

Final Test, Form E

1. $x = 18, y = 147$  
2. D  
3. Hypothesis: if a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment. Conclusion: it is equidistant from the endpoints of the segment.

4. J  
5. A  
6. 150  
7. 78  
8. 76  
9. G  
10. A  
11. D  
12. C  
13. B  
14. $A = 907.9 \text{ m}^2, C = 106.8 \text{ m}$  
15. $(-6, 2)$  
16. $y = -x - 8$  
17. SAS Postulate  
18. not possible  
19. SSS Postulate  
20. HL Theorem  
21. AAS Theorem  
22. $x = 12, y = 6\sqrt{3}$  
23. F  
24. $540\pi \text{ ft}^3$  
25. 8  
26. D  
27. 180  
28. 128 cm$^2$  
29. 14 ft  
30. 25  
31. G  
32. 66  
33. 68  
34. 174  
35. obtuse  
36. center: $(-6, 5)$; radius: 8  
37. $MW \approx 43.5, MX \approx 34.3$  
38. 80  
39. 15  
40. 173.21 in.$^2$
Test-Taking Strategies

Writing Gridded Responses
1. 6 2. 5 3. 11.4 4. 326.7 5. 615.75 6. 2.5 7. 12 8. 51 9. 87 10. 690 11. 552.9 12. 13.30

Writing Short Responses
1a. 2 points; all parts are solved, work is shown, and answers are correct. 1b. 0 points; no work shown and answer is incorrect. 1c. 1 point; error in procedure, but problem is set up correctly. Incorrect answer. 2a. 0 points; incorrect and incomplete response. 2b. 2 points; the measure of each angle is found, all work is shown, and a diagram is complete and correct. 2c. 1 point; correct answer, but did not complete a drawing. 3a. 0 points; incorrect and incomplete response. 3b. 1 point; work is shown, however it contains a math error.

Writing Extended Responses
1. No steps are shown and no justification is made about why \( m \angle 1 = m \angle 2 \).
   \[
   \begin{array}{c}
   A \quad 4x - 30 \quad B \\
   D \quad 2x \\
   C \\
   F
   \end{array}
   \]

   Answers may vary: Sample: \( BD \) bisects \( \angle ABC \), therefore \( m \angle CBD = m \angle ABD \).
   \[2x = 4x - 30\]
   \[x = 15\]
   \[m \angle CBD = 30\]
   \[m \angle BDC = 90\]

   The measure of an exterior angle is the sum of the two remote interior angles therefore the
   \[
   m \angle BCF = m \angle CBD + m \angle BDC
   \]
   \[= 30 + 90\]
   \[= 120\].

3. Answers may vary. Sample: Find the slope of the line.
   \[m = \frac{1 - 3}{3 - (-3)} = -\frac{2}{6} = -\frac{1}{3}\]

   Parallel lines have the same slope.
   \[y + 1 = -\frac{1}{3}(x + 1)\]
   Substitute \( m = -\frac{1}{3} \) and \((-1, -1)\) in the point-slope formula.
   \[y + 1 = -\frac{1}{3}x - \frac{4}{3}\]
   Solve for \( y \) to write the equation in slope-intercept form.

Interpreting Data

Using a Variable
1. 108, 78, and 94 2. 40, 60, and 80 3. (6.5, 0) 4. \((-\frac{7}{19}, 0)\)
5. 8 or 12 6. 40 7. 60 8. 22\frac{1}{2} 9. 4 in. 10. \frac{1}{2}
11. 3 12. 7 13. 4 14. 108°

Drawing a Diagram
1. rhombus 2. \( BE = ED = 8 \) and \( AE = EC = 24 \)
3. Two side are 22 and two sides are 14 4. \( Y(-8, 2); Z(2, 6) \)
5. 35, 35 6. 79, 79 7. (0, 1), (3, 0) 8. (5, 5) 9. (-6, -6), \( m = 1 \)
10. 12 + 6\sqrt{2} or 20.49 11. \( x = 1 \); each diagonal is 8 12. 120° 13. Two sides are 12 and two sides are 32

Finding Multiple Correct Answers

Testing Multiple Choices

Eliminating Answers
1a. Answers may vary. Sample: \( D \) can be eliminated because \( 14^2 = 196 \) and the length of the side will be less than 14. \( A \) can be eliminated because the length of the side will be longer than 7. 1b. C 2a. The angle is just a bit larger than a 30° angle, so using the special right triangle relationships you can eliminate \( F \). The hypotenuse is the longest side so you can eliminate \( J \). 2b. G 3a. The angle is just a bit smaller than a 30° angle, so using the special right triangle relationships you can eliminate \( A \) and \( D \). 3b. B 4a. \( H \) and \( J \) are obtuse angles. 4b. G 5a. \( A \) and \( B \) are too small since the area will be greater than \( 7 \cdot 7 - 48 \).

5b. C 6a. The first coordinate has to be \(-9\) so choices \( G \) and \( J \) can be eliminated. 6b. F

Choosing “Cannot Be Determined”

Using Estimation

Answering the Question Asked
1. the sum of the \( x \)- and the \( y \)-coordinates of the reflected point; \( D \) 2. the \( y \)-coordinate of the reflected point; \( J \) 3. the distance of the translated point to the \( x \)-axis; \( B \) 4. the vector of the translation; \( G \) 5. the sum of the \( x \)- and \( y \)-coordinates of the rotated point; \( D \) 6. the difference between the \( y \)-coordinate and the \( x \)-coordinate of the image after translation; \( H \) 7. The length \( AB' \); \( B \) 8. lines of symmetry in the figure; \( H \) 9. a false property of a reg. hexagon; \( C \) 10. image of \((8, -5)\); \( G \) 11. the equation of the line after reflection; \( B \) 12. the image after the dilation; \( H \)
### Answers: SAT/ACT Practice Test

#### Multiple Choice

1. **A** **B** **C** **D** **E**
2. **A** **B** **C** **D** **E**
3. **A** **B** **C** **D** **E**
4. **A** **B** **C** **D** **E**
5. **A** **B** **C** **D** **E**
6. **A** **B** **C** **D** **E**
7. **A** **B** **C** **D** **E**
8. **A** **B** **C** **D** **E**
9. **A** **B** **C** **D** **E**
10. **A** **B** **C** **D** **E**
11. **A** **B** **C** **D** **E**
12. **A** **B** **C** **D** **E**
13. **A** **B** **C** **D** **E**
14. **A** **B** **C** **D** **E**
15. **A** **B** **C** **D** **E**
16. **A** **B** **C** **D** **E**
17. **A** **B** **C** **D** **E**
18. **A** **B** **C** **D** **E**
19. **A** **B** **C** **D** **E**
20. **A** **B** **C** **D** **E**
21. **A** **B** **C** **D** **E**
22. **A** **B** **C** **D** **E**
23. **A** **B** **C** **D** **E**
24. **A** **B** **C** **D** **E**
25. **A** **B** **C** **D** **E**
26. **A** **B** **C** **D** **E**
27. **A** **B** **C** **D** **E**
28. **A** **B** **C** **D** **E**
29. **A** **B** **C** **D** **E**
30. **A** **B** **C** **D** **E**
31. **A** **B** **C** **D** **E**
32. **A** **B** **C** **D** **E**

#### Student-Produced Responses

1. \[2/5\]
2. \[5\]
3. \[7\]
4. \[108\]
5. \[3\]
6. \[720\]
7. \[20\]
8. \[78\]
9. \[64\]
10. \[6\]
Answer Sheet

1. A B C D
2. F G H J
3. A B C D
4. F G H J
5. A B C D
6. F G H J
7. A B C D
8. F G H J
9. A B C D
10. F G H J
11. A B C D
12. F G H J
13. A B C D
14. F G H J
15. A B C D
16. F G H J
17. A B C D
18. F G H J
19. A B C D
20. F G H J
21. A B C D
22. F G H J
23. A B C D
24. F G H J
25. A B C D
26. F G H J
27. A B C D
28. F G H J
29. A B C D
30. F G H J
31. A B C D
32. F G H J
33. A B C D
34. F G H J
35. A B C D
36. F G H J
37. A B C D
38. F G H J
39. A B C D
40. F G H J
41. A B C D
42. F G H J
43. A B C D
44. F G H J
45. A B C D
46. F G H J
47. A B C D
48. F G H J
49. A B C D
50. F G H J
51. A B C D
52. F G H J
Synchronized Strut

1. Lines have different positive slopes and have the same y-intercept.
2. parallel with different y-intercepts
3. Same line

4–5. Check students’ work.
6. y-intercept; represents initial distance from the motion detector

7–9. Check students’ work.
10. Speed each student walks
11. They walk at different speeds.
12. Check students’ work.
13. the student whose graph has a steeper slope

14–15. Check students’ work.
16. The slopes should be about the same because they walked at the same speed.
17. They have the same slope.
18. \( m \)
19. The difference is constant because they are walking at the same speed.
20. They are the same line.

Coming and Going

1. No, a motion detector is like a function machine because there is only one distance for any given time.
2. Students should sketch a line of positive slope with a y-intercept near zero. They should also sketch a line of negative slope with a y-intercept near the top of the screen. The negative slope indicates a person walking toward the detector. The positive slope indicates a person walking away from the detector. The y-intercepts show how far from the detectors each person started.
3. Not necessarily. It can be approximated from the data, however.

4–7. Check students’ work.

8. Check students’ work. If an equation models the data well, the line should overlap the plot. If an equation does not model the data well, the line may either have a different slope or an incorrect y-intercept.

9–10. Check students’ work.

11. The measurements should agree closely with the calculations. Discrepancies might be caused by delays due to the reaction times of the marker and timer.
12. A motion detector records only the distance to the closest object in front of it. A motion detector cannot record two objects at once. A function has only one y-value for each x-value.
13. Check students’ work.

Keep Your Eye on the Ball

1. The student should sketch a series of parabolas opening downward with maximum values that gradually decrease.
2. decrease
3. Yes, each rebound is shorter than the previous bounce.
4–5. Check students’ work.
Answers (continued)

Race Cars Activity 6

1. Yes; the battery-powered car gives a linear graph because it travels at a constant speed. The speed of the wind-up car changes, so its graph will show a curve as it speeds up and a curve as it slows down.
2. Students should sketch a line for the battery-powered car and a curve showing the acceleration and deceleration of the wind-up car.
3. The battery-powered car can be modeled by a line. The wind-up car can be modeled by two parabolas, one for acceleration and one for deceleration.
4. Check students’ work; a line
5. Check students’ work; the slope is the speed and the y–intercept is the initial distance from the motion detector.
6. Check students’ work. The equations may not model the data after the car initially starts driving because there is some acceleration.
7. Check students’ work.
8. The values should be very close. The slope might be higher than the average speed from the graph because the slope ignores the brief acceleration of the car.
9. 0 ≤ x ≤ 1.5; students may want the domain to extend beyond x = 1.5 because the car continues to drive.
   Any domain that they can justify is reasonable, but it should not be very long, since the car may stop.
10. The collected data would span a longer period of time.
    Everything else should remain constant.
11. Check students’ work. Should be approximately the same as the previous model.
12. Check students’ work; yes.
13. 0 ≤ x ≤ 6
14. Check students’ work.
15. The data will appear linear but you would expect some type of curve. You will see a line because you are recording data for a very short time. But the spring-loaded car must be either speeding up or slowing down since it does not go at a constant speed as does the battery-powered car. Thus you would expect to see a curve.
16. A longer time interval will give a better look at the car’s motion. You will probably see s-shaped data. You will see a parabola opening upward for when the car starts and is speeding up and a parabola opening downward for when it is slowing at the end.

Back and Forth It Goes Activity 7

1. Sketch should resemble a cosine curve with negative amplitude.
2. decrease; increase
3. Check students’ work.
4. Students should label the minimums and maximums of the graph. The motion detector records points at regular intervals. It probably does not record the actual minimum and maximum but records points near the maximum and minimum.
5–9. Check students’ work.
10. Check students’ work; yes; non-linear
11. \( y = \frac{x}{2\pi}^2 \); check students’ work; it should be a good model.
12. 1.74; the two methods should give close answers but there will be some experimental discrepancy.
13. \( l = 0.64 \); the answer is reasonable, although it does not agree exactly with the measurement from the activity.
14. \( t = 2\pi\sqrt{\frac{l}{g}} \); independent; as you change the length of the pendulum, the period changes.
15. Check students’ work. The model gives \( T = 2.84 \) s.
    The model and the experiment should give similar answers.
16. You should shorten it.

Charge It! Activity 8

1. \( f(x); g(x) \)
2. \( a \) is the starting amount or the \( y \)–intercept. \( b \) is the base, the growth factor, or the decay factor.
3. \( b > 1; 0 < b < 1 \)
4. 1; 2; \( y = 2^x \)
5. 30; 0.75; \( y = 30(0.75)^x \)
6–7. Check students’ work.
8. Check students’ work; it is the voltage of the battery.
9. The two answers should be close. The \( y \)–intercept may be lower because the capacitor may still have been collecting charge.
10. They are the same.
11. exponential decay
12–15. Check students’ work.
16. They should be very close.
17. No, exponential functions never reach their asymptotes. No, the voltage of the capacitor will eventually reach zero.
18. 0.9782; they should be close.

Falling Objects Activity 9

1. Students should graph a horizontal line with a positive \( y \)–intercept that changes to the right side of a downward-opening parabola. There should be a horizontal line once the parabola reaches the \( x \)–axis.
2. quadratic
3. Check students' work. First the book was held over the motion detector. Next it fell. Finally it lies on top of the motion detector.
4–11. Check students' work.
12. The vertex represents the time at which the student dropped the book and the book's initial height above the motion detector.
13. Check students' work. Only the part of the parabola that corresponds to the actual time the book was falling should be included in the domain. At the other times, the book was actually stationary.
14. Check students' work.
15. The model should not agree exactly. The physics model assumes that there is no air resistance. The book is affected by air resistance. The physics model assumes that the book starts to fall when \( t = 0 \). In the activity, the book was held briefly so that it started to fall after \( t = 0 \).

Bouncing Ball, Part 1 Activity 10

1. The student should sketch a series of parabolas that open upward that all stop about 5 units above the \( x \)-axis.
2. The student should sketch a series of parabolas opening downward with maximum values that gradually decrease. Students who understand that the motion detector measures distance from itself should understand that the final result will be a graph of the distance from the floor instead of the distance from the motion detector.
3. The ball will bounce to a maximum height that is a percent of the previous height. The percent should be constant.
4–5. Check students' work.
6. One method is to substitute the coordinates of the vertex for \( h \) and \( k \); next substitute the coordinates of a point on the parabola for \( x \) and \( y \); then solve for \( a \). Another method is to find three points on the parabola and substitute their coordinates into the equation \( y = ax^2 + bx + c \); then solve for \( a \), \( b \), and \( c \).
7. Check students' work. The curve is too wide and opens up instead of down.
8. A parabola with a negative \( a \) opens down. A parabola with a positive \( a \) opens up; negative.
9. The curve is narrower. A larger \( a \) means a narrower curve. A smaller \( a \) means a wider curve.
10. Check students' work, but \( a \) should be near \(-16\).
11. No; if students use an actual data point, their models may be off in height or not centered on the bounce horizontally.
12. Check students' work. They should be similar.
13. \( a \) is the acceleration due to gravity, \( h \) is the time when the ball is at the maximum height, \( k \) is the maximum height of the ball for the bounce. \( a \) should remain constant because gravity is constant.

Bouncing Ball, Part 2 Activity 11

1. The student should sketch a series of parabolas opening downward with maximum values that gradually decrease.
2. Eventually comes to a stop
3. decrease
4. remains constant
5–8. Check students' work.
9. exponential or possibly quadratic
10. A quadratic equation does not have an asymptote. After an infinite amount of time, a quadratic equation predicts that the ball is at 0 ft, or resting on the floor.
11–13. Check students' work.
14. Check students' work. A well-fitting model should approximately touch the vertex of each bounce.
15–16. Check students' work.
17. coefficient of restitution

When's the Tea Ready? Activity 12

1. 100°C; room temperature. A liquid cannot cool below room temperature.
2. Students should draw an exponential function with an asymptote at room temperature.
3. The graphs will be similar, but the probe should cool more quickly because it has less mass.
4–5. Check students' work.
6. exponential
7. the \( x \)-axis
8. Check students' work; no.
9. By adding a constant \( c \), the graph of the exponential equation is translated vertically and the asymptote is no longer the \( x \)-axis.
10. \( c \) should be given the value found in Exercise 8; it is the temperature of the classroom.
11–13. Check students' work.
14. Check students' work. The model should fit well because it overlaps most of the data points. The model may not fit the first few points.
15. Check students' work.
16. No, \( a \) is the difference between the initial value and the room temperature, \( c \).
17. Check students' work. This model is a vertical translation of the previous model. The translation is necessary because the room temperature is greater than zero.
18. yes; yes
19. yes; yes
20–21. Check students' work.
22. \( a \) is the difference between room temperature and the initial temperature of the water. \( b \) is the rate at which the water cools; it does not change. \( c \) is room temperature; it does not change. Check students' work.
Full Speed Ahead  Activity 13

1. Students’ should sketch a line.
2. Students should sketch a curve opening upward.
3. Students should sketch a curve opening downward.
4. Check students’ work; yes.
5. Check students’ work; the slope is the speed and the $y$-intercept is the initial distance from the motion detector.
6. The model should fit reasonably well, the residual plot should show a definite parabolic pattern.
7. quadratic because the speed is increasing
8. Check students’ work.
9. The model should fit better. The residuals should confirm the quality of the model.
10. The speed is increasing because as $x$ increases the change in $y$ divided by the change in $x$ is getting larger. This means that the car is traveling farther during each $x$-unit.
11. The sketch should show a parabola opening upward representing an increase in speed joined to a parabola opening downward. The data shows the car speeding up at the start and then slowing down.
12. The model is reasonable only over the domain $0 \leq x \leq 1.5$. The time interval was too short to see the car slow down.
13. Check students’ work. Students should label when the car is accelerating and decelerating.
14–15. Check students’ work. Models should fit well.
16. The magnitude of $a$ is determined by the angle at which the string is pulled back and by the length of the string. $b$ decreases with the length of the string. $c$ increases when the motion detector is started after the pendulum is swinging. $d$ increases when the string is pulled back farther and when the string is longer.

Playing With Numbers  Activity 15

Investigate

1. See screen
2. The digits are the same but in a different order.
3. The digits are the same but in a different order.

Exercises 1–3

4. Yes
5. Same digits with zero included

Exercise 4

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</tr>
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<td>493827156</td>
<td>246913570</td>
</tr>
</tbody>
</table>

Exercise 5

6. Multiples of 3 have been omitted from the list.
7. 22A, 23A, 25A, 26A
8. The same patterns occur.

In the Swing of Things  Activity 14

1. The graph should be a periodic function with a maximum or minimum at the $y$-axis.
2. The length of the string
3. cosine or sine
4. $a = \text{amplitude}; b = \frac{2\pi}{\text{period}}$ divided by the period; $c = \text{horizontal translation}; d = \text{vertical translation}$
5–7. Check students’ work.
8. Take half the difference between the $y$-value of a peak and the $y$-value of a trough. Check students’ work.
9. Take the difference between the $x$-values of two peaks or two trough. Check students’ work.
10. Find the $x$-value of the first peak. Check students’ work.
11. Take the average of the $y$-values of one peak and one trough. Check students’ work.
12–13. Check students’ work.
14. The amplitude is greater, because the pendulum is pulled back farther. The graph has a greater vertical translation because the motion detector sill needs to be 1.5 m away from the closest swing of the pendulum.
15. The period decreases because the string is longer. The amplitude and vertical translation increase because a longer string means that the pendulum starts farther away from the center of its swing.
6. Using paper and pencil, it can be seen that there is a column of 10 one’s, whose sum is 10. This results in a “carry-over” which causes the pattern to break down.

Investigate
1. Answers may vary. Sample: 82471
2. Answers may vary. Sample: 27814
3. Answers may vary. Sample: 82471 − 27814 = 54657
4. Answers may vary. Sample: 
   \[ \begin{align*}
   &4 + 5 + 6 + 5 + 7 = 27; \\
   &72 - 27 = 45; \\
   &4 + 5 = 9
   \end{align*} \]
5. A sum of 9 always results.
6. Variables can take on any numerical value, therefore proving a result for any number.

Exercises
1. The result contains five copies of the two-digit number (i.e. 48 · 101010101 = 4848484848).
2. The number is repeated as a decimal (i.e. \( \frac{9}{9} = 1 \) and \( \frac{99}{99} = 1 \) as well.
3. The (\( x, y \)) coordinates of the graphs are real numbers, so graphical methods can only reveal real answers.

Solving Absolute Value Inequalities Graphically

Investigate
1–3. Check student’s work.

Exercises
1. They are approximations.
2. \( \frac{36}{99} = \frac{36}{99} \)
3. The (\( x, y \)) coordinates of the graphs are real numbers, so graphical methods can only reveal real answers.

Magic Pricing Numbers

Investigate
1. or 24
2. 1.16
3. .9
4. 1.028
5. \( \frac{1}{24} \cdot 1.16 \cdot .9 \cdot 1.028 = .044718 \)
6. \( y = .044718x \)
7. Yes
8. Yes
9. $28.81

Exercises
1. Multiply 857.98 by \( \frac{1}{36} \):
2. 2. Multiply 857.98 by \( \frac{1}{36} \):
3. 3. \( w = (.044718)^{-1} = 22.36326x \), where \( w \) = wholesale price and \( x \) = retail price

Mrs. Murphy’s Algebra Test Scores

Investigate
1. first line: 30 units; second line: 20 units
2. \( x = 42 \)
3. \( x = 42 \)
4. \( y = 78; \frac{y}{20} = \frac{78}{20} \)
5. \( \frac{x}{30} = \frac{y}{20} \)
6. The graph is a straight line.

Exercises
1. 60
Exploring Point-Slope Form  Activity 20

Investigate
1. \( y - 1 = 3(x - 2) \)
2-4. Check student’s work.
5. \( L3 = \{-2, -1, -0.5, 0, 0.5, 1, 2\} \)

Exercises
1.  
2.  
3.  
4.  
5. Use \( L3 = \{-2, -0.5, 0, 0.5, 2, 200\} \) and 
\( Y1 = L3 \times (X - 12) + 12 \)

Fast Food Follies  Activity 21

Investigate
1. \( 3x + 4y + 4z = 16.45 \)
\( 5x + 2y + 5z = 19.30 \)
\( 4x + 6y + 6z = 23.50 \)
2. The product of matrix \( A \) and matrix \( X \) will produce matrix \( B \).
3. If \( AX = B \), then \( X = A^{-1}B \).
4-6. Check student’s work.

Exercises
1. 
2. 
3. 
4. 

Burgers cost $2.95; Fries cost $1.80; Soft Drinks cost $1.10
5. If it costs $25.04 for 4 burgers, 4 fries, and 4 soft drinks, then it should cost \( \frac{25.04}{4} = 6.25 \) for 2 burgers, 2 fries, and 2 soft drinks. The screen at right shows the error message that results when \( [A]^{-1}[C] \) is executed.

Wheat on a Chessboard  Activity 22

Investigate
1. | Square | Num. of Grains |
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2. | Num. of Squares | Total Num. of Grains |
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<td>511</td>
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<td>1023</td>
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</table>
3. Raise 2 to the number of squares and subtract 1.
4. \( T(n) = 2^n - 1 \) where \( n \) = number of squares and 
   \( T(n) \) = total number of grains
   about \( 1.8 \times 10^{19} \) grains of wheat
5. about \( 1.7 \times 10^{13} \) or 17 trillion bushels
6. 6728.3 years

**Exercises**
1. 1,075,000
2. 21
3. 20
4. 255
5. The wheat covering 51 squares provides a close match 
to the 1998 U.S. wheat production.
6. about 2.6 inches

**Manipulating a Polynomial Activity 23**

**Investigate**
1. Check student’s work.
2. It moves up.
3. Evaluate \( f(0) = 12 \) to find the \( y \)-intercept, \((0, 12)\), and \( 12 > 10 \).
4. The \( y \)-values range from \(-20\) to \(20\) and the scale is 1, 
   which results in 40 tick marks on the \( y \)-axis.
5. Check student’s work.

**Exercises**
1. Answers may vary. Sample: \( f(x) = (x + 3)(x - 1)(x - 3)(x - 5) \); graph shown with \( Xmin = -4.7, Xmax = 4.7, Xscl = 1 \), \( Ymin = -120, Ymax = 20, Yscl = 0 \)

2. No portion of the graph can be seen in the ZDecimal window.

**Writing a Simple Program Activity 24**

**Investigate**
1–8. Check student’s work.

**Exercises**
1. Answers may vary. Sample: \( A = 1, B = 1, C = -20 \)
2. Answers may vary. Sample: \( A = 4, B = -12, C = 9 \)
3. Answers may vary. Sample: \( A = 2, B = -5, C = 7 \)
4. Answers may vary. Sample: \( A = 6, B = 7, C = -5 \)
5. Answers may vary. Sample: \( A = 3, B = 8, C = -7 \)

6. a. \$625.72
   b. \$2236.58

**Repeated Radicals Activity 25**

**Exercises**
1. 9
2. \( x = 6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots}}}} \)
   \( (x - 6)^2 = \left( \sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots}}} \right)^2 \)
   \( x^2 - 12x + 36 = 6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots}} \)
   \( x^2 - 12x + 36 = x \)
   \( x^2 - 13x + 36 = 0 \)
   \( (x - 4)(x - 9) = 0 \)
   \( x = 4 \) and \( x = 9 \)
   The solution \( x = 4 \) is extraneous, since \( x \) is clearly 
greater than 6.
3. 5.302775638; the number is an approximation
4. \( 7 + \sqrt{13} \), irrational
5. The calculator can only give an approximate answer, whereas algebraic techniques give an exact answer.
Asymptotes and Holes  Activity 26

Investigate

1. The function is undefined at \( x = \pm 2 \). This makes sense algebraically since the denominator is equal to zero at these two values of \( x \).

2. It disappears.

3. The function approaches \( \pm \infty \) on either side of the singularity. This makes sense because the denominator approaches zero as \( x \) approaches the singularity.

4. Vertical lines appear at the singularities.

5. The \( y \)-value ‘jumps’ from negative to positive (or vice versa) as you trace through a singularity.

6. The calculator was drawing a line between two points on either side of the singularity.

7. Yes; the singularity at \( x = -2 \) appears as a hole in the graph.

8. The function is undefined when the denominator is equal to zero.

9. As you approach the singularity at \( x = -2 \) from the left or right, the corresponding \( y \)-values approach the same number. As you approach the singularity at \( x = 2 \) from the left, the \( y \)-values approach \( -\infty \). As you approach the singularity at \( x = 2 \) from the right, the \( y \)-values approach \( +\infty \).

10. No; the simplified function is defined at \( x = -2 \) and the hole disappears.

11. When graphing \( Y_1 \), the graphing ball ‘breaks’ when it reaches \( x = -2 \). This does not happen when you graph \( Y_2 \).

Exercises

1. The first graph has a hole at \( x = 1 \) whereas the second graph does not. The domain of \( y = \dfrac{x^2 - 1}{x - 1} \) is all real numbers except 1 and the domain of \( y = x + 1 \) is all real numbers.

2. Answers may vary. Sample: \( f(x) = \dfrac{2}{(x + 1)(x - 1)} \)

3. Answers may vary. Sample: \( f(x) = \dfrac{x^3 - x^2}{x - 1} \) or \( f(x) = \dfrac{x^2(x - 1)}{x - 1} \)

4. Check student’s work.

5. Type ‘.05 \rightarrow \triangle X’ on the home screen to eliminate false asymptotes.

Stepping Out  Activity 27

Investigate

1. The graph resembles a series of steps.

2. The range values are \([1, 2, 4, 3, 4]\). Answers may vary. Sample: ‘Int’ rounds numbers down to the nearest integer.

3. The range values are \([-2, -2, -5, -4, -5]\). Answers may vary. Sample: ‘Int’ rounds numbers to the nearest integer that is less than or equal to the number.

4. The calculator will connect the endpoints of each step to the next step.

5. \( f(x) = \dfrac{2}{(x + 1)(x - 1)} \)

6. Answers may vary. Sample: \( Y_1 = 3.5 \rightarrow 2.5\text{int}(-x) \)

7. There can only be one matrix \([A]^{-1}[B]\), so the calculator could not produce multiple correct answers in this way.

Exercises

1. \( C(x) = .34 + .23\text{int}(x + 1) \)

2. The two functions are identical except at the integer values. A careful graph of \( y = \text{int}(x + 1) \) would show a closed dot on the left of each step and an open dot on the right, while a careful graph of \( y = -\text{int}(-x) \) would show an open dot on the left of each step and a closed dot on the right.

3. Answers may vary. Sample: \( Y_1 = 3.5 \rightarrow 2.5\text{int}(-x) \)

4. Yes; the prices given in the previous question solve the system.

5. \$6.85; \$11.65; \$9.25

6. \$6.85; \$11.65; \$9.25

7. Yes; the prices given in the previous question solve the system.

Solutions

1. \( C(x) = .34 + .23\text{int}(x + 1) \)

2. The two functions are identical except at the integer values. A careful graph of \( y = \text{int}(x + 1) \) would show a closed dot on the left of each step and an open dot on the right, while a careful graph of \( y = -\text{int}(-x) \) would show an open dot on the left of each step and a closed dot on the right.

3. Answers may vary. Sample: \( Y_1 = 3.5 \rightarrow 2.5\text{int}(-x) \)

4. Yes; the prices given in the previous question solve the system.

5. \$6.85; \$11.65; \$9.25

6. \$6.85; \$11.65; \$9.25

7. Yes; the prices given in the previous question solve the system.

8. \( C(x) = 3.50 + 2.50\text{int}(x + 1) \)

9. Yes; the prices given in the previous question solve the system.

10. Answers may vary. Sample: \( Y_1 = 3.5 \rightarrow 2.5\text{int}(-x) \)

11. Answers may vary. Sample: \( Y_1 = 3.5 \rightarrow 2.5\text{int}(-x) \)
Exercises

1. Check students’ work.
2. Any values of \( x, y, \) and \( z \) satisfy the equation.
3. \( 3 \left( y - \frac{1}{3} \right) = 3 \left( \frac{41}{60} \right) \rightarrow 3y - z = \frac{41}{20} \rightarrow 3y - z = 2.05 \)
4. \( x + z = \frac{12}{5} \)
5. \( x = 2.2; z = 0.2 \)
6. \( 2(2.2) + 3(0.75) + 2 = 6.85 \rightarrow 6.85 = 6.85 \)
   \( 4(2.2) + 3(0.75) + 3(2) = 11.65 \rightarrow 11.65 = 11.65 \)
   \( 3(2.2) + 3(0.75) + 2(2) = 9.25 \rightarrow 9.25 = 9.25 \)

Extend

7. The cost of a burger is too low; some of the numbers are not rounded to the nearest penny.
8. Answers may vary. Sample: $1.30, $1.05, $1.10; $1.60, $0.95, $0.80; $1.90, $0.85, $0.50

Parabolic Reflectors

Investigate

1–4. Check students’ work.
5. They are equal.
6. Check students’ work.

Exercises

1. \((0, 4)\)
2. closer
3. By placing the lamp at the focus of a parabolic reflector, all rays that strike the reflector are directed outward parallel to the axis. This helps ‘concentrate’ the light in a specific direction.

Exploring Powers of \(-1\)

Investigate

1. Check students’ work.
2. \((-1)^{\frac{1}{2}} = -1; (-1^{2})^{\frac{1}{2}} = 1\); The calculator evaluates \((-1)^{\frac{1}{2}}\) as \((-1)^{\frac{1}{2}} = -1\), whereas the law of exponents would seem to imply that \((-1)^{\frac{1}{2}} = (1^{2})^{\frac{1}{2}} = 1^{\frac{1}{2}} = 1\). The answer depends on the fraction being reduced to lowest terms before the laws of exponents are applied.
3. It reduces \(\frac{2}{3}\) to \(\frac{1}{3}\), which is the same as taking the cube root.
4. Values of \(x\) are considered as fractions in lowest terms before being used as exponents. For example, \(f(\frac{1}{2})\) is evaluated as \(f\left(\frac{2}{4}\right) = (-1)^{\frac{1}{2}} = 1\). \(f(\frac{1}{2})\) is evaluated as \(f\left(\frac{2}{5}\right) = (-1)^{\frac{1}{2}}\) which is undefined and \(f(\frac{2}{6})\) is evaluated as \(f\left(\frac{3}{5}\right) = (-1)^{\frac{1}{2}} = -1\).
5. Using this window, the calculator attempts (but fails) to evaluate \(y = (-1)^{x}\) for values of \(x\) that are irrational. This explains why no graph results.
6. The calculator only evaluates \(y = (-1)^{x}\) for values of \(x\) that only have real answers (such as \(x = \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \ldots\)). These values cause \(y\) to alternate between \(-1\) and \(1\).
7. The calculator only evaluates \(y = (-1)^{x}\) for values of \(x\) whose fractions reduce such that the numerators are always even numbers (such as \(x = \frac{2}{3}, \frac{4}{5}, \frac{6}{7}, \ldots\)). These \(x\)-values always produce \(y\)-values of \(1\).
8. The calculator only evaluates \(y = (-1)^{x}\) for values of \(x\) whose fractions reduce such that the numerators are always odd numbers (such as \(x = \frac{1}{3}, \frac{3}{5}, \frac{5}{7}, \ldots\)). These \(x\)-values always produce \(y\)-values of \(-1\).

Exercises

1. \(y = (-2)^{x}; y = (-2)^{x}; y = 2(-1)^{x}\)
2. \(\pi\) is irrational—it cannot be written as the quotient of two integers; use a rational approximation for \(\pi\) such as \(3.14159\)

Bull’s-eyes

Investigate

1. Check students’ work.
2. Check students’ work.
3. No graph results.
4. The second and third one
5. Type the function shown below
Answers (continued)

Exercises

1. \( D: -2 \leq x \leq 2 \)  
   \( R: 0 \leq y \leq 2 \)

2. \( D: -4 \leq x \leq 4 \)  
   \( R: 0 \leq y \leq 4 \)

3. Check students’ work.

The Natural Logarithm

Investigate

1. Check students’ work.
2. 1
3. Check students’ work.
4. Upper limit = \( e \)

Exercises

1. \( .6931471806; \ln(2) = .6931471806; \) the two numbers are equal
2. they are equal
3. they are equal
4. it is negative
5. For any \( a > 0, \ln(a) = \) the area under the graph of \( y = \frac{1}{x} \) between \( x = 1 \) and \( x = a \), with “area” being negative for \( 0 < a < 1 \).

Can a Graph Cross Its Own Asymptote?

Investigate

1. The \( x \)–axis
2. no
3. yes; \( r = \frac{x}{x^2 + 1} \)
4. twice
5. three times
6. yes

Exercises

1. Answers may vary. Sample: \( y = \frac{(x^2 - 1)(x^2 - 4)}{x^6 + 1} \)
2. The graph crosses its own asymptote an infinite number of times.
3. Yes, because the function gets arbitrarily close to \( y = 2 \) as \( x \) approaches \( \pm \infty \). The fact that the function actually equals 2 is technically not an issue.
4. Answers may vary. Sample: \( x^2 \) if \( x \leq 0 \)

Making Ellipses out of Circles

Activity 34

Investigate

1–3. Check students’ work.
4. It looks identical to the graph shown in the introduction.
5. A horizontal ellipse with major axis of length 6 and minor axis of length 4 is a circle of radius 1 that is stretched horizontally by a factor of 3 and vertically by a factor of 2. These scale factors can be produced by dividing the \( x \)–axis window values by 3 and the \( y \)–axis window values by 2.

Exercises

1. Divide \( X_{\text{min}}, X_{\text{max}}, \) and \( X_{\text{scl}} \) by 2; divide \( Y_{\text{min}}, Y_{\text{max}}, \) and \( Y_{\text{scl}} \) by 3.
2. Divide \( X_{\text{min}}, X_{\text{max}}, \) and \( X_{\text{scl}} \) by 4; divide \( Y_{\text{min}}, Y_{\text{max}}, \) and \( Y_{\text{scl}} \) by 3.
3. Divide the \( X \)–Window values by 6 and the \( Y \)–Window values by 2. The graph does not fit in the window.
Extend

4. First, multiply Xmin, Xmax, Ymin, and Ymax by 2. Then follow the steps in the answer to Exercise #3 above.

5. First, multiply Xmin, Xmax, Ymin, and Ymax by 2. Then divide Xmin, Xmax, and Xscl by 2 and divide Ymin, Ymax, and Yscl by 6. Finally, add 3/6 to Ymin and Ymax to re-center the screen vertically. The circle command will be ‘Circle(2/2, 3/6, 1).’

The Harmonic Series

Investigate

1. The graph rises (slowly) without bound.
2. \( y = e^x; \) all reals
3. \( 1, \frac{1}{2}, \frac{1}{3}, \ldots; 1, \frac{1}{2}, \frac{1}{3}, \ldots \)
4. In both figures, the area of the rectangles represents the terms of the harmonic sequence. The expression \( \ln(a) \) represents the area under the curve from \( x = 1 \) to \( x = a. \) Since the rectangles are all above the curve, the inequality \( 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{a} > \ln(a) \) must be true. In the second inequality, since the rectangles are entirely contained within the region below the curve, the inequality \( 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{a} < 1 + \ln(a), \) must be true.
5. between 9.21 and 10.21

Exercises

1. Check students’ work.
2. \( 5.187377518; \ln(100) = \frac{4.605172418}{\ln(100)} = 5.605 \)
3. \( 6.72982343; \ln(500) = \frac{6.215127066}{\ln(500)} = 7.215 \)
4. No; it’s between \( \ln(10^k) = 20.723 \) and \( 1 + \ln(10^k) = 21.723. \)
5. \( e^{100} = 2.688 \times 10^{13} \) terms

Monte Carlo π

Activity 36

Investigate

1. The probability that a randomly selected point lies within the shaded region is equal to the ratio of the area of the shaded region to the area of the square, that is, \( \frac{\pi}{4} = \frac{\pi}{4} \)
2. Check students’ work.
3. Because the coordinates, A and B, will always be less than or equal to 1.
4. Answers may vary. Sample: The ratio will most likely fall between 2 and 5, with most results clustered near 3.2.
5. Answers may vary. Sample: The ratio should consistently be very close to 3.14.

Exercises

1. Line 1: Clears any drawings
   Line 2: Turns off all plots
   Line 3: Turns off any functions located in the ‘Y=’ editor
   Line 4: Sets the viewing window, graphs square, graphs circle
   Line 5: Set the value of \( H \) equal to 0
   Line 6: Prompts the user to input the number of points for the simulation
   Line 7: Sets up a loop that will execute the next 4 lines \( N \) times; \( K \) is a counter
   Line 8: Stores random numbers between 0 and 1 as \( A \) and \( B \)
   Line 9: Plots the ordered pairs \( (A, B) \) on the graph
   Line 10: Calculates the distance from the origin to point \( (A, B) \) and tests if this distance is less than or equal to 1 (determines if the point is within the circle)
   Line 11: Adds 1 to the value of \( H \) if the point falls within the circle.
   Line 12: Specifies where the loop ends, which happens when \( K \) is equal to \( N \)
   Lines 13 and 14: Displays “4 * RATIO IS:” along with the value of \( 4H/N \) (which should be approximately equal to \( \pi \))
2. Check students’ work.

The Way the Ball Bounces

Activity 37

Investigate

1–2. Check students’ work.
3. A ball bouncing on a spring.
Answers (continued)

4. The ball moves quicker as it passes through the origin because its motion is mostly in the vertical direction (keeping in mind that that ball is really traveling along a unit circle). The ball moves slower at the top and bottom of its path because its motion is mostly in the horizontal direction.

5. Yes

Exercises

1. The ball moves twice as fast.
2. Change Tmax to 18π.
3. 3 cycles of a sine curve.
4. Nothing; the scale along the x-axis is so large that the graph is entirely contained along the y-axis and cannot be seen.
5. The wave has been compressed horizontally (because of the large x-axis scale), giving a result identical to step 3 of the investigation.
6. The vertical position of a point traveling along a unit circle at a constant speed can be used to model harmonic motion. If the vertical motion is graphed as a function of time, it generates a sine wave.

Graphing Ellipses and Activity 38

Hyperbolas

Investigate

1-2. Check students’ work.
3. \( \frac{x^2}{9} + \frac{y^2}{4} = \frac{(3\cos T)^2}{9} + \frac{(2\sin T)^2}{4} = \frac{9\cos^2 T}{9} + \frac{4\sin^2 T}{4} = \cos^2 T + \sin^2 T = 1; \) horizontal stretch factor = \( a \), vertical stretch factor = \( b \)
4. \( x^2 - y^2 = \frac{1}{\cos^2 T} - \frac{\sin^2 T}{\cos^2 T} = \frac{1 - \sin^2 T}{\cos^2 T} = \frac{\cos^2 T}{\cos^2 T} = 1; \) a hyperbola
5. Check students’ work.

Exercises

1. \( \tan T \)
2. 3; 2
3. \( X_{1T} = \frac{3}{\cos T}; Y_{1T} = \frac{2\sin T}{\cos T} = 2\tan T \)

Vertical Angles Activity 39

Investigate

1. They are congruent.
2. \( m\angle EAD + m\angle EAB = 180 \)
3. \( m\angle EAD + m\angle EBC = 180 \)
4. \( (\angle EAD + \angle EAB) - (\angle EAB + \angle BAC) = 0 \)
5. \( (\angle EAD + \angle EAB) - (\angle EAB + \angle BAC) = 0 \)
   simplifies to \( \angle EAD - \angle BAC = 0 \) which means \( \angle EAD = \angle BAC \)

Extend

6. Check students’ work.
7. They are perpendicular.
8. Answers may vary. Sample: Adjacent angles are supplementary. The angle bisectors of these angles create a pair of angles that are complementary.

Exterior Angles of Triangles Activity 40

Investigate

1. The measure of an exterior angle of a triangle is equal to the sum of the remote interior angles.
2. supplementary; \( \angle A + \angle B + \angle ACB = 180 \) and \( \angle ACB + \angle BCD = 180 \); rewrite both equations as \( \angle A + \angle B = 180 - \angle ACB \) and \( \angle BCD = 180 - \angle ACB \); by substitution \( \angle A + \angle B = \angle BCD \)

Extend

3. their sum is 90; they must be complementary since the angles of a triangle have a sum of 90 and \( \angle B = 90 \)
4. their sum is 270:
   \( \angle DAC + \angle ACE = (\angle BAC + \angle B) \) + 
   \( (\angle ACB + \angle B) \) = \( (\angle BAC + \angle ACB) + (\angle B + \angle B) = 90 + 180 = 270 \)

Investigate Further

5. \( \angle ADE - (\angle A + \angle B + \angle C) = 180 \);
   \( \angle ADE + 180 = \angle A + \angle B + \angle C \);
   \( \angle A + \angle B + \angle C + \angle ADC = 360 \) and \( \angle ADC + \angle ADE = 180 \);
   \( \angle A + \angle B + \angle C = 360 - \angle ADC \) and
   \( \angle ADE = 180 - \angle ADC \); by substitution
   \( \angle ADE + 180 = \angle A + \angle B + \angle C \)
6. the sum of the remote interior angles of a pentagon is equal to the sum of the exterior angle and 360.
7. Answers may vary. Sample:

<table>
<thead>
<tr>
<th>No. of Sides</th>
<th>Ext. ( \angle )</th>
<th>Sum of Remote Int. ( \angle )</th>
<th>Ext. ( \angle ) − Sum of Remote Int. ( \angle )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>80</td>
<td>80</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
<td>260</td>
<td>180</td>
</tr>
<tr>
<td>5</td>
<td>80</td>
<td>440</td>
<td>360</td>
</tr>
<tr>
<td>6</td>
<td>80</td>
<td>620</td>
<td>540</td>
</tr>
</tbody>
</table>

The sum of the remote interior angles of any convex polygon is equal to \( E + (n - 3)180 \) where \( E \) is the measure of the exterior angle and \( n \) is the number of sides of the polygon.

Relationships in Triangles Activity 41

Investigate

1. They are congruent.
2. The triangles are congruent by SAS.
3. \( BE \cong AD \) by CPCTC
Extend
4. They are congruent.
5. \( \triangle FAC \cong \triangle BAE \) by SAS so \( \overline{CF} \cong \overline{BE} \) by CPCTC

Investigate Further
6. they are equal to 120
7. Each angle of the triangle must be less than 120.
8. yes
9. It is equal to the sum of the lengths of the two shortest sides of the triangle.

Isosceles Triangles Activity 42
Investigate
1. isosceles
2. \( \angle DBC \) and \( \angle DCB \) are congruent therefore \( \triangle BCD \) remains isosceles
3. \( AD \) is the perpendicular bisector of \( BC \)

Investigate
4. The area of \( \triangle ABC \) is twice the area of \( ADFE \).
5. the height is half that of \( \triangle ABC \).
6. \( DE = \frac{1}{2} BC \).
7. Each of the smaller triangles, \( \triangle ADE \) and \( \triangle DEF \), have base and height that are half those of \( \triangle ABC \). Therefore, the areas of each smaller triangle is \( \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \triangle ABC \). Together, the areas of \( \triangle ADE \) and \( \triangle DEF \) must be half the area of \( \triangle ABC \).
8. They have the same area.
9. The slopes of \( \triangle DEF \) are equal to the corresponding slopes of \( \triangle ABC \). The measures of the sides of \( \triangle DEF \) are one-half the measure of the corresponding sides of \( \triangle ABC \). This is a demonstration of the midsegment theorem.
10. yes

Orthocenters Activity 43
Investigate
1. a. when the triangle is acute;
   b. when the triangle is right;
   c. when the triangle is obtuse
2. the orthocenter of \( \triangle ABD \) is \( C \); the orthocenter of \( \triangle BCD \) is \( A \); the orthocenter of \( \triangle ACD \) is \( B \)

Investigate
3. The circumferences and areas of all four circles are equal.

Quadrilaterals Activity 44
Investigate
1. parallelogram
2. They are equal; They are parallel to \( BD \).
3. One pair of opposite sides of \( EHG \) are congruent and parallel.

Regular Polygons Activity 47
Investigate
1. rhombus
2. 108
3. isosceles
4. 36

Using Different Menus to Construct Special Quadrilaterals Activity 45
Investigate
1–2. kite; two pairs of congruent, adjacent sides

Investigate Further
3. rectangle
4. Check students’ work.
5. rhombus
6. rhombus

Extend
rectangle; congruent diagonals bisect each other

Parallelograms and Triangles Activity 46
Investigate
1. Check students’ work.
2. \( \angle BAD \) is twice \( \angle AFD \); \( \angle ABC \) is twice \( \angle BEC \)
3. \( \angle BAD = \angle AFD + \angle ADF \) by the Exterior Angle Theorem; \( \angle AFD \cong \angle ADF \) by the 3.
   Isosceles Triangle Theorem; therefore, by substitution, \( \angle BAD = 2\angle AFD \) or \( \frac{1}{2} \angle BAD = \angle AFD \); same reasoning used to show \( \frac{1}{2} \angle ABC = \angle BEC \)
4. \( \angle BEC + \angle AFD = 90 \), therefore \( \angle CHD = 90 \)
5. \( \angle HCD \cong \angle BEC \) and \( \angle CDH \cong \angle AFD \)
6. Corresponding Angles Postulate
5. \(m \angle GDE = m \angle GFE = 72; m \angle E = 108\)
6. 108, since the measures of a quadrilateral have a sum of 360
7. \(DE \equiv EF\)
8. Since opposite angles are congruent, \(DGFE\) is a parallelogram. Because a pair of adjacent sides is congruent, \(DGFE\) is a rhombus.

Extend
9. rhombus
10. Check students’ work.
11. Check students’ work.
12. Draw the first diagonal such that it is parallel to one of the sides of the regular polygon. The second diagonal will join vertices that are adjacent to the first diagonal such that the diagonals intersect.

Area Activity 48
Investigate
1. both ratios are 2:1
2. the area of \(ABCD\) is \(3x \cdot 3x = 9x^2\); the area of \(\triangle ABE = \text{area } \triangle ADF = \frac{1}{2} \cdot 2x \cdot 3x = 3x^2\); the area of \(\triangle AECF = 9x^2 - 3x^2 = 3x^2\); the areas are equal
3. Check students’ work.
4. \(E\) and \(G\) are midpoints of \(BC\) and \(CD\), respectively; \(F\) coincides with \(C\)
5. Using \(x\) to represent the length of one side of \(ABCD\), each triangle has measure \(\frac{1}{2} \cdot x \cdot 2x = x^2\). The areas are equal.
6. The sides to which the segments are drawn are divided into segments in a ratio of 2:2:1.
7. The sides to which the segments are drawn are divided into segments in a ratio of 1:1:1.
8. The sides to which the segments are drawn are divided into equal segments.
9. The sides to which the segments are drawn are divided into segments in a ratio of 2:2:...:2:1.

Pythagorean Triples Activity 49
Investigate
1. Plato’s method uses the formulas

<table>
<thead>
<tr>
<th>(n)</th>
<th>(\frac{n^2}{4} - 1)</th>
<th>(\frac{n^2}{4} + 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>(= A2 \cdot A2/4 - 1)</td>
<td>(= A2 \cdot A2/4 + 1)</td>
</tr>
<tr>
<td>6</td>
<td>(= A2 + 2)</td>
<td>(= A3 \cdot A3/4 - 1)</td>
</tr>
<tr>
<td>6</td>
<td>(= A3 + 2)</td>
<td>(= A4 \cdot A4/4 - 1)</td>
</tr>
</tbody>
</table>

which produces the following triples

<table>
<thead>
<tr>
<th>(n)</th>
<th>(\frac{n^2}{4} - 1)</th>
<th>(\frac{n^2}{4} + 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

2. Pythagoras’ method uses the formulas


which produces the following triples

<table>
<thead>
<tr>
<th>(n)</th>
<th>(\frac{n^2}{4} - 1)</th>
<th>(\frac{n^2}{4} + 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>(= (A2 \cdot A2 - 1)/2)</td>
<td>(= A2)</td>
</tr>
<tr>
<td>5</td>
<td>(= A2 + 2)</td>
<td>(= (A3 \cdot A3 - 1)/2)</td>
</tr>
<tr>
<td>7</td>
<td>(= A3 + 2)</td>
<td>(= (A4 \cdot A4 - 1)/2)</td>
</tr>
</tbody>
</table>

3. The formulas


produce the following triples

<table>
<thead>
<tr>
<th>(n)</th>
<th>(2n + 1)</th>
<th>(2n^2 + 2n)</th>
<th>(2n^2 + 2n + 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(= 2 \cdot A2 + 1)</td>
<td>(= 2 \cdot A2 \cdot A2 + 2 \cdot A2)</td>
<td>(= 2 \cdot A2 \cdot A2 + 2 \cdot A2 + 2)</td>
</tr>
<tr>
<td>3</td>
<td>(= 2 \cdot A2 + 1)</td>
<td>(= 2 \cdot A3 + 2 \cdot A3 + A3)</td>
<td>(= 2 \cdot A3 + 2 \cdot A3 + 2 \cdot A3 + A3)</td>
</tr>
<tr>
<td>5</td>
<td>(= 2 \cdot A4 + 1)</td>
<td>(= 2 \cdot A4 + 4 \cdot A4 + 2 \cdot A4)</td>
<td>(= 2 \cdot A4 + 4 \cdot A4 + 2 \cdot A4 + A4)</td>
</tr>
</tbody>
</table>

4. Plato’s method and Pythagoras’ method generate triples from the same ‘family’ of integers, but not necessarily the exact same numbers. For example, the Pythagoras method produces the triple 5, 12, 13 whereas Plato’s method produces 10, 24, 26, which is a multiple of 5, 12, 13. The unattributed method produces the same triples as the Pythagoras method.

Similarity Activity 50
Investigate
1. parallelogram
2. The sides of \(PQRS\) are midsegments, and therefore parallel, to the sides of \(ABCD\)
3. corresponding angles are congruent; corresponding sides are in proportion
4. 2:1; 4:1
5. yes

Extend
6. corresponding sides are in a ratio of 2:1; corresponding angles are congruent
7. the ratio of the areas is 4:1
8. Check student’s work.

Investigate Further
9. \(ILMJ \sim KOPN\); \(KIJN \sim OLMP\)
10. Answers may vary. Sample: \(IL : KO\) (for \(ILMJ \sim KOPN\)); \(KI : OL\) (for \(KIJN \sim OLMP\))
11. Check students’ work.
Web-Based Activities

Using the Web to Study Mathematics
Check students’ work.

Mining Data Found on the Internet
Check students’ work.

Step 1: Winning Times for the Olympics Women’s 4 × 100–meter Relay

<table>
<thead>
<tr>
<th>Year</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1952</td>
<td>45.9</td>
</tr>
<tr>
<td>1956</td>
<td>44.5</td>
</tr>
<tr>
<td>1960</td>
<td>44.5</td>
</tr>
<tr>
<td>1964</td>
<td>43.6</td>
</tr>
<tr>
<td>1968</td>
<td>42.8</td>
</tr>
<tr>
<td>1972</td>
<td>42.81</td>
</tr>
<tr>
<td>1976</td>
<td>42.55</td>
</tr>
<tr>
<td>1980</td>
<td>41.6</td>
</tr>
<tr>
<td>1984</td>
<td>41.65</td>
</tr>
<tr>
<td>1988</td>
<td>41.98</td>
</tr>
<tr>
<td>1992</td>
<td>42.11</td>
</tr>
</tbody>
</table>

Step 2 and 3: There is a moderate linear relationship.

Step 4: Actual 1996 Winning time: 41.95; Line of best fit prediction = 40.80927273; The data appears to be leveling off from 1980–1992. This would seem reasonable since winning times cannot continue to decrease at the same rate.

Algebra 1

Answers (continued)

Volume Activity 51

Investigate
1. $12 - 4x$
2. $V(x) = x \cdot x \cdot (12 - 4x) = 12x^2 - 4x^3$
3. 2
4. 16
5. $S(x) = 2 \cdot x \cdot x + 4 \cdot x \cdot (12 - 4x) = -14x^2 + 48x$
6. 1.714
7. no
8. (2.606, 30); maximum volume 10.695

Extend
9. $V(x) = \pi x^2 (12 - 2\pi x)$ where $x$ is the radius of the container
10. $r = 1.273$ and $h = 4; 20.372$
11. $S(x) = 2\pi x^2 + 2\pi x(12 - 2\pi x)$
12. $r = 1.136$ and $h = 4.864$
13. The shape of a package, in addition to its weight, is an important consideration—Can the package be stacked on other packages? Can it fit into certain spaces on a truck or plane? Is it difficult to carry? etc.

Rotations Activity 52

Investigate
1. each angle = 120; hexagon
2. each angle 90; square
3. 100 is not a factor of 360
4. Any number that is a factor of 360 can used with a whole number amount of rotations to produce a regular polygon.

Extend
5. It is supplementary.
6. Yes
7. The angles that have a vertex at $A$ are equal to the size of rotation. Any other pair of angles formed will be supplementary to the angle of rotation.
8. A square can be used to form a tessellation.
9. Their angles must be factors of 360.
10. The sum of the angles around any point equals 360.
11. Regular pentagons and rhombuses can be used to form a semi-pure tessellation.

Web-Based Activities

Answers Technology Activities
Answers (continued)

Generating Quizzes on the Web
Check students’ work.

Graphing the Solar System

Investigate
1. \(9.538\)
2. \(0.5341\)
3. \(b = 9.523; \frac{x^2}{9.538^2} + \frac{y^2}{9.523^2} = 1\)
4. \(\frac{(x + 0.5341)^2}{9.538^2} + \frac{y^2}{9.523^2} = 1\)

6. An eccentricity close to zero, such as 0.056, gives the appearance of a circle. Notice that the major axis, \(2 \cdot 9.538 = 19.076\), is slightly larger than the minor axis, \(2 \cdot 9.523 = 19.046\).

Exercises
1. The average distances to the Sun for the terrestrial planets are much closer to the Sun than the Jovian planets. A scale drawing of all nine planets would make it difficult to see the terrestrial planets.
2. Answers may vary: Pluto’s average distance to the Sun is about 100 times greater than Mercury’s average distance to the Sun. A sketch would need to be about 25 feet wide in order to reasonably see the orbits of all nine planets.
3. \(\frac{x^2}{39.785^2} + \frac{y^2}{38.542^2} = 1\)
4. Pluto’s orbit is much more elliptical \((e = 0.248)\) than Neptune’s (which is nearly circular with \(e = 0.009\)). Because of this, there are times when Pluto’s orbit brings the planet closer to the sun than Neptune’s.
5. The earth is tilted more towards the Sun during the summer months, thereby creating a warmer climate.
6. There would be extreme temperature changes if the Earth’s orbit was more elliptical.

Building a Web Page About Buildings
Check students’ work.

Creating Dynamic Images for the Web
Check students’ work.